WS-9 - oblique Projectiles Advanced ①

Task

8/4

①

let 'θ' be the angle of Projection

'u' be the velocity of Projection

$H_{max} = \frac{u^2 \sin^2\theta}{2g} \Rightarrow gH_{max} = \frac{u^2 \sin^2\theta}{2}$

Now $V_H^2 = u^2\cos^2\theta$. & Given $V_H = \sqrt{\frac{2}{5}}\, V_{\frac{H}{2}}$

From $v^2 - u^2 = 2as$ $= \sqrt{\frac{2}{5}}$ ucos.

$\Rightarrow V_{\frac{H}{2}}^2 - u^2 = -\cancel{2}g\frac{H}{\cancel{2}}$ [$a=-g$; $S=\frac{H}{2}$]

$\Rightarrow V_{\frac{H}{2}}^2 = u^2 - gH$ [$\because V_{\frac{H}{2}} = \sqrt{\frac{5}{2}}\, V_H$]

$\Rightarrow \frac{5}{2}V_H^2 = u^2 - g\frac{u^2\sin^2\theta}{2g}$

$\Rightarrow \frac{5}{2}u^2\cos^2\theta = u^2 - \frac{u^2\sin^2\theta}{2}$

$\Rightarrow \frac{5}{2}\cancel{u^2}[1-\sin^2\theta] = \cancel{u^2}\{1-\frac{\sin^2\theta}{2}]$

$\Rightarrow \frac{5}{2} - \frac{5}{2}\sin^2\theta = 1 - \frac{\sin^2\theta}{2}$

$\Rightarrow \frac{\sin^2\theta}{2} - \frac{5}{2}\sin^2\theta = 1 - \frac{5}{2}$

$\Rightarrow \cancel{-}4\frac{\sin^2\theta}{\cancel{2}} = \frac{\cancel{-}3}{\cancel{2}}$

$\Rightarrow \sin^2\theta = \frac{3}{4} \Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 60^\circ$

muzzle velocity $V_1 = 2000$ m/s

Angle of Projection $\theta = 45°$

let V_2 be the vertical velocity of Projection for 2nd body.

Given that

Maximum height reached by 2nd body = Maximum height reached by Projectile

$$\Rightarrow \frac{u_2^2}{\cancel{2g}} = \frac{V_1^2 \sin^2\theta}{\cancel{2g}}$$

$$\Rightarrow u_2^2 = V_1^2 \sin^2\theta$$

$$\Rightarrow V_2 = V_1 \sin\theta$$

$$\Rightarrow V_2 = 2000 \times \sin 45° = 2000 \times \frac{1}{\sqrt{2}} = 1000\sqrt{2} = 1414 \text{ m/s}$$

3)

Given Parabolic equation $y = x - \frac{1}{40}x^2$ m

Compare with

$$y = \tan\theta\, x - \frac{g}{2u^2\cos^2\theta}x^2$$

$\tan\theta = 1$; $\frac{g}{2u^2\cos^2\theta} = \frac{1}{40}$

$\Rightarrow \theta = 45°$

$$\Rightarrow \frac{10}{2u^2\cos^2 45} = \frac{1}{40} \Rightarrow 40 \times 10 = 2u^2 \times \left(\frac{1}{\sqrt{2}}\right)^2$$

$$\Rightarrow 400 = u^2 \times \cancel{2} \times \frac{1}{\cancel{2}}$$

$$\Rightarrow u = \sqrt{400} = 20 \text{ m/s}$$

$\therefore$

$$H_{max} = \frac{u^2 \sin^2\theta}{2g} = \frac{(20)^2 \times \sin^2 45}{2 \times 10}$$

$$= 10 \text{ m.}$$

(4)

Angle of Projection $\theta = 45°$

Time taken to reach highest point = time of ascent

$$t_a = 2\sqrt{2} \text{ sec}$$

$$\Rightarrow \frac{\cancel{2}u\sin\theta}{g} = \cancel{2}\sqrt{2}$$

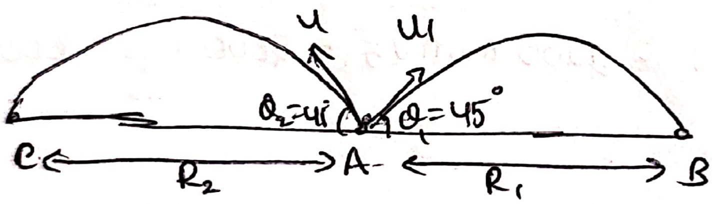
$$\Rightarrow u\sin\theta = 2\sqrt{2}\, g.$$

