

TRIGONOMETRIC EQUATIONS-I

①

Class: IX, Mathematics

IIT ADVANCED

TEACHING TASK

01. $\csc \theta + 2 = 0$

$$\Rightarrow \csc \theta = -2$$

$$\Rightarrow \csc \theta = (\cancel{180}^\circ \csc (180^\circ + 60^\circ) = \csc 240^\circ$$

$$(\text{or}) = \csc (360^\circ - 60^\circ) = \csc 300^\circ$$

$$\therefore \theta = 240^\circ \text{ or } 300^\circ \quad \therefore \text{Ans: B}$$

02. $7 \sin^2 x + 3 \cos^2 x = 4$

$$\Rightarrow 7 \sin^2 x + 3(1 - \sin^2 x) = 4$$

$$\Rightarrow 4 \sin^2 x = 1$$

$$\Rightarrow \sin^2 x = \left(\frac{1}{2}\right)^2$$

$$\Rightarrow \sin x = \sin \frac{\pi}{6}$$

$$\Rightarrow x = n\pi \pm \frac{\pi}{6}, n \in \mathbb{Z}$$

Ans: D

03. $\cos 2\theta = 2 \sin^2 \theta$

$$\Rightarrow 1 - 2 \sin^2 \theta = 2 \sin^2 \theta$$

$$\Rightarrow 4 \sin^2 \theta = 1$$

$$\Rightarrow \sin^2 \theta = \left(\frac{1}{2}\right)^2$$

$$\sin \theta = \sin \frac{\pi}{6}$$

$$\therefore \theta = n\pi \pm \frac{\pi}{6}, n \in \mathbb{Z}$$

Ans: C



04 $\cot \theta = -\sqrt{3}$, $\csc \theta = 2$ (2)

$\Rightarrow \tan \theta = -\frac{1}{\sqrt{3}}$, $\sin \theta = \frac{1}{2}$

$\Rightarrow \tan \theta = \tan \frac{5\pi}{6}$, $\sin \theta = \sin \frac{5\pi}{6}$

$\therefore \theta = 2n\pi + \frac{5\pi}{6}$ Ans: D

05 $\tan \theta = +1$, $\cos \theta = \frac{1}{\sqrt{2}}$

$\tan \theta = \tan \left(\frac{7\pi}{4}\right)$, $\cos \theta = \cos \frac{7\pi}{4}$

$\theta = \frac{7\pi}{4}$

hence $\theta = 2n\pi + \frac{7\pi}{4}$ Ans: B

06 $2 \sin x + \csc x = 3$

$\Rightarrow 2 \sin x + 1 = 3 \sin x$

$\Rightarrow 2 \sin^2 x - 3 \sin x + 1 = 0$

$\Rightarrow (\sin x - 1)(2 \sin x - 1) = 0$

$\Rightarrow \sin x = 1$ or $\sin x = \frac{1}{2}$

$\Rightarrow \sin x = \sin \frac{\pi}{2}$ or $\sin x = \sin \frac{\pi}{6}$

$\Rightarrow x = n\pi + (-1)^n \frac{\pi}{2}$ or $x = n\pi + (-1)^n \frac{\pi}{6}$ Ans: D

07 option verification

let $x = \frac{\pi}{4}$

LHS $= \cos 2x = \cos \left(\frac{\pi}{2}\right) = \cos \frac{\pi}{2} = 0$

RHS $= (\sqrt{2} + 1) \left(\cos x - \frac{1}{\sqrt{2}}\right) = (\sqrt{2} + 1) \left(\cos \frac{\pi}{4} - \frac{1}{\sqrt{2}}\right) = 0$

hence $x = \left\{2n\pi \pm \frac{\pi}{4}\right\}$ Ans: D

Q8

$$\sin^{10} x - \cos^{10} x = 1$$

(3)

opt: A. $\sin^{10}(n\pi + \frac{\pi}{2}) - \cos^{10}(n\pi + \frac{\pi}{2})$

let $n=1$
 $= \sin^{10}(\pi + \frac{\pi}{2}) - \cos^{10}(\pi + \frac{\pi}{2})$

$$= \left[-\sin \frac{\pi}{2}\right]^{10} - \left[-\cos \frac{\pi}{2}\right]^{10}$$

$$= [-1]^{10} - [0]^{10} = 1 - 0 = 1$$

Ans: A

Q9

$$r \sin \theta = \sqrt{3}$$

$$r + 4 \sin \theta = 2(\sqrt{3} + 1)$$

$$\Rightarrow r = \frac{\sqrt{3}}{\sin \theta}$$

$$\frac{\sqrt{3}}{\sin \theta} + 4 \sin \theta = 2(\sqrt{3} + 1)$$

$$\Rightarrow \sqrt{3} + 4 \sin^2 \theta = 2 \sin \theta (\sqrt{3} + 1)$$

$$\Rightarrow 4 \sin^2 \theta - 2 \sin \theta + \sqrt{3} - 2\sqrt{3} \sin \theta = 0$$

$$\Rightarrow 2 \sin \theta (2 \sin \theta - 1) + \sqrt{3} (2 \sin \theta - 1) = 0$$

$$\Rightarrow (2 \sin \theta - 1) (2 \sin \theta - \sqrt{3}) = 0$$

$$\Rightarrow \sin \theta = \frac{1}{2} \text{ or } \sin \theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin \theta = \sin \frac{\pi}{6} \text{ or } \sin \theta = \sin \frac{\pi}{3}$$

$$\Rightarrow \theta = \frac{\pi}{6}, \frac{\pi}{3}, \frac{5\pi}{6}, \frac{2\pi}{3} \text{ satisfies the}$$

above equations

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$$4 \cos^2 x \sin x - 2 \sin^2 x - 3 \sin x = 0$$

