WS-2 8^{th} foundation-f

Task

① $\cos(270-\theta) + \sin(180-\theta) =$ ___

$\therefore$ $-\sin\theta + \sin\theta = 0$

② Given $\sin(-\theta) + \sin(90^\circ+\theta) =$ ___

$\Rightarrow$ $-\sin\theta + \cos\theta$

$\Rightarrow$ $\cos\theta - \sin\theta$

③ Given $\tan 135^\circ + \sin 240^\circ + \cot 420^\circ = ?$

$\Rightarrow \tan(90+45^\circ) + \sin(180+60) + \cot(360+60^\circ)$

$\Rightarrow -\cot 45^\circ - \sin 60^\circ + \cot 60^\circ$

$\Rightarrow -1 - \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{-2\sqrt{3} - 3 + 2}{2\sqrt{3}}$

$= \frac{-2\sqrt{3} - 1}{2\sqrt{3}} = -\frac{(2\sqrt{3}+1)}{2\sqrt{3}}$

④ Given $\cos^2 75^\circ + \sin^2 75^\circ + \operatorname{cosec}^2 80^\circ - \cot^2 80^\circ = ?$

we know that $\cos^2\theta + \sin^2\theta = 1$; $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\therefore$ $\cos^2 75 + \sin^2 75 = 1$; $\operatorname{cosec}^2 80 - \cot^2 80 = 1$

From the question $\cos^2 75 + \sin^2 75 + \operatorname{cosec}^2 80 - \cot^2 80$

$\Rightarrow 1 + 1$

$= 2$

(5) Given $\sec A + \tan A = x \rightarrow (1)$

we know that $\sec^2 A - \tan^2 A = 1$

$\Rightarrow (\sec A - \tan A)(\sec A + \tan A) = 1$

$\Rightarrow (\sec A - \tan A)\, x = 1$

$\Rightarrow \sec A - \tan A = \frac{1}{x} \rightarrow (2)$

From (1) + (2)

$$\begin{array}{r} \sec A + \cancel{\tan A} = x \\ \sec A - \cancel{\tan A} = \frac{1}{x} \\ \hline 2\sec A = x + \frac{1}{x} \end{array}$$

$\Rightarrow 2\sec A = \frac{x^2+1}{x}$

$\Rightarrow \sec A = \frac{x^2+1}{2x}$

(6) Given $\frac{1-\cos A}{\sin A} = ?$

$\Rightarrow \frac{\cancel{2}\sin^{\cancel{2}} \frac{A}{2}}{\cancel{2}\cancel{\sin\frac{A}{2}}\cos\frac{A}{2}}$

$= \frac{\sin\frac{A}{2}}{\cos\frac{A}{2}} = \tan\frac{A}{2}$

(7) Given $\sin^2 5^\circ + \sin^2 10^\circ + \sin^2 15 + \sin^2 20 + \cdots + \sin^2 90^\circ = ?$

$\Rightarrow \sin^2 5 + \sin^2 85 + \sin^2 10^\circ + \sin^2 80^\circ + \cdots + \sin^2 45^\circ + \cdots + \sin^2 90^\circ$

$\Rightarrow \sin^2 5 + \sin^2(90-5) + \sin^2 10 + \sin^2(90-10) + \cdots + \left(\frac{1}{\sqrt{2}}\right)^2 + \cdots + 1$

$\Rightarrow \underbrace{\sin^2 5^\circ + \cos^2 5^\circ} + \underbrace{\sin^2 10^\circ + \cos^2 10^\circ} + \cdots + \frac{1}{2} + 1$

$\Rightarrow 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + \frac{1}{2} + 1$

$\Rightarrow 9 + \frac{1}{2} = \frac{19}{2} = 9.5$

(8). Given $\frac{1+\cot^2 A}{1+\tan^2 A} = ?$

We know that $\operatorname{cosec}^2 A - \cot^2 A = 1$

$\Rightarrow \operatorname{cosec}^2 A = 1+\cot^2 A$

and $\sec^2 A - \tan^2 A = 1$

$\Rightarrow \sec^2 = 1+\tan^2 A$

$\Rightarrow \frac{\operatorname{cosec}^2 A}{\sec^2 A}$

$\Rightarrow \frac{\cos^2 A}{\sin^2 A}$

$\Rightarrow \cot^2 A \Rightarrow \boxed{\operatorname{cosec}^2 A - 1}$

(9) Given If $\sec A = \frac{15}{7}$ and $A+B = 90^\circ$

$B = 90 - A$

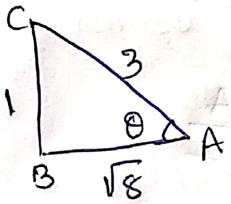
then $\operatorname{cosec} B = \operatorname{cosec}(90-A)$