$\therefore$ Maximum height reached $H_{max} = \dfrac{u^2\sin^2\theta}{2g}$

$$= \frac{(u\sin\theta)^2}{2g}$$

$$\Rightarrow \frac{(2\sqrt{2}\,g)^2}{2g} = \frac{\cancel{2}g^{\cancel{2}}\times 4}{\cancel{2g}} = 4g = 40m$$

(5)



let 'A' be the point of Projection; For maximum range $\theta = 45°$. Given both are Projecting with same velocity (i.e) $u_1 = u_2 = g$.

The seperation between them = $R_1 + R_2$

$$\Rightarrow \frac{u_1^2}{2g} + \frac{u_2^2}{2g} = \frac{g^2}{2g} + \frac{g^2}{2g}$$

$$= \frac{2g^2}{g}$$

$$= 2g.$$

⑧

⑥

Given angle of Projection $\theta_1 = 15°$

Range $R_1 = 1.5$ km.

For angle of projection $\theta_2 = 45°$

Range $R_2 = ?$

we know that Range $= \frac{u^2 \sin 2\theta}{g}$

$\Rightarrow$ Range $\propto \sin 2\theta$

$$\Rightarrow \frac{R_1}{R_2} = \frac{\sin 2\theta_1}{\sin 2\theta_2}$$

$$\Rightarrow \frac{1.5}{R_2} = \frac{\sin 2(15)}{\sin 2(45)} \Rightarrow \frac{1.5}{R_2} = \frac{\sin 30}{\sin 90}$$

$$\Rightarrow \frac{1.5}{R_2} = \frac{\frac{1}{2}}{1} \Rightarrow \frac{1.5}{R_2} = \frac{1}{2} \Rightarrow R_2 = 3\text{km}$$

⑦

Let initial velocity of Projectile $u = 30$ m/s

Angle of Projection $\theta = 30°$,

Angle made by the Projectile to the horizontal after 1.5 sec is $\tan\alpha = \frac{V_y}{V_x} = \frac{u\sin\theta - gt}{u\cos\theta}$

$$\Rightarrow \tan\alpha = \frac{30 \times \sin 30 - 10 \times 1.5}{30 \times \cos 30} = \frac{30 \times \frac{1}{2} - 15}{30 \times \frac{1}{2}}$$

$$\Rightarrow \tan\alpha = \frac{15 - 15}{15} = 0$$

$\Rightarrow \alpha = 0°$

(8)

Mass of A $M_A = M$ = Mass of B (M_B)

Angle of Projection for A $\theta_A = 30^\circ$

for B $\theta_B = 45^\circ$

velocity of Projection for A = velocity of Projection for B

$\therefore$ From horizontal range $= R = \frac{u^2 \sin 2\theta}{g}$

$$\therefore \frac{R_A}{R_B} = \frac{\sin 2\theta_A}{\sin 2\theta_B} = \frac{\sin 2(30^\circ)}{\sin 2(45^\circ)}$$

$$\Rightarrow \frac{R_A}{R_B} = \frac{\sin 60^\circ}{\sin 90^\circ} = \frac{\sqrt{3}}{2}$$

(9)

Speed of the Projectile at greatest height

$u\cos\theta = 10$ m/s.

It is staying in air for 5 sec means it is time of flight $T = \frac{2u\sin\theta}{g} = 5 \Rightarrow \cancel{\frac{u\sin\theta}{g}} =$

$$\therefore \text{Range} = R = \frac{u^2 \sin 2\theta}{g}$$

$$= u^2 \frac{2\sin\theta\cos\theta}{g}$$

$$\Rightarrow \frac{2u\sin\theta}{g} \times u\cos\theta$$

$$\Rightarrow 5 \times 10$$

$$\Rightarrow 50\text{m}$$

(10) (5)

Given Angle of Projection $\theta = 30°$

After 2 sec the projectile is reaching the ground

$T = 2$ sec.