(4)

$$\sin x (4 \cos^2 x - 2 \sin x - 3) = 0$$

$$\sin x = 0 \quad \text{or} \quad 4(1 - \sin^2 x) - 2 \sin x - 3 = 0$$

$$\Rightarrow x = n\pi \quad \Rightarrow 4 - 4 \sin^2 x - 2 \sin x - 3 = 0$$

$$\Rightarrow 4 \sin^2 x + 2 \sin x - 1 = 0$$

$$\Rightarrow \sin x = \frac{-1 \pm \sqrt{5}}{4}$$

$$\sin x = \frac{\sqrt{5}-1}{4} \quad \text{or} \quad \sin x = \frac{-1-\sqrt{5}}{4}$$

$$\sin x = \sin \frac{\pi}{10} \quad \sin x = \sin \left(-\frac{3\pi}{10}\right)$$

$$x = n\pi + (-1)^n \cdot \frac{\pi}{10} \quad x = n\pi + (-1)^n \cdot \left(-\frac{3\pi}{10}\right)$$

Hence the required solution.

$$\{n\pi, n \in \mathbb{Z}\} \cup \left\{n\pi + (-1)^n \frac{\pi}{10}\right\} \cup \left\{n\pi + (-1)^n \left(-\frac{3\pi}{10}\right)\right\}$$

$$10 \quad 4 \cos^2 x \sin x - 2 \sin^2 x - 3 \sin x = 0 \quad (4)$$

$$\sin x (4 \cos^2 x - 2 \sin x - 3) = 0$$

$$\sin x = 0 \quad \text{or} \quad 4(1 - \sin^2 x) - 2 \sin x - 3 = 0$$

$$\Rightarrow 4 - 4 \sin^2 x - 2 \sin x - 3 = 0$$

$$\Rightarrow x = n\pi \quad \Rightarrow 4 \sin^2 x + 2 \sin x - 1 = 0$$

$$\Rightarrow \sin x = \frac{-1 \pm \sqrt{5}}{4}$$

$$\sin x = \frac{\sqrt{5}-1}{4} \quad \text{or} \quad \sin x = \frac{-1-\sqrt{5}}{4}$$

$$\sin x = \sin \frac{\pi}{10} \quad \sin x = \sin \left(-\frac{3\pi}{10}\right)$$

$$x = n\pi + (-1)^n \cdot \frac{\pi}{10} \quad x = n\pi + (-1)^n \cdot \left(-\frac{3\pi}{10}\right)$$

Hence the required solution.

$$\{n\pi, n \in \mathbb{Z}\} \cup \left\{n\pi + (-1)^n \frac{\pi}{10}\right\} \cup \left\{n\pi + (-1)^n \left(-\frac{3\pi}{10}\right)\right\}$$

Ans: A

$$11 \quad a \cos 2\theta + b \sin 2\theta = c$$

$$\Rightarrow a \left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right) + b \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right) = c$$

$$\Rightarrow a - a \tan^2 \theta + 2b \tan \theta = c + c \tan^2 \theta$$

$$\Rightarrow (a+c) \tan^2 \theta - 2b \tan \theta + (c-a) = 0$$

$$\therefore \tan \alpha \cdot \tan \beta = \frac{c-a}{c+a}$$

Ans: C

12

12. $\sin^2 \theta + \sin \theta \cos \theta + \cos^2 \theta = 1$

(5)

$$(\sin^2 \theta + \cos^2 \theta) + \sin \theta \cos \theta = 1$$

$$(\sin \theta + \cos \theta)(1 - \sin \theta \cos \theta) - (1 - \sin \theta \cos \theta) = 0$$

$$(1 - \sin \theta \cos \theta) [\sin \theta + \cos \theta - 1] = 0$$

$$\sin \theta \cos \theta = 1$$

$$2 \sin \theta \cos \theta = 2$$

$$\sin 2\theta = 2$$

↓
Not possible

$$\frac{1}{\sqrt{2}} (\sin(\theta - \frac{\pi}{4})) = \frac{1}{\sqrt{2}}$$

$$(\cos(\theta - \frac{\pi}{4})) = \frac{1}{\sqrt{2}}$$

$$\cos(\theta - \frac{\pi}{4}) = \cos \frac{\pi}{4}$$

$$\therefore \theta - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$$

$$\theta = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{4}$$

$$\theta = 2n\pi + \frac{\pi}{2} \text{ or } \theta = 2n\pi$$

Ans. B

13. Statement I: $\cos \theta = \frac{3}{2} = 1.5 \rightarrow$ This is not possible. (False)

Statement II: Conceptual (True)

Ans: D

14. $\sin x$

14 $\sin x \cdot \cos 2y = (a^2 - 1)^2 + 1 \rightarrow (1)$ (6)
 $\cos x \cdot \sin 2y = a + 1 \rightarrow (2)$
 Since the L.H.S parts of (1) and (2) does not exceed 1
 We have $(a^2 - 1)^2 + 1 \leq 1$ and $a + 1 \leq 1$
 $a \leq 0 \rightarrow (3)$

$$(a^2 - 1)^2 \leq 0$$

$$\Rightarrow (a^2 - 1)^2 = 0$$

$$\Rightarrow a = \pm 1 \rightarrow (4)$$

from (3) and (4) $\Rightarrow a = -1$ is the only ~~solution~~ value, which satisfies (1) & (2) Ans. A

15 from the solution of problem no: 14.

