ws-4

T bask

① let F_x, F_y are horizontal and vertical components

Given $F_x = 8N$ and $F_y = 15N$.

$\therefore$ The force $F = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{8^2 + 15^2}$

$= \sqrt{64 + 225} = \sqrt{289} = 17N.$

② let the vectors are $\vec{A} = 4\hat{i} - 3\hat{j} + 5\hat{k}$

$\vec{B} = 2\hat{i} - 4\hat{j} + 4\hat{k}$; $\vec{C} = \hat{i} + \sqrt{6}\hat{j} + 3\hat{k}$

let $\vec{D}$ is a vector whose length is average of $\vec{B}$ and $\vec{C}$

$\Rightarrow |\vec{D}| = \dfrac{|\vec{B}| + |\vec{C}|}{2}$

$|\vec{B}| = \sqrt{2^2 + (-4)^2 + 4^2}$

$= \sqrt{36} = 6$

$\therefore |\vec{D}| = \dfrac{6 + 4}{2} = 5$

$|\vec{C}| = \sqrt{1^2 + (\sqrt{6})^2 + 3^2}$

$= \sqrt{1 + 6 + 9}$

$= \sqrt{16} = 4$

Given $\vec{D}$ is parallel to $\vec{A}$

So we can write

$\vec{D} = |\vec{D}|\,\hat{A}$

$|\vec{A}| = \sqrt{4^2 + (-3)^2 + 5^2}$

$= \sqrt{16 + 9 + 25}$

$= 5\sqrt{2}$

$\Rightarrow \vec{D} = 5 \dfrac{\vec{A}}{|\vec{A}|}$

$= \dfrac{\cancel{5}\,(4\hat{i} - 3\hat{j} + 5\hat{k})}{\cancel{5}\sqrt{2}}$

$\Rightarrow \vec{D} = \dfrac{4\hat{i} - 3\hat{j} + 5\hat{k}}{\sqrt{2}}$

(3) Given Directional cosines $\cos\alpha = \frac{4}{5\sqrt{2}}$; $\cos\beta = \frac{1}{\sqrt{2}}$; $\cos\gamma = \frac{3}{5\sqrt{2}} = \frac{A_z}{|\vec{A}|}$

$$= \frac{A_x}{|\vec{A}|}$$

$$\cos\beta = \frac{5}{5\sqrt{2}} = \frac{A_y}{|\vec{A}|}$$

$\therefore$ the vector $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$$= 4\hat{i} + 5\hat{j} + 3\hat{k}.$$

(4)

Given $\vec{A} = \hat{i} - 2\hat{j} - 3\hat{k}$ & $\vec{B} = 4\hat{i} - 2\hat{j} + 6\hat{k}$.

$$\therefore \vec{A} + \vec{B} = \hat{i} - 2\hat{j} - 3\hat{k} + 4\hat{i} - 2\hat{j} + 6\hat{k}$$

$$= 5\hat{i} - 4\hat{j} + 3\hat{k}$$

$$|\vec{A} + \vec{B}| = \sqrt{5^2 + (-4)^2 + 3^2} = \sqrt{25 + 16 + 9} = 5\sqrt{2}$$

Angle made by the ~~A+~~ vector $\vec{A} + \vec{B}$ with x-axis

$$\cos\alpha = \frac{\text{x-component}}{\text{Magnitude}} = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\alpha = 45^\circ.$$

(5)

let one component of a force $F_x = \frac{3P}{5}$

We know that $F = \sqrt{F_x^2 + F_y^2}$

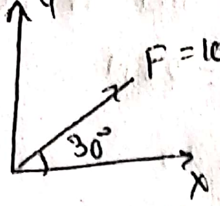
$$\Rightarrow P = \sqrt{\left(\frac{3P}{5}\right)^2 + F_y^2}$$

$$\Rightarrow P^2 = \frac{9P^2}{25} + F_y^2$$

$$\Rightarrow F_y^2 = P^2 - \frac{9P^2}{25} = \frac{16}{25}P^2$$

$$\Rightarrow F_y = \frac{4}{5}P$$

(6)



The components are $F_x = F\cos\theta$; $F_y = F\sin\theta$

$\Rightarrow F_x = 100\cos 30$; $F_y = 100\sin 30$

$= 100 \frac{\sqrt{3}}{2}$; $= 100 \times \frac{1}{2}$

$= 50\sqrt{3}\,N$; $= 50N.$

(7)

let the components of $\vec{A}$ are $A_x = 4m$; $A_y = 6m$.

The components of $(\vec{A}+\vec{B})$ are $(A+B)_x = 10m$; $(A+B)_y = 9m$.

$\therefore \vec{A}+\vec{B} = A_x\hat{i} + A_y\hat{j} + B_x\hat{i} + B_y\hat{j}$

$\Rightarrow (A+B)_x\hat{i} + (A+B)_y\hat{j} = A_x\hat{i} + A_y\hat{j} + B_x\hat{i} + B_y\hat{j}$

$\Rightarrow 10\hat{i} + 9\hat{j} = 4\hat{i} + 6\hat{j} + B_x\hat{i} + B_y\hat{j}$

$\Rightarrow B_x\hat{i} + B_y\hat{j} = 6\hat{i} + 3\hat{j}$

Angle made by $\vec{B}$ with x-axis is $\tan\theta = \frac{B_y}{B_x} = \frac{3}{6}$

$\tan\theta = \frac{1}{2}$

$\Rightarrow \theta = \tan^{-1}(\frac{1}{2})$

(8)