$\therefore$ change in velocity $= |du| = |v - u|$

$\Rightarrow |(u\cos\theta\,\hat{i} + (u\sin\theta - gT)\hat{j}) - (u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j})|$

$\Rightarrow |\cancel{u\cos\theta\,\hat{i}} + (u\sin\theta - gT)\hat{j} - \cancel{u\cos\theta\,\hat{i}} - u\sin\theta\,\hat{j}|$

$= |\cancel{u\sin\theta\,\hat{j}} - gT\hat{j} - \cancel{u\sin\theta\,\hat{j}}|$

$= |-gT\,\hat{j}| = gT = 10 \times 2 = 20$ (or)

$9.8 \times 2 = 19.6$ m/s

(13)

$135°$ & $45°$ are complimentary angles.

For complimentary angles Ranges are same.

(14) From diagram it is clear that

$H_{max_1} = H_{max_2}$ and $\theta_1 > \theta_2$

$\Rightarrow \dfrac{u_1^2 \sin^2\theta_1}{2g} = \dfrac{u_2^2 \sin^2\theta_2}{2g} \Rightarrow u_1 \sin\theta_1 = u_2 \sin\theta_2 \rightarrow (1)$

we know that Time of flight $T = \dfrac{2u\sin\theta}{g}$ i.e)

$T \propto u\sin\theta \rightarrow (2)$

From (1) and (2) we can conclude that T is same for both Particles

From (1) $u_1 = u_2 \dfrac{\sin\theta_2}{\sin\theta_1}$ $\left[\because \theta_1 > \theta_2\right] \; \dfrac{\sin\theta_2}{\sin\theta_1} < 1$

$\therefore u_2 < u_2$.

(15)

From $T = \frac{2u\sin\theta}{g}$ and $H = \frac{u^2\sin^2\theta}{2g}$

As θ increases from $30 \to 60^\circ$ T will also increases because $\sin\theta$ is an increasing function

so $\sin^2\theta$ also increases $\Rightarrow$ H also increases.

But from $R = \frac{u^2 \sin 2\theta}{g}$

As θ changes from $30^\circ \to 60^\circ$ then 2θ changes from $60^\circ \to 120^\circ$

$\sin 2\theta$ will increase from $60^\circ \to 90^\circ$ but decreases from 90° to 120°

$\therefore$ R will increase first and then decreases.

So option 'B' is correct.

(16)

We know that $H = \frac{R}{4}\tan\theta$

(a) For $R = H \Rightarrow H = \frac{H}{4}\tan\theta \Rightarrow 4 = \tan\theta \Rightarrow \theta = \tan^{-1}(4)$

(b) For $R = 2H \Rightarrow \cancel{H} = \frac{2\cancel{H}}{4}\tan\theta \Rightarrow 4 = 2\tan\theta$

$\Rightarrow 2 = \tan\theta$

$\Rightarrow \theta = \tan^{-1}(2)$

(c) For $R = 3H \Rightarrow \cancel{H} = \frac{3\cancel{H}}{4}\tan\theta \Rightarrow 4 = 3\tan\theta \Rightarrow \tan\theta = \frac{4}{3}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{4}{3}\right)$

(d) For $R = 4H \Rightarrow \cancel{H} = \frac{4\cancel{H}}{4}\tan\theta \Rightarrow \tan\theta = 1$

(17), (18), (19)

velocity of projection $u = 60$ m/s

Angle of projection $\theta = 30^\circ$

Initial velocity vector $\vec{u} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

$= 60\cos 30\,\hat{i} + 60\sin 30\,\hat{j}$

$= 60 \times \frac{\sqrt{3}}{2}\,\hat{i} + 60 \times \frac{1}{2}\,\hat{j}$

$\vec{u} \Rightarrow 30\sqrt{3}\,\hat{i} + 30\hat{j}$

velocity after '3' sec $\vec{v} = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\hat{j}$