We get $a = -1$.

$$(1) \Rightarrow \cos x \cdot \cos 2y = 1$$

When $\cos x = 1$

$$\cos x = \cos 0$$

$$x = 2n\pi$$

and (2) $\Rightarrow \cos x \cdot \sin 2y = 0$

$$\Rightarrow \sin 2y = 0$$

$$\Rightarrow 2y = 0, \pi, 2\pi, 3\pi, 4\pi$$

$$\Rightarrow y = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$$

15. From the solution of problem no: 14 ⑦

We get ~~cos~~ $\sin x \cdot \cos 2y = 1$ and

$$\cos x \cdot \sin 2y = 0$$

Case I: When $\sin x = \cos 2y = 1$

$$\sin x = 1$$

$$\Rightarrow x = \frac{\pi}{2} \quad [\because \text{in } [0, 2\pi]]$$

$$\text{for } \cos 2y = 1$$

$$\Rightarrow 2y = 2n\pi, n \in \mathbb{Z}$$

$$\Rightarrow y = n\pi$$

$$\Rightarrow \boxed{y = 0, \pi, 2\pi} \quad [\text{in } [0, 2\pi]]$$

Case II: When $\sin x = \cos 2y = -1$

$$\text{for } \sin x = -1$$

$$\Rightarrow x = \frac{3\pi}{2}$$

$$\text{for } \cos 2y = -1$$

$$\Rightarrow 2y = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow \boxed{y = \frac{\pi}{2}, \frac{3\pi}{2}} \quad \text{in } [0, 2\pi]$$

Ans: D

So, we total 5 values of y .

$$16. (2\sin x - \cos x)(1 + \cos x) = 1 - \cos x$$

$$(2\sin x - \cos x)(1 + \cos x) = (1 + \cos x)(1 - \cos x)$$

$$16. (2\sin x - \cos x)(1 + \cos x) = \sin^2 x \quad (8)$$

$$(2\sin x - \cos x)(1 + \cos x) = 1 - \cos^2 x$$

$$(2\sin x - \cos x)(1 + \cos x) = (1 + \cos x)(1 - \cos x)$$

$$(1 + \cos x) [2\sin x - \cos x - 1 + \cos x] = 0$$

$$1 + \cos x = 0 \quad \text{or} \quad 2\sin x - 1 = 0$$

$$\cos x = -1 \quad \text{or} \quad \sin x = \frac{1}{2}$$

$$x = \pi \quad (\times)$$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$\boxed{\cos x = \frac{\sqrt{3}}{2}}$$

$$3\cos^2 x - 10\cos x + 3 = 0$$

$$\Rightarrow 3\cos^2 x - 9\cos x - \cos x + 3 = 0$$

$$\Rightarrow 3\cos x [\cos x - 3] - [\cos x - 3] = 0$$

$$\Rightarrow \cos x - 3 = 0 \quad \cos x = \frac{1}{3}$$

$$\Rightarrow \cos x = 3 \quad (\times) \quad \boxed{\cos \beta = \frac{1}{3}}$$

$$\text{Again } 1 - \sin 2x = \cos x - \sin x$$

$$\Rightarrow (\sin x - \cos x)^2 + (\sin x - \cos x) = 0$$

$$\Rightarrow (\sin x - \cos x) [\sin x - \cos x + 1] = 0$$

$$\Rightarrow \sin x - \cos x = 0 \quad \therefore \boxed{\cos \beta = \frac{1}{\sqrt{2}}}$$

$$\Rightarrow \tan x = 1$$

$$\Rightarrow \cos x = \frac{1}{\sqrt{2}}$$

$$\text{Now } \cos x + \cos \beta + \cos \gamma$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{3} + \frac{1}{\sqrt{2}}$$

$$= \frac{3\sqrt{6} + 2\sqrt{2} + 6}{6\sqrt{2}}$$

Ans: A

17. from the solution of problem 16. (9)

$$\cos \alpha = \frac{\sqrt{3}}{2} \quad \left| \quad \cos \beta = \frac{1}{3} \quad \left| \quad \cos \gamma = \frac{1}{\sqrt{2}} \right. \right.$$
$$\Rightarrow \sin \alpha = \frac{1}{2} \quad \Rightarrow \sin \beta = \frac{2\sqrt{2}}{3} \quad \left| \quad \sin \gamma = \frac{1}{\sqrt{2}} \right.$$

$$\therefore \sin \alpha + \sin \beta + \sin \gamma$$

$$= \frac{1}{2} + \frac{2\sqrt{2}}{3} + \frac{1}{\sqrt{2}}$$

$$= \frac{14 + 3\sqrt{2}}{6\sqrt{2}}$$

Ans: A

18. $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$

$$= \frac{1}{2} \cdot \frac{1}{3} - \frac{\sqrt{3}}{2} \cdot \frac{2\sqrt{2}}{3}$$

$$= \frac{1 - 2\sqrt{6}}{6}$$

Ans: C

19. $|\sin x \cdot \cos x| + \sqrt{2 + \tan^2 x + \cot^2 x} = \sqrt{3}$

$$\Rightarrow |\sin x \cdot \cos x| + \sqrt{(\tan x + \cot x)^2} = \sqrt{3}$$

$$\Rightarrow |\sin x \cdot \cos x| + |\tan x + \cot x| = \sqrt{3}$$

$$\Rightarrow |\sin x \cdot \cos x| + \left| \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right| = \sqrt{3}$$

$$\Rightarrow |\sin x \cdot \cos x| + \frac{1}{|\sin x \cdot \cos x|} = \sqrt{3}$$