$= \sec A$ In First quadrant all are positive

$= \frac{15}{7}$

(10) Given $\sin\theta = \frac{1}{3}$ ~~mee~~

From



From Pythagorous theorem

$AC^2 = AB^2 + BC^2$

$\Rightarrow 3^2 = AB^2 + 1^2$

$\Rightarrow 9 = AB^2 + 1$

$\Rightarrow AB^2 = 9-1 = 8$

$\Rightarrow AB = \sqrt{8}$

~~$\cot\theta = \frac{1}{\sqrt{8}}$~~

$\cot\theta = \sqrt{8}$

$\therefore$ ~~$2\cot^2\theta + 2 = 2\left(\frac{1}{\sqrt{8}}\right)^2 + 2$~~

~~$= 2 \times \frac{1}{8\,4} + 2$~~

~~$= \frac{1}{4} + 2$~~

$\therefore 2\cot^2\theta + 2 =$

$\Rightarrow 2(\sqrt{8})^2 + 2$

$= 2 \times 8 + 2$

$= 16 + 2 = 18$

(11) Given $\sin(A+B) = \frac{\sqrt{3}}{2}$ and $\cos(A-B) = \frac{\sqrt{3}}{2}$

$\Rightarrow A+B = 60^\circ \rightarrow (1)$ and $A-B = 30^\circ \rightarrow (2)$

Because $\sin 60^\circ = \frac{\sqrt{3}}{2}$

& $\cos 30^\circ = \frac{\sqrt{3}}{2}$

From (1) & (2)

$$A+B = 60^\circ$$
$$A-B = 30$$
$$\overline{2A = 90^\circ}$$
$$\Rightarrow A = 45^\circ$$

By substituting A in (1)

we get $45^\circ + B = 60^\circ$

$\Rightarrow B = 60 - 45^\circ = 15^\circ$

(13) Given $\sec\theta - \tan\theta = P \rightarrow (1)$

$\therefore$ From the Relation $\sec^2\theta - \tan^2\theta = 1$ we get

$(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$\Rightarrow P(\sec\theta + \tan\theta) = 1$

$\Rightarrow \sec\theta + \tan\theta = \frac{1}{P} \rightarrow (2)$

From (1) & (2) we get

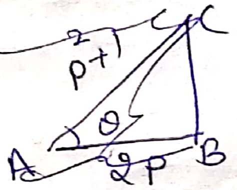
$$\sec\theta - \tan\theta = P$$
$$\sec\theta + \tan\theta = \frac{1}{P}$$
$$\overline{\Rightarrow 2\sec\theta = P + \frac{1}{P}}$$

$\Rightarrow 2\sec\theta = \frac{P^2+1}{P} \Rightarrow \sec\theta = \frac{P^2+1}{2P}$

$\cancel{\therefore \cos\theta = \frac{1}{\sec\theta} = \frac{2P}{P^2+1}}$ By ~~using right angle triangle~~

~~we know $AC^2 = AB^2 + BC^2$~~

$\Rightarrow$



By subtracting (1) & (2) we get

$$\sec\theta + \tan\theta = \frac{1}{p}$$
$$\sec\theta - \tan\theta = p$$

$$2\tan\theta = \frac{1}{p} - p$$

$$\Rightarrow \tan\theta = \frac{1-p^2}{2p}$$

$$\therefore \frac{\tan\theta}{\sec\theta} = \frac{\frac{1-p^2}{\cancel{2p}}}{\frac{1+p^2}{\cancel{2p}}} = \frac{1-p^2}{1+p^2}$$

$$\Rightarrow \frac{\frac{\sin\theta}{\cancel{\cos\theta}}}{\frac{1}{\cancel{\cos\theta}}} = \frac{1-p^2}{1+p^2} \Rightarrow \sin\theta = \frac{1-p^2}{1+p^2}$$