$= 60\cos 30\,\hat{i} + (60\sin 30 - 10 \times 3)\hat{j}$

$= 60\frac{\sqrt{3}}{2}\,\hat{i} + (60 \times \frac{1}{2} - 30)\hat{j}$

$\Rightarrow 30\sqrt{3}\,\hat{i} + (30 - 30)\hat{j}$

$= 30\sqrt{3}\,\hat{i}$

Displacement after '2' sec $= x\hat{i} + y\hat{j}$

$= (u\cos\theta t)\hat{i} + (u\sin\theta t - \frac{1}{2}gt^2)\hat{j}$

$= 60 \times \cos 30 \times 2\,\hat{i} + (60\sin 30 \times 2 - \frac{1}{2} \times 10 \times 2^2)\hat{j}$

$\Rightarrow 60 \times \frac{\sqrt{3}}{\cancel{2}} \times \cancel{2}\,\hat{i} + (\overset{30 \times 2}{\cancel{60 \times \frac{1}{2}}} - 20)\hat{j})$

$\Rightarrow 60\sqrt{3}\,\hat{i} + 40\hat{j}$.

(20)

Given $H_{max} = 4m$, $R = 12m$

we know that $H_{max} = \frac{u^2 \sin^2\theta}{2g}$

$\Rightarrow 4 = \frac{(u \sin\theta)^2}{2g} \Rightarrow 8g = u_y^2 \quad [u_y = u\sin\theta]$

Range = 12

$\Rightarrow u^2 \frac{\sin 2\theta}{g} = 12 \Rightarrow \cancel{2} u\cos\theta \frac{u\sin\theta}{g} = \cancel{12}6$

$\Rightarrow u_x \frac{u_y}{g} = 6$

$\Rightarrow u_x = \frac{6g}{u_y} = \frac{6g}{\sqrt{8g}}$

$\therefore$ velocity of projection $u = \sqrt{u_x^2 + u_y^2}$

$= \sqrt{\frac{36 g^{\cancel{2}}}{8\cancel{g}} + 8g} = \sqrt{\frac{36g + 64g}{8}}$

$= \sqrt{\frac{100g}{8}} = \frac{\cancel{10}^5 \sqrt{g}}{\cancel{2}\sqrt{2}} = 5\sqrt{\frac{10}{2}} = 5\sqrt{5}$

where $g = 10\ m/s^2$

(21)

Given $x = 36t$; $y = 48t - 4.9t^2$

compare with $x = u\cos\theta\, t$; $y = u\sin\theta\, t - \frac{g}{2} t^2$

$\therefore u\cos\theta = 36$

$\therefore u\sin\theta = 48$

$\therefore$ velocity of projection $= \sqrt{(u\cos\theta)^2 + (u\sin\theta)^2}$

$= \sqrt{(36)^2 + (48)^2}$

$= 60\ m/s$

L Task

CUQ's

(7)

(2)

As the angle of projection increases from $0 \to 90°$

since $R = \frac{u^2 \sin 2\theta}{g}$

$\therefore$ $\sin 2\theta$ value changes from $0° \to 180°$

$\therefore$ $\sin 2\theta$ value increases from $0° \to 90°$ and then decreases from $90° \to 180°$

(3)

As the angle of Projection increases from $0 \to 90°$

$\therefore$ $\sin\theta$ value changes from $0° \to 90°$ (increases)

we know that $H_{max} = \frac{u^2 \sin^2\theta}{2g}$

$\Rightarrow H_{max} \propto \sin^2\theta$

$\therefore$ H_{max} value increases from $0 \to 90°$

(4)

At maximum height the velocity of Projectile is minimum and is equal to $u\cos\theta$.