$$\text{let } |\sin x \cdot \cos x| = a$$

$$\Rightarrow a + \frac{1}{a} = \sqrt{3}$$

$$\Rightarrow a^2 - \sqrt{3}a + 1 = 0$$

This equation does not have any real roots since $\Delta < 0$

hence no. of solutions = 0

Ans: 0



$$20 \quad (\sqrt{3}+1)^{2x} + (\sqrt{3}-1)^{2x} = 2^{3x} \quad (10)$$

$$\Rightarrow (\sqrt{3}+1)^{2x} + (\sqrt{3}-1)^{2x} = (2\sqrt{2})^{2x}$$

$$\Rightarrow \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)^{2x} + \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^{2x} = 1$$

$$\Rightarrow (\sin 75^\circ)^{2x} + (\cos 75^\circ)^{2x} = 1$$

$$\Rightarrow x = 1$$

Ans: 1

No. of solutions: 1

$$21 \quad \cos 4x = \cos 2 \cdot 2x$$

$$= (2\cos^2 x - 1)$$

$$= 2[2(2\cos^2 x - 1)^2 - 1]$$

$$= 8[2(4\cos^4 x - 4\cos^2 x + 1) - 1]$$

$$= 8\cos^4 x - 8\cos^2 x + 1$$

$$= 1 + 8\cos^2 x + 8\cos^4 x$$

$$= a_0 + a_1 \cos^2 x + a_2 \cos^4 x$$

$$a_0 = 1, \quad a_1 = -8, \quad a_2 = 8$$

$$5a_0 + a_1 + a_2 = 5(1) - 8 + 8 = 5 \quad \text{Ans: 5}$$

22 a) $4\sin^2 x + \sin^2 x = 3$ (11)
 $\Rightarrow 4\sin^2 x + 4\sin^2 x \cos^2 x = 3$
 $\Rightarrow 4\sin^2 x + 4\sin^2 x (1 - \sin^2 x) = 3$
 $\Rightarrow 4\sin^4 x - 8\sin^2 x + 3 = 0$
 $\Rightarrow (2\sin^2 x - 3)(2\sin^2 x - 1) = 0$
 $\Rightarrow \sin^2 x = \frac{3}{2}$ or $\sin^2 x = \frac{1}{2}$
 $\Rightarrow \sin x = \frac{1}{\sqrt{2}} \Rightarrow x = \frac{\pi}{4}$
 $\therefore \tan x = 1 \Rightarrow x = \tan^{-1}(1)$

b) $4\cos^2 x + 8\cos x = 7$
 $\Rightarrow 4(2\cos^2 x - 1)^2 + 8\cos x = 7$
 $\Rightarrow 4(4\cos^4 x - 4\cos^2 x + 1) + 8\cos x - 7 = 0$
 $\Rightarrow 16\cos^4 x - 8\cos^2 x - 3 = 0$
 $\Rightarrow (4\cos^2 x + 1)(4\cos^2 x - 3) = 0$
 $\Rightarrow \cos x = \frac{\sqrt{3}}{2} \Rightarrow x = 30^\circ \Rightarrow \sin x = \frac{1}{2}$
 $\Rightarrow x = \sin^{-1}(\frac{1}{2})$

c) $3(1 + \sin x) = 1 + \cos 2x$
 $\Rightarrow 3 + 3\sin x = 1 + 1 - 2\sin^2 x$
 $\Rightarrow 2\sin^2 x + 3\sin x + 1 = 0$
 $\Rightarrow (\sin x + 1)(2\sin x + 1) = 0$
 $\Rightarrow \sin x + 1 = 0 \Rightarrow x = \sin^{-1}(-1)$

d) $4\sin^3 x - 2\sin^2 x - 2\sin x + 1 = 0$
 $= 2\sin^2 x (2\sin x - 1) - 1(2\sin x + 1) = 0$
 $= (2\sin x - 1)(2\sin^2 x - 1) = 0$
 $\Rightarrow \sin x = \frac{1}{2}$

$\Rightarrow x = \sin^{-1}(\frac{1}{2})$

Ans: P, S, Q, S

23 a) $\tan^2 x + \cot^2 x = 2$ (12)

$$\Rightarrow (\tan x - \cot x)^2 = 0$$

$$\Rightarrow \tan x - \cot x = 0$$

$$\Rightarrow \tan^2 x = 1$$

$$\Rightarrow \tan x = \tan \frac{\pi}{4}$$

$$\Rightarrow x = n\pi + \frac{\pi}{4}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

No. of solutions = 4

b) $\sin^2 x - \cos x = \frac{1}{2}$

$$\Rightarrow 1 - \cos^2 x - \cos x = \frac{1}{2}$$

$$\Rightarrow 2 - 2\cos^2 x - 2\cos x = 1$$

$$\Rightarrow 2\cos^2 x + 2\cos x - 1 = 0$$

$$\Rightarrow (\cos x + 1)(2\cos x - 1) = 0$$

$$\Rightarrow \cos x = -1 \text{ or } \cos x = \frac{1}{2}$$

$$x = \pi, 2\pi$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

No. of solutions = 4

c) $4\sin^2 x - 6\cos^2 x = 10$

$$\Rightarrow 4\sin^2 x - 6(1 - \sin^2 x) = 10$$

$$\Rightarrow 10\sin^2 x - 16 = 0$$

$$\Rightarrow \sin^2 x = \frac{8}{5} \Rightarrow \sin x = \frac{2\sqrt{2}}{\sqrt{5}} \approx 1.26$$

No. of solutions = 0

d) $\sin x = 1$

$$\Rightarrow \sin x = \sin \frac{\pi}{2}$$

$$\Rightarrow x = n\pi + (-1)^n \frac{\pi}{2}$$

$$\Rightarrow x = \frac{\pi}{2}$$

No. of solutions = 1

Ans: S, S, 0, Y

LEARNERS TASK (13)