(15) Given $\tan 20^\circ = p$

$$\therefore \frac{\tan 160^\circ - \tan 110^\circ}{1+\tan 160^\circ \tan 110^\circ} = \frac{\tan(180-20^\circ) - \tan(90+20^\circ)}{1+\tan(180-20^\circ)\tan(90+20^\circ)}$$

$$= \frac{-\tan 20^\circ - (-\cot 20^\circ)}{1+(-\tan 20^\circ)(-\cot 20^\circ)}$$

$$= \frac{-p+\frac{1}{p}}{1+(-p)(-\frac{1}{p})} = \frac{\frac{-p^2+1}{p}}{1+1}$$

$$= \frac{1-p^2}{2p}$$

(16) $$\frac{\tan 610^\circ + \tan 700^\circ}{\tan 560^\circ - \tan 470^\circ} = \frac{\tan[7\times90+20] + \tan[8\times360-20]}{\tan(6\times90+20] - \tan[5\times90+20]}$$

We know $\tan(n\times90+\theta)$

For even n $= \pm\tan\theta$

(or)

For odd n $= \pm\cot\theta$

$$= \frac{\cot 20 + \tan 20}{-\tan 20 - (-\cot 20)}$$

$$= \frac{\frac{1}{p} + p}{-p + \frac{1}{p}} = \frac{\frac{1+p^2}{p}}{\frac{-p^2+1}{p}}$$

$$= \frac{1+p^2}{1-p^2}$$

(17) Given $\sin(45^\circ+\theta) - \cos(45^\circ-\theta) = ?$

we can write it as $\sin[90-45+\theta] - \cos(45-\theta]$

we know $\cos(-\theta) = \cos\theta$ $\Rightarrow$ $\cos(\theta-45] - \cos(45-\theta)$

$\Rightarrow \cos(45-\theta) - \cos(45-\theta)$

$= 0.$

(18) Given $\operatorname{cosec}(75^\circ+\theta) - \sec(15^\circ-\theta) - \tan(55^\circ+\theta) + \cot(35^\circ-\theta)$

$\Rightarrow \operatorname{cosec}(90-15+\theta) - \sec(15^\circ-\theta) - \tan(55+\theta)$

$+ \cot(90-55-\theta)$

$\Rightarrow \sec(-15+\theta) - \sec(15-\theta) - \tan(55+\theta)$

$+ \tan(55+\theta)$

$\Rightarrow \sec(\theta-15) - \sec(15-\theta) - \tan(55+\theta) + \tan(55+\theta)$

$= 0$

[$\because$ $\sec(-\theta) = \sec\theta$ and $\cot(90-55-\theta)$

$= \cot(90-(55+\theta))$

$= \tan(55+\theta)$)

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(19) $\sin^{19}x + \operatorname{cosec}^{20}x$

From given relation $\sin x + \operatorname{cosec} x = 2$

$\Rightarrow \sin x + \frac{1}{\sin x} = 2$

$\Rightarrow \sin^2 x + 1 = 2\sin x$

$\Rightarrow \sin^2 x - 2\sin x + 1 = 0$

$\Rightarrow (\sin x - 1)^2 = 0$

$\Rightarrow \sin x - 1 = 0 \Rightarrow \sin x = 1$

$= \operatorname{cosec} x = 1$

$\therefore \sin^{19}x + \operatorname{cosec}^{20}x = \sin^{19}x + \operatorname{cosec}^{19}x \cancel{+ \operatorname{cosec} x}$

$\Rightarrow [\sin x]^{19} + (\operatorname{cosec} x)^{20} \cancel{+ \operatorname{cosec} x}$

$\Rightarrow 1^{19} + 1^{20} = 1 + 1 = 2$

~~⇒ 1+1+1 = 3~~

L Task

CUQ'x

(5) $\sin 135^\circ = \sin(90 + 45^\circ)$

$= \cos 45^\circ$ In II^{nd} quadrant

sin is positive

$= \frac{1}{\sqrt{2}}$.

(6) $\sin(-45^\circ) + \cos(-45^\circ)$

$\Rightarrow -\sin 45^\circ + \cos 45^\circ$ $\quad \sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\Rightarrow -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$= 0$

(7) They are asking $\frac{\sin A}{2} = ?$

we know $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$$\therefore \frac{\sin A}{2} = \frac{\cancel{2}\sin\frac{A}{2}\cos\frac{A}{2}}{\cancel{2}}$$

$$= \sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)$$

(8) SAQ's

(2) $\tan(90+\theta) + \tan(90-\theta) = -\cot\theta + \cot\theta = 0$

In II^{nd} quadrant tan is Negative.