(5)

$H_{max} = \frac{u^2 \sin^2\theta}{2g}$ $\therefore H_{max} \propto \sin^2\theta$

Given $H_1 > H_2 \Rightarrow \sin^2\theta_1 > \sin^2\theta_2 \Rightarrow \theta_1 > \theta_2$

$T = \frac{2u\sin\theta}{g} \Rightarrow T \propto \sin\theta \Rightarrow T_1 > T_2$ [$\sin\theta_1 > \sin\theta_2$]

SAQ's

①

~~Velelocal~~ Velocity of Projection $u = 147$ m/s

Angle of Projection $\theta = 60°$

After 't' sec the angle made by the projectile with horizontal is,

$$\tan\alpha = \frac{V_y}{V_x} \quad ; \text{Given } \alpha = 45°$$

$$\Rightarrow \tan 45° = \frac{u\sin\theta - gt}{u\cos\theta}$$

$$\Rightarrow 1 = \frac{u\sin\theta - gt}{u\cos\theta} \Rightarrow u\sin\theta - gt = u\cos\theta$$

$$\Rightarrow t = \frac{u\sin\theta - u\cos\theta}{g} = \frac{147\{\sin 60 - \cos 60\}}{9.8}$$

$$\Rightarrow t = \frac{147\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)}{9.8} = \frac{73.5(\sqrt{3}-1)}{9.8}$$

$$\Rightarrow t = \frac{73.5}{9.8} \times 0.732 = 5.5 \text{ sec}$$

④

velocity of Projection $u = 20$ m/s

$H_{max} = 5$ m.

$$\therefore H_{max} = \frac{u^2\sin^2\theta}{2g}$$

$$\Rightarrow 5 = \frac{20^2 \sin^2\theta}{2 \times 10}$$

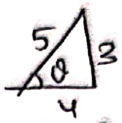
$$\Rightarrow 5 = \frac{\cancel{400}\,20}{\cancel{20}}\sin^2\theta \Rightarrow \sin^2\theta = \frac{5}{20} = \frac{1}{4}$$

$$\Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$$

(2) (8)

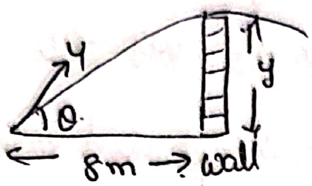
Velocity of Projection $u = 10\,m/s$

$\tan\theta = \frac{3}{4}$,



$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$



∴ The vertical distance travelled by the body

$$y = \tan\theta\, x - \frac{g}{2u^2\cos^2\theta} x^2$$

$$\Rightarrow y = \frac{3}{\cancel{4}} \times \cancel{8}^{2} - \frac{10}{2\times 10^{\cancel{2}} \times \left(\frac{4}{5}\right)^2} 8^2$$

$$= 6 - \frac{64}{4\,\cancel{20}\times\frac{16}{\cancel{25}}} = 6 - \frac{\cancel{64}\times 5}{\cancel{4}\times\cancel{16}}$$

$$= 6 - 5 = 1\,m$$

(3)

Angle of Projection $\theta = 30^\circ$

Time of flight $T = 2$ sec

$$\Rightarrow \frac{2u\sin\theta}{g} = 2$$

$$\Rightarrow \frac{2\times u\times \sin 30}{10} = 2$$

$$\Rightarrow \cancel{2}\times u\times\frac{1}{\cancel{2}} = 2\times 10$$

$\Rightarrow u = 20\,m/s$ (or) $19.6\,m/s$ [if $g = 9.8\,m/s^2$]

(5)

From given question Range = 80 m

$$\Rightarrow \frac{2u_x u_y}{g} = 80m$$

$$\Rightarrow u_x u_y = 40g \rightarrow (1)$$

Given that after '1' sec vertical distance $y = 15m$

$\therefore$ From $y = u_y t - \frac{1}{2}gt^2$

$$\Rightarrow 15 = u_y \times 1 - \frac{1}{2} \times \overset{5}{10} \times 1^2$$

$$\Rightarrow 15 = u_y - 5$$

$$\Rightarrow u_y = 20m \rightarrow (2)$$

From (1) $u_x \times 20 = 400 \Rightarrow u_y = 20$ m.