CUB'S

01. $\sin \theta = \frac{1}{2}$
 $\Rightarrow \theta = \frac{\pi}{6}$ Ans: C
02. $\sin \theta = 1$
 $\Rightarrow \theta = \frac{\pi}{2}$ Ans: A
03. $\sin \theta = 0$
 $\Rightarrow \theta = n\pi, n \in \mathbb{Z}$ Ans: B
04. $\sec \theta = \sqrt{2}$
 $\Rightarrow \theta = \frac{\pi}{4}$ Ans: B
05. $\tan \theta = -\sqrt{3}$
 $\Rightarrow \tan \theta = \tan\left(-\frac{\pi}{3}\right)$
 $\Rightarrow \theta = -\frac{\pi}{3}$ Ans: D
06. $\cos \theta = 0$
 $\Rightarrow \theta = (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$ Ans: C
07. $\tan \theta = 0$
 $\Rightarrow \theta = n\pi, n \in \mathbb{Z}$ Ans: A
08. $\alpha \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ Ans: C
09. $\alpha \in [0, \pi]$ Ans: B
10. $\tan \theta = \tan \alpha \Rightarrow \alpha = R$ Ans: D

JEE MAINS LEVEL

01. $\cos \theta = \cos \frac{5\pi}{4}$
 $\Rightarrow \theta = 2n\pi \pm \frac{5\pi}{4}$ Ans: C

02. $\sqrt{3} \cos \theta = \sin \theta$ (14)
 $\Rightarrow \tan \theta = \sqrt{3}$
 $\Rightarrow \tan \theta = \tan \frac{\pi}{3}$
 $\Rightarrow \theta = n\pi + \frac{\pi}{3}, n \in \mathbb{Z}$ Ans: A

03. $\sqrt{\sin x} + \cos x = 0$
 $\Rightarrow \sqrt{\sin x} = -\cos x$
 $\Rightarrow \sin x = \cos^2 x$
 $\Rightarrow \sin x = 1 - \sin^2 x$
 $\Rightarrow \sin^2 x + \sin x - 1 = 0$
 $\Rightarrow \sin x = \frac{-1 + \sqrt{5}}{2} = \frac{\sqrt{5}-1}{2}$ Ans: D

04. $4 \cos \theta - \sec \theta = 4 \tan \theta$
 $\Rightarrow 4 \cos \theta - \frac{1}{\cos \theta} = \frac{4 \sin \theta}{\cos \theta}$
 $\Rightarrow 4 \cos^2 \theta - 1 = 4 \sin \theta$
 $\Rightarrow 4(1 - \sin^2 \theta) - 1 = 4 \sin \theta$
 $\Rightarrow 4 \sin^2 \theta + 4 \sin \theta - 3 = 0$
 $\Rightarrow (2 \sin \theta + 3)(2 \sin \theta - 1) = 0$
 $\Rightarrow \sin \theta = \frac{1}{2}$
 $\Rightarrow \sin \theta = \sin \frac{\pi}{6}$
 $\Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{6}, n \in \mathbb{Z}$ Ans: A

05. $3(1 - 2 \sin^2 \theta) + 2 = 7 \sin \theta$
 $\Rightarrow 6 \sin^2 \theta + 7 \sin \theta - 5 = 0$
 $\Rightarrow (2 \sin \theta - 1)(3 \sin \theta + 5) = 0$
 $\Rightarrow \sin \theta = \frac{1}{2}$ $\sin \theta = -\frac{5}{3}$ not possible
 $\Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{6}, n \in \mathbb{Z}$ Ans: B

06

$$\frac{\cos 3\theta}{2\cos 2\theta - 1} = \frac{1}{2}$$

(15)

$$\Rightarrow \frac{4\cos^3\theta - 3\cos\theta}{2(2\cos^2\theta - 1) - 1} = \frac{1}{2}$$

$$\Rightarrow \frac{4\cos^3\theta - 3\cos\theta}{4\cos^2\theta - 3} = \frac{1}{2}$$

$$\Rightarrow \frac{\cos\theta(4\cos^2\theta - 3)}{4\cos^2\theta - 3} = \frac{1}{2}$$

$$\Rightarrow 2\cos\theta(4\cos^2\theta - 3) - (4\cos^2\theta - 3) = 0$$

$$\Rightarrow (4\cos^2\theta - 3)(2\cos\theta - 1) = 0$$

$$\Rightarrow \cos^2\theta = \frac{\sqrt{3}}{2} \quad \cos\theta = \frac{1}{2}$$

$$\Rightarrow \cos\theta = \cos \frac{\pi}{3}, \quad \cos\theta = \cos \frac{\pi}{3}$$

$$\Rightarrow \theta = n\pi \pm \frac{\pi}{3}, \quad \theta = 2n\pi \pm \frac{\pi}{3}$$

Ans: B

07

$$\tan^2\theta - (1+\sqrt{3})\tan\theta + \sqrt{3} = 0$$

$$\Rightarrow \tan^2\theta - \tan\theta - \sqrt{3}\tan\theta + \sqrt{3} = 0$$

$$\Rightarrow (\tan\theta - 1)(\tan\theta - \sqrt{3}) = 0$$

$$\Rightarrow \tan\theta = 1 \quad \text{or} \quad \tan\theta = \sqrt{3}$$

$$\Rightarrow \tan\theta = \tan \frac{\pi}{4} \quad \text{or} \quad \tan\theta = \tan \frac{\pi}{3}$$

$$\Rightarrow \theta = n\pi + \frac{\pi}{4} \quad \theta = n\pi + \frac{\pi}{3}$$