(3) Given $\sec^2\theta = 1$

From the relation $\sec^2\theta - \tan^2\theta = 1$

$$\Rightarrow \tan^2\theta = \sec^2\theta - 1$$

$$\Rightarrow \tan^2\theta = 1 - 1$$

$$\Rightarrow \tan^2\theta = 0 \Rightarrow \tan\theta = 0$$

(4) They are asking $\frac{\cos\theta + \cos(-\theta)}{\tan\theta + \cot\theta} = ?$

we know $\cos(-\theta) = \cos\theta$

$$\therefore \frac{\cos\theta + \cos(-\theta)}{\tan\theta + \cot\theta} = \frac{\cos\theta + \cos\theta}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}$$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$$= \frac{2\cos\theta}{\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}}$$

$$= 2\sin\theta\cos\theta\cos\theta$$

$$= 2\sin\theta\cos^2\theta$$

(5) $\cot\theta - \tan\theta = \dfrac{\cos\theta}{\sin\theta} - \dfrac{\sin\theta}{\cos\theta}$

$$= \frac{\cos^2\theta - \sin^2\theta}{\cos\theta \sin\theta}$$

$$= \frac{1}{\sin\theta \cos\theta}$$

(6) $\cos(A-B) = \dfrac{3}{5}$ & $\tan A \tan B = 2$

$$\Rightarrow \frac{\sin A}{\cos A} \frac{\sin B}{\cos B} = 2$$

$$\Rightarrow \sin A \sin B = 2\cos A \cos B \rightarrow (1)$$

$\therefore \cos(A-B) = \cos A \cos B + \sin A \sin B$

$$\Rightarrow \frac{3}{5} = \cos A \cos B + 2\cos A \cos B$$

$$\Rightarrow \frac{\cancel{3}}{5} = \cancel{3}\cos A \cos B$$

$$\Rightarrow \cos A \cos B = \frac{1}{5} \quad \text{From (1)}$$

$\therefore \sin A \sin B = 2 \times \dfrac{1}{5}$

$$\Rightarrow \sin A \sin B = \frac{2}{5}$$

$\therefore \cos(A+B) = \cos A \cos B - \sin A \sin B$

$$= \frac{1}{5} - \frac{2}{5} = \frac{1-2}{5} = \frac{-1}{5}.$$

(7) $\cos\theta = \dfrac{\sqrt{3}}{2}$ (i.e) $\theta = 30^\circ$

$$\sin^2\theta + \tan^2\theta = \sin^2 30^\circ + \tan^2 30$$

$$= \left[\frac{1}{2}\right]^2 + \left(\frac{1}{\sqrt{3}}\right]^2$$

$$= \frac{1}{4} + \frac{1}{3} = \frac{7}{12}$$

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(8) Given $5(\tan^2 A) - 5\sec^2 A + 1 = ?$

$\because \sec^2 A - \tan^2 A = 1$

$\therefore 5\tan^2 A - 5\sec^2 A + 1 = -5(\sec^2 A - \tan^2 A) + 1$

$= -5(1) + 1$

$= -5 + 1$

$= -4.$

(9) Given $\cos(\alpha + \beta) = 0$

i.e) $\alpha + \beta = 90^\circ$

$\Rightarrow \alpha = 90^\circ - \beta$ (or) $\beta = 90^\circ - \alpha.$

$\therefore \sin(\alpha - \beta) = \sin(90 - \beta - \beta)$

$= \sin(90 - 2\beta)$

$= \cos 2\beta$

(10) Given $\sin\theta - \cos\theta = 0$

$\Rightarrow \sin\theta = \cos\theta \Rightarrow \theta = 45^\circ$

Because $\sin 45 = \frac{1}{\sqrt{2}}$; $\cos 45 = \frac{1}{\sqrt{2}}$

$\therefore \sin^4\theta + \cos^4\theta = (\sin\theta)^4 + (\cos\theta)^4$

$= (\sin 45^\circ)^4 + (\cos 45^\circ)^4$

$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4$

$= \frac{1}{4} + \frac{1}{4}$

$= \frac{1+1}{4} = \frac{2}{4} = \frac{1}{2}$

(11) $\sin x + \sin^2 x = 1$

$\Rightarrow \sin x = 1 - \sin^2 x$

$\Rightarrow \sin x = \cos^2 x$

$\{\because \sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x\}$

Given $\cos^2 x + \cos^4 x = ?$

$\Rightarrow (\cos^2 x)^2 + (\cos^2 x)^2$

$\Rightarrow \sin x + (\sin x)^2$

$= 1$

(12) Given $\cot\theta - \tan\theta = ?$

$\Rightarrow \dfrac{\cos\theta}{\sin\theta} - \dfrac{\sin\theta}{\cos\theta} \Rightarrow \dfrac{\cos^2\theta - \sin^2\theta}{\cos\theta \sin\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$