$\therefore$ we know that $\tan\theta = \frac{u_y}{u_x} = \frac{20}{20} = 1$

$$\Rightarrow \theta = 45°$$

(7)

Given that Range = 2 Hmax. $\Rightarrow R = 2H$

From $H = \frac{R}{4}\tan\theta \Rightarrow H = \frac{2H}{4_2}\tan\theta$

$$\Rightarrow \tan\theta = 2$$

$$\Rightarrow \frac{u_y}{u_x} = 2$$

$$\Rightarrow u_y = 2u_x \rightarrow (1)$$

velocity of Projection $\vec{u} = a\hat{i} + b\hat{j}$ m/s

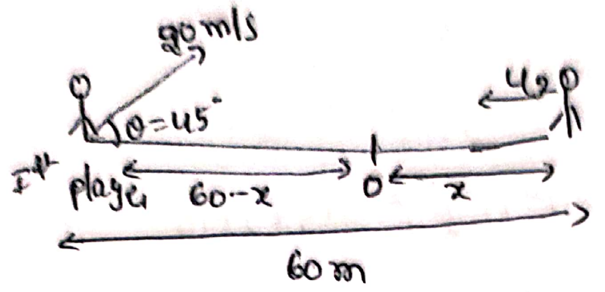
compare with $\vec{u} = u_x\hat{i} + u_y\hat{j} \Rightarrow u_x = a ; u_y = b$

From (1) $\boxed{b = 2a}$

(6)

Angle of Projection $\theta = 45^\circ$

velocity of Projection $u = 20$ m/s



From the given data time taken by the two players is same

Time of flight $T_1 = \frac{2u\sin\theta}{g} = \frac{2\times 20\times \sin 45}{10}$

$= 2\times 2\times \frac{1}{\sqrt{2}} = 2\sqrt{2}$ sec.

From $s = ut + \frac{1}{2}at^2$

Ist player	For 2nd player
$u=20$; $a=-g$	$s=x$, $a=g$
$60-x = \underset{20\sin 45}{20}t - \frac{1}{2}gt^2$	$x = u_2 t + \frac{1}{2}gt^2$

$\therefore$ substituting 'x' value

$60 - (u_2 t + \frac{1}{2}gt^2) = \frac{20t}{\sqrt{2}} - \frac{1}{2}gt^2$

$\Rightarrow 60 - u_2 t - \cancel{\frac{1}{2}gt^2} = \frac{20t}{\sqrt{2}} - \cancel{\frac{1}{2}gt^2}$

$\Rightarrow 60 = \frac{20t}{\sqrt{2}} + u_2 t$

$\Rightarrow 60 = \frac{20}{\cancel{\sqrt{2}}}\times 2\cancel{\sqrt{2}} + u_2(2\sqrt{2})$

$\Rightarrow 60 = 20\times 2 + u_2 2\sqrt{2}$

$\Rightarrow u_2 \times 2\sqrt{2} = 60-40$

$\Rightarrow u_2 \times \cancel{2}\sqrt{2} = \cancel{20}\,10$

$u_2 = \frac{10}{\sqrt{2}} = 5\sqrt{2}$ m/s

(8)

Angle of Projection $\theta = 30°$

$$H_{max} = 15\,m$$

$$\Rightarrow \frac{u^2 \sin^2\theta}{2g} = 15$$

$$\Rightarrow \frac{u^2 \sin^2 30}{2\times10} = 15$$

$$\Rightarrow u^2 \times (\tfrac{1}{2})^2 = 15\times2\times10$$

$$\Rightarrow u^2 \times \tfrac{1}{4} = 300 \Rightarrow u^2 = 1200\,m$$

If the body projected vertically upwards

Maximum height $= H = \frac{u^2}{2g} = \frac{1200}{2\times10} = 60\,m$

(9)

Given

Range (R) = maximum height (H)

we know that $H = \frac{R}{4}\tan\theta$

$$\Rightarrow \cancel{H} = \frac{\cancel{H}}{4}\tan\theta$$

$$\Rightarrow 1 = \tfrac{1}{4}\tan\theta \Rightarrow \tan\theta = 4 \Rightarrow \theta = \tan^{-1}(4)$$

(10)

we know that $H_{max} = \frac{u^2\sin^2\theta}{2g} = \frac{(u\sin\theta)^2}{2g} = \frac{u_y^2}{2g}$