Ans: C

08

$$\sqrt{1 - \cos x} = \sin x \Rightarrow 1 - \cos x = 1 - \cos^2 x$$

$$\Rightarrow \cos^2 x - \cos x = 0$$

$$\Rightarrow \cos x (\cos x - 1) = 0$$

$$\Rightarrow \cos x = 0 \quad \text{or} \quad \cos x = 1$$

$$x = (2n+1)\frac{\pi}{2}$$

$$\cos x = 1$$

$$x = (2n+1)\pi$$

$$x = 2n\pi \pm 0$$

$$x = 2n\pi$$

Ans: A



09

$$\tan \theta - \cot \theta = 0$$

(16)

$$\Rightarrow \tan \theta - \frac{1}{\tan \theta} = 0$$

$$\Rightarrow \tan^2 \theta - 1 = 0$$

$$\Rightarrow \tan^2 \theta = 1$$

$$\Rightarrow \tan \theta = \tan \frac{\pi}{4}$$

$$\Rightarrow \theta = n\pi \pm \frac{\pi}{4}$$

Ans: D

10

$$\sin 6\theta = \sin 4\theta - \sin 2\theta$$

$$= 2 \cos \left(\frac{4\theta + 2\theta}{2} \right) \cdot \sin \left(\frac{4\theta - 2\theta}{2} \right)$$

10

$$\sin 6\theta = \sin 4\theta - \sin 2\theta$$

$$\Rightarrow \sin 6\theta + \sin 2\theta = \sin 4\theta$$

$$\Rightarrow 2 \sin 4\theta \cdot \cos 2\theta - \sin 4\theta = 0$$

$$\Rightarrow \sin 4\theta (2 \cos 2\theta - 1) = 0$$

$$\Rightarrow \sin 4\theta = 0 \quad \text{or} \quad \cos 2\theta = \frac{1}{2}$$

$$\Rightarrow 4\theta = n\pi \quad \text{or} \quad \cos 2\theta = \cos \frac{\pi}{3}$$

$$\Rightarrow \theta = \frac{n\pi}{4}$$

$$2\theta = 2n\pi \pm \frac{\pi}{3}$$

$$\theta = n\pi \pm \frac{\pi}{6}$$

Ans: B

$$11. a \cos \alpha + b \sin \alpha = c \quad (19)$$

$$\Rightarrow a \cos \alpha - c = -b \sin \alpha$$

$$\Rightarrow a^2 \cos^2 \alpha + c^2 - 2ac \cos \alpha = b^2 (1 - \cos^2 \alpha)$$

$$\Rightarrow a^2 \cos^2 \alpha + c^2 - 2ac \cos \alpha - b^2 + b^2 \cos^2 \alpha = 0$$

$$\Rightarrow (a^2 + b^2) \cos^2 \alpha - 2ac \cos \alpha + c^2 - b^2 = 0$$

$$\cos \alpha + \cos \beta = \frac{2ac}{a^2 + b^2} \quad \text{Ans: B}$$

$$12. a \tan \alpha + b \sec \alpha = c$$

$$\Rightarrow a \tan \alpha - c = -b \sec \alpha$$

$$\Rightarrow a^2 \tan^2 \alpha - 2ac \tan \alpha + c^2 = b^2 (1 + \sec^2 \alpha)$$

$$\Rightarrow a^2 \tan^2 \alpha - 2ac \tan \alpha + c^2 - b^2 - b^2 \sec^2 \alpha = 0$$

$$\Rightarrow (a^2 - b^2) \tan^2 \alpha - 2ac \tan \alpha + c^2 - b^2 = 0$$

$$\tan \alpha + \tan \beta = \frac{2ac}{a^2 - b^2}$$

$$\tan \alpha \cdot \tan \beta = \frac{c^2 - b^2}{a^2 - b^2}$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta}$$

$$= \frac{\left(\frac{2ac}{a^2 - b^2} \right)}{1 - \left(\frac{c^2 - b^2}{a^2 - b^2} \right)} =$$

$$= \frac{2ac}{a^2 - c^2}$$

$$\text{Ans: A}$$

13

Statement I: $\sin x = -1$

(18)

$$\sin x = \sin\left(-\frac{\pi}{2}\right)$$

$$x = n\pi + (-1)^n\left(-\frac{\pi}{2}\right) \quad (\text{False})$$

Statement II: Conceptual (True)

Ans: D

14.

Statement I: $3\sin x + 4\cos x = 7$

$$\textcircled{D} \quad 7 > \sqrt{3^2 + 4^2} \quad (\text{False}) \quad (\text{True})$$

Statement II: Conceptual (True)

Ans: A

15

Statement I: $\frac{\tan 3x - \tan 2x}{1 + \tan 3x \cdot \tan 2x} = 1$

$$\Rightarrow \tan(3x - 2x) = 1$$

$$\Rightarrow \tan x = 1$$

$$\Rightarrow \tan x = \tan \frac{\pi}{4}$$

$$\Rightarrow x = n\pi \pm \frac{\pi}{4} \quad (\text{False})$$

Statement II: Conceptual (True)