$\Rightarrow \dfrac{\cos^2\theta - (1 - \cos^2\theta)}{\cos\theta \sin\theta}$

$= \dfrac{\cos^2\theta - 1 + \cos^2\theta}{\cos\theta \sin\theta}$

$\cot\theta - \tan\theta = \dfrac{2\cos^2\theta - 1}{\cos\theta \sin\theta}$

(13) (A) we know that $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \sec^2\theta = 1 + \tan^2\theta$

(B) $1 + \dfrac{\sin^2\theta}{\cos^2\theta} = 1 + \left[\dfrac{\sin\theta}{\cos\theta}\right]^2$

$= 1 + \tan^2\theta$

$= \sec^2\theta$

(C) $\sin^2\theta + \cos^2\theta = 1$ & $\sec^2\theta \cos^2\theta = \dfrac{1}{\cos^2\theta} \times \cos^2\theta$

$\sec^2\theta \cos^2\theta = 1$

$\therefore \sin^2\theta + \cos^2\theta = \sec^2\theta \cos^2\theta$

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(14) $\sin 1470^\circ = \sin[16\times 90^\circ + 30^\circ] = \sin 30^\circ = \frac{1}{2}$

we know that $\sin(n\times 90 \pm \theta) = \pm \sin\theta$ For

'n' is even.

If 'n' is odd $\sin(n\times 90 \pm \theta) = \pm \cos\theta$.

and $\cos 60^\circ = \frac{1}{2}$

(16) Given $\sin A + \cos A = \sqrt{2}$

By doing squaring on both sides we get

$\Rightarrow [\sin A + \cos A]^2 = (\sqrt{2})^2$

$\Rightarrow \sin^2 A + \cos^2 A + 2\sin A \cos A = 2$

$\Rightarrow 1 + 2\sin A\cos A = 2$

$\Rightarrow 2\sin A\cos A = 2-1$

$\Rightarrow 2\sin A \cos A = 1$

$\Rightarrow \sin A \cos A = \frac{1}{2}$

(17) Value of $\sec A \operatorname{cosec} A = \frac{1}{\cos A}\frac{1}{\sin A}$

$= \frac{1}{\sin A \cos A}$

$= \frac{1}{\frac{1}{2}}$

$\sec A \operatorname{cosec} A = 2$.

(18) The value of $\sin A \cdot \operatorname{cosec} A = \sin A \cdot \frac{1}{\sin A}$

$= 1.$

(21) Take solution from 11^{th} problem

(19) Given $x \tan 45^\circ \sin 30^\circ = \cos 30^\circ \tan 30^\circ$

$$\Rightarrow x \times 1 \times \frac{1}{2} = \frac{\cancel{\sqrt{3}}}{2} \times \frac{1}{\cancel{\sqrt{3}}}$$

$$\Rightarrow x \frac{\cancel{1}}{\cancel{2}} = \frac{\cancel{1}}{\cancel{2}}$$

$$\Rightarrow x = 1$$

(20) Given $\sin 2B = 2 \sin B$

$$\Rightarrow \cancel{2 \sin B} \cos B = \cancel{2 \sin B}$$

$$\Rightarrow \cos B = 1$$

$$\Rightarrow B = 0^\circ$$

(21) Given $\cos 9\alpha = \sin \alpha$

$$\Rightarrow \cancel{\cos} 9\alpha = \cancel{\cos}(90 - \alpha)$$

$$\Rightarrow 9\alpha = 90 - \alpha$$

$$\Rightarrow 9\alpha + \alpha = 90^\circ \Rightarrow 10\alpha = 90^\circ$$

$$\Rightarrow \alpha = 9^\circ$$

$\therefore \tan 5\alpha = \tan 5(9^\circ)$

$= \tan 45^\circ$

$= 1$