Time of flight $T = \frac{2u\sin\theta}{g} \Rightarrow \frac{2}{g}u_y$

$$\frac{H_{max}}{T^2} = \frac{\frac{u_y^2}{2g}}{(\frac{2}{g}u_y)^2} = \frac{\frac{\cancel{u_y^2}}{2g}}{\frac{4}{g^{\cancel{2}}}\times \cancel{u_y^2}} = \frac{\frac{1}{2}}{\frac{4}{g}} = \frac{g}{8} = \frac{10}{8}$$

$$= \frac{5}{4}.$$

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(17), (18)
(19) Given Angle of Projection $\theta_1 = 30°$ & $\theta_2 = 60°$

we know that Time of flight $T = \frac{2u\sin\theta}{g}$

$T \propto \sin\theta$

$$\Rightarrow \frac{T_1}{T_2} = \frac{\sin\theta_1}{\sin\theta_2} = \frac{\sin 30}{\sin 60}$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

$$H_{max} = \frac{u^2\sin^2\theta}{2g} \Rightarrow H_{max} \propto \sin^2\theta$$

$$\Rightarrow \frac{H_1}{H_2} = \frac{\sin^2\theta_1}{\sin^2\theta_2} = \left(\frac{\sin\theta_1}{\sin\theta_2}\right)^2$$

$$\Rightarrow \frac{H_1}{H_2} = \left[\frac{T_1}{T_2}\right]^2 = \left[\frac{1}{\sqrt{3}}\right]^2 = \frac{1}{3}$$

$$\text{Range} = \frac{u^2\sin 2\theta}{g} \Rightarrow \text{Range} \propto \sin 2\theta$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{\sin 2\theta_1}{\sin 2\theta_2} = \frac{\sin 2[30]}{\sin 2[60]} = \frac{\sin 60}{\sin 120}$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}} \Rightarrow \frac{1}{1}$$

(21) Given $R = 4\sqrt{3}\,H$

we know that $H = \frac{R}{4}\tan\theta$

$$\Rightarrow \cancel{H} = \frac{\cancel{4}\sqrt{3}\cancel{H}}{\cancel{4}}\tan\theta$$

$$\Rightarrow 1 = \sqrt{3}\tan\theta \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30°$$

(8)

Given Ranges of two stones are same.

$R_1 = R_2$

$\theta_1 = \frac{\pi}{3}$, $\theta_2 = \frac{\pi}{6}$ [Because $R_1 = R_2$ this is possible for complimentary angles]

$H_1 = 102$ m

$\Rightarrow \frac{u^2 \sin^2\theta_1}{2g} = 102$

$\Rightarrow \frac{u^2}{2g} \sin^2(60) = 102 \times \cancel{2g}$

$\Rightarrow \frac{u^2}{2g} \times \left(\frac{\sqrt{3}}{2}\right)^2 = 102$

$\Rightarrow \frac{u^2}{2g} \times \frac{3}{4} = 102$

$\Rightarrow \frac{u^2}{2g} = 102 \times \frac{4}{3}$

$H_2 = \frac{u^2 \sin^2\theta_2}{2g}$

$= \cancel{102}^{34} \times \frac{4}{\cancel{3}} \sin^2(30^\circ)$

$= 34 \times 4 \times \left[\frac{1}{2}\right]^2$

$= 34 \times \cancel{4} \times \frac{1}{\cancel{4}}$

$= 34$ m

(14)

Given angles of projection $\theta_1 = 60^\circ$; $\theta_2 = 30^\circ$

So $R_1 = R_2$ [Because θ_1 & θ_2 are complimentary angles]

$H = \frac{(u\sin\theta)^2}{2g} \Rightarrow H \propto \sin^2\theta$ As $\theta_1 > \theta_2 \Rightarrow \sin\theta_1 > \sin\theta_2$

$\Rightarrow H_1 > H_2$

$T = \frac{u\sin\theta}{g} \Rightarrow T \propto \sin\theta$ clearly $T_1 > T_2$

$\therefore \frac{H_1}{R_1} > \frac{H_2}{R_2}$ $\propto$ $\frac{H}{T} \propto \frac{\sin^2\theta}{\sin\theta} \Rightarrow \frac{H}{T} \propto \sin\theta$

As $\sin\theta_1 > \sin\theta_2$

$\frac{H_1}{T_1} > \frac{H_2}{T_2}$