Ans: D

$$16. \cos \theta \cdot \cos 2\theta \cdot \cos 3\theta = \frac{1}{4} \quad (19)$$

$$\Rightarrow 2 \cos 2\theta [2 \cos \theta \cdot \cos 3\theta] = 1$$

$$\Rightarrow 2 \cos 2\theta [\cos 4\theta + \cos 2\theta] = 1$$

$$\Rightarrow 2 \cos 2\theta \cdot \cos 4\theta + 2 \cos^2 2\theta - 1 = 0$$

$$\Rightarrow 2 \cos 2\theta \cdot \cos 4\theta + \cos 4\theta = 0$$

$$\Rightarrow \cos 4\theta (2 \cos 2\theta + 1) = 0$$

$$\Rightarrow \cos 4\theta = 0 \quad \text{or} \quad \cos 2\theta = -\frac{1}{2}$$

$$4\theta = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow \theta = (2n+1)\frac{\pi}{8}$$

$$\cos 2\theta = \cos \frac{2\pi}{3}$$

$$2\theta = 2n\pi \pm \frac{2\pi}{3}$$

$$\theta = n\pi \pm \frac{\pi}{3}$$

$$\text{here } 0 \leq \theta \leq \pi$$

$$\therefore \theta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8} \text{ Also}$$

$$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$$

$$\text{Sum} = \frac{\pi}{8} + \frac{3\pi}{8} + \frac{5\pi}{8} + \frac{7\pi}{8} + \frac{\pi}{3} + \frac{2\pi}{3}$$

$$= \frac{16\pi}{8} + \frac{3\pi}{3} = 2\pi + \pi = 3\pi$$

Ans: C

$$17 \quad 3 \sec \theta = 4 \tan \theta + 5$$

$$9(1 + \tan^2 \theta) = 16 \tan^2 \theta + 40 \tan \theta + 25$$

$$\Rightarrow 7 \tan^2 \theta + 40 \tan \theta + 16 = 0$$

$$17. \quad 3 \sec \theta - 5 = 4 \tan \theta$$

(20)

$$\frac{3}{\cos \theta} - 5 = \frac{4 \sin \theta}{\cos \theta}$$

$\sin \theta \neq 0$

$$\Rightarrow 3 - 5 \cos \theta = 4 \sin \theta$$

$$\Rightarrow 5 \cos \theta + 4 \sin \theta = 3$$

$$\Rightarrow \frac{5}{\sqrt{41}} \cos \theta + \frac{4}{\sqrt{41}} \sin \theta = \frac{3}{\sqrt{41}}$$

$$\Rightarrow \cos \alpha \cos \theta + \sin \alpha \sin \theta = \frac{3}{\sqrt{41}}$$

$$\Rightarrow \cos(\theta - \alpha) = \frac{3}{\sqrt{41}}$$

$$\cos \alpha = \frac{5}{\sqrt{41}}$$

$$\sin \alpha = \frac{4}{\sqrt{41}}$$

$$\Rightarrow \theta - \alpha = 2n\pi \pm \beta$$

$$\Rightarrow \theta = 2n\pi \pm \beta + \alpha$$

$$\beta = \cos^{-1}\left(\frac{3}{\sqrt{41}}\right)$$

$$\therefore \theta = 2n\pi + \alpha + \beta$$

$$\theta = 2n\pi + \alpha - \beta$$

Now in $[0, 4\pi]$ $\theta = 2n\pi + (\alpha - \beta)$ gives two solutions

Also $\theta = 2n\pi + (\alpha + \beta)$ gives two more solutions

$$\therefore \text{total no. of solutions} = 2 + 2 = 4$$

Ans 4



18. a) $\tan \theta = 1$ (21)

$$\Rightarrow \tan \theta = \tan \frac{\pi}{4}$$

$$\Rightarrow \theta = n\pi \pm \frac{\pi}{4}$$

b) $\cos^2 \theta = \left(\frac{1}{2}\right)^2$

$$\Rightarrow \cos^2 \theta = \cos^2 \frac{\pi}{3}$$

$$\Rightarrow \theta = n\pi \pm \frac{\pi}{3}$$

c) $\sin^2 \theta = \left(\frac{1}{2}\right)^2$

$$\Rightarrow \sin^2 \theta = \sin^2 \left(\frac{\pi}{6}\right)$$

$$\Rightarrow \theta = n\pi \pm \frac{\pi}{6}$$

d) $\sec \theta = 1$

$$\Rightarrow \sin \theta = 1$$

$$\Rightarrow \sin \theta = \sin \frac{\pi}{2}$$

$$\Rightarrow \theta = n\pi \pm \frac{\pi}{2}$$

Ans. $\theta, \pi, \frac{\pi}{2}, \frac{3\pi}{2}$

19 a) $1 + \sin 2x = \sin x + \cos x$ (22)

$$1 + 2 \sin x \cos x = \sin x + \cos x$$

$$\text{Let } \cos x = 0 \Rightarrow x = \frac{\pi}{2}$$

$$1 + 2 \sin(0) = \sin \frac{\pi}{2} + 0$$

$$1 = \sin \frac{\pi}{2} \text{ (True)}$$

b) $(2 \sin x - \cos x)(1 + \cos x) = \sin^2 x$

$$\text{Let } \sin x = \frac{1}{2} \Rightarrow \cos x = \frac{\sqrt{3}}{2}$$

$$\text{LHS} = \left(2 \times \frac{1}{2} - \frac{\sqrt{3}}{2}\right) \left(1 + \frac{\sqrt{3}}{2}\right)$$

$$= \left(1 - \frac{\sqrt{3}}{2}\right) \left(1 + \frac{\sqrt{3}}{2}\right) = 1 - \frac{3}{4} = \frac{1}{4}$$

$$\text{RHS} = \sin^2 x = \left(\frac{1}{2}\right)^2 = \frac{1}{4} \text{ (True)}$$

c) $\tan \theta + \cot \theta = 2$

$$\therefore \theta = \frac{\pi}{4}$$

d) $\sin 7\theta = \sin 4\theta - \sin \theta$

$$\Rightarrow \sin 7\theta + \sin \theta = \sin 4\theta$$

$$\Rightarrow 2 \sin 4\theta \cdot \cos 3\theta = \sin 4\theta$$

$$\text{Let } \theta = \frac{\pi}{9}$$

$$\text{LHS} = 2 \sin 4\theta \cdot \cos 3\theta$$

$$= 2 \sin 4\theta \cdot \cos \frac{\pi}{3}$$

$$= 2 \sin 4\theta \cdot \frac{1}{2} = \sin 4\theta$$

Ans: a, b, c, d

ADDITIONAL PRACTICE QUESTION

(23)

01. $2\sin^2\theta + 3\sin\theta + 1 = 0$

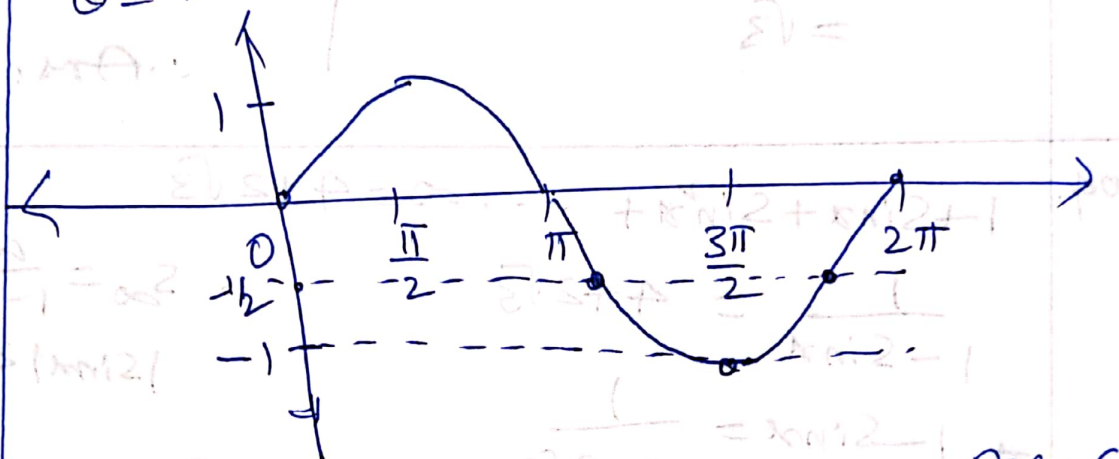
$$(\sin\theta + 1)(2\sin\theta + 1) = 0$$

$$\Rightarrow \sin\theta = -1 \quad \text{or} \quad \sin\theta = -\frac{1}{2}$$

$$\Rightarrow \sin\theta = \sin\left(-\frac{\pi}{2}\right) \quad \sin\theta = \sin\left(-\frac{\pi}{6}\right)$$

$$\theta = n\pi + (-1)^n\left(-\frac{\pi}{2}\right)$$

$$\theta = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$$



Number of solutions = 3

Ans: C

~~02. $\sin(x + 28^\circ) = \cos(3x - 78^\circ)$~~

~~$$x + 28^\circ = 3x - 78^\circ$$~~

~~$$\Rightarrow 2x = 106^\circ$$~~

~~$$\Rightarrow x = 53^\circ$$~~

02. $\sin(x + 28^\circ) = \cos(3x - 78^\circ)$

$$\Rightarrow x + 28^\circ + 3x - 78^\circ = 90^\circ$$

$$\Rightarrow x = 35^\circ$$

Also $x = 8^\circ \Rightarrow \sin 36^\circ = \cos 54^\circ$ (true)

Ans: B



03 $3 \tan(\theta - 15^\circ) = \tan(\theta + 15^\circ)$ (24)

Let $\theta = 45^\circ$

LHS = $3 \tan(45^\circ - 15^\circ)$ | RHS = $\tan(\theta + 15^\circ)$

= $3 \tan 30^\circ$

= $3 \times \frac{1}{\sqrt{3}}$

= $\sqrt{3}$

= $\tan(45^\circ + 15^\circ)$

= $\tan 60^\circ$

= $\sqrt{3}$

∴ Ans: B

04. $1 + \sin x + \sin^2 x + \dots \infty = 4 + 2\sqrt{3}$

$\frac{1}{1 - \sin x} = 4 + 2\sqrt{3}$

$S_\infty = \frac{a}{1 - r}$
 $|\sin x| < 1$

⇒ $1 - \sin x = \frac{1}{4 + 2\sqrt{3}}$

⇒ $\sin x = 1 - \frac{1}{4 + 2\sqrt{3}} = \frac{3 + 2\sqrt{3}}{4 + 2\sqrt{3}}$
 $= \frac{\sqrt{3}(\sqrt{3} + 2)}{2(2 + \sqrt{3})} = \frac{\sqrt{3}}{2}$

∴ $x = \frac{\pi}{3}, \frac{2\pi}{3}$

Ans: D

05 $(1 + \tan \alpha)(1 + \tan 4\alpha) = 2$

⇒ $1 + \tan 4\alpha + \tan \alpha + \tan \alpha \cdot \tan 4\alpha = 2$

⇒ $\tan \alpha + \tan 4\alpha = 1 - \tan \alpha \cdot \tan 4\alpha$

⇒ $\frac{\tan \alpha + \tan 4\alpha}{1 - \tan \alpha \cdot \tan 4\alpha} = 1$

⇒ $\tan(\alpha + 4\alpha) = \tan \frac{\pi}{4}$

⇒ $5\alpha = \frac{\pi}{4}$

$\alpha = \frac{\pi}{20}$

Ans: A