Given velocity $v = 10\sqrt{3}\,m/s$; $\theta = 60°$

It's rectangular components are $V_x = V\cos\theta$

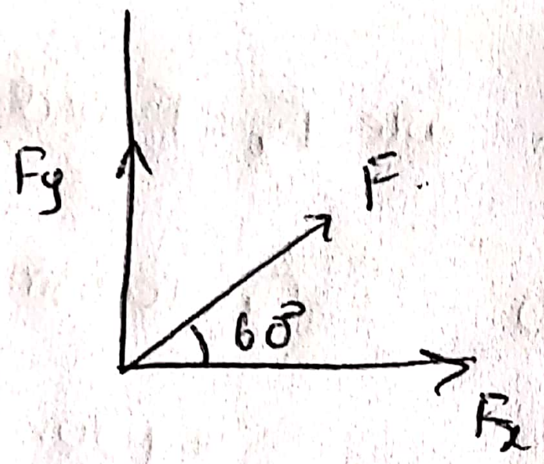
$\Rightarrow V_x = 10\sqrt{3}\cos 60 = 10\sqrt{3} \times \frac{1}{2} = 5\sqrt{3}\,m/s$

$V_y = V\sin\theta$

$= 10\sqrt{3}\sin 60$

$= 10\sqrt{3} \times \frac{\sqrt{3}}{2} = 5 \times 3 = 15\,m/s.$

(9)



let the force be F and it makes an angle 60° with $F_x = 25N$

$\Rightarrow F\cos\theta = 25$

$\Rightarrow F\cos 60 = 25$

$\Rightarrow F \frac{1}{2} = 25 \Rightarrow F = 50N.$

the other component is $F_y = F\sin\theta$

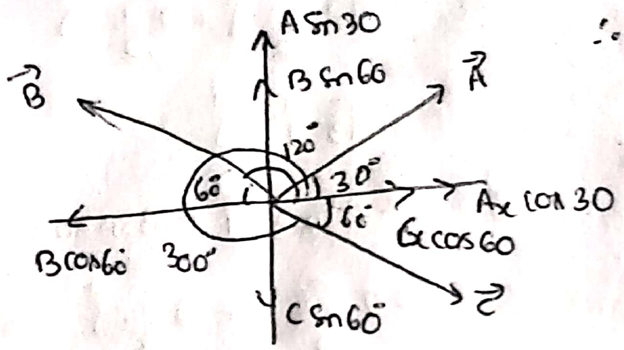
$= 50\sin 60$

$= 50 \frac{\sqrt{3}}{2} = 25\sqrt{3}N.$

(10)

(10)

Given $|\vec{A}| = 10$ units ; $|\vec{B}| = 5$ units ; $|\vec{C}| = 10$ units



∴ The resultant of $\vec{A}$, $\vec{B}$ and $\vec{C}$

is $= (A\cos 30 + C\cos 60 - B\cos 60)\hat{i}$

$+ (A\sin 30 + B\sin 60 - C\sin 60)\hat{j}$

$= [10\times\frac{\sqrt{3}}{2} + 10\times\frac{1}{2} - 5\times\frac{1}{2}]\hat{i} + \cancel{[10\times\frac{1}{2} + 5\frac{\sqrt{3}}{2} - 10\frac{\sqrt{3}}{2}]}$

$[10\times\frac{1}{2} + 5\frac{\sqrt{3}}{2} - 10\frac{\sqrt{3}}{2}]\hat{j}$

$\Rightarrow (10\frac{\sqrt{3}}{2} + \frac{5}{2})\hat{i} + (\frac{10}{2} - 5\frac{\sqrt{3}}{2})\hat{j}$

$\Rightarrow \frac{5}{2}[2\sqrt{3}+1]\hat{i} + \frac{5}{2}[2-2\sqrt{3}]\hat{j}$

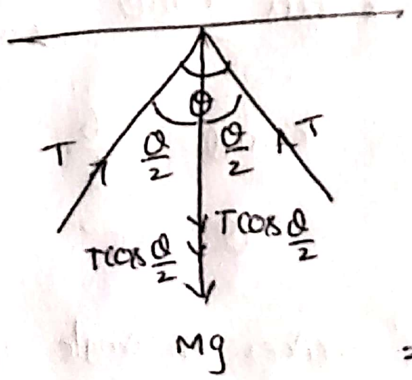
∴ It's magnitude $= \frac{5}{2}\sqrt{(2\sqrt{3}+1)^2 + (2-2\sqrt{3})^2}$

$= \frac{5}{2}\sqrt{12+1+\cancel{4\sqrt{3}}+4+3-\cancel{4\sqrt{3}}}$

$= \frac{5}{2}\sqrt{20} \Rightarrow \frac{5}{2}\times 2\sqrt{5} = 5\sqrt{5}$ units

Advanced level

(2)



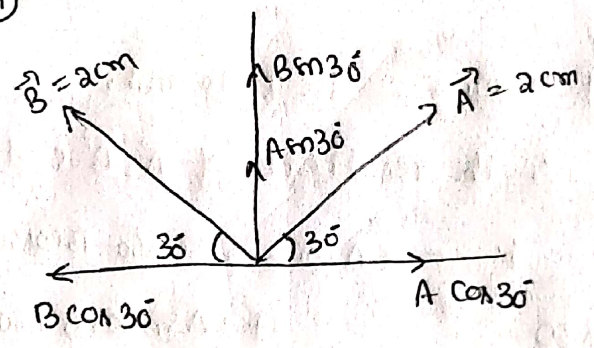
$2T\cos\frac{\theta}{2} = Mg$

$\Rightarrow T = \frac{Mg}{2\cos\frac{\theta}{2}}$ For $\theta = 120°$

$\Rightarrow T = \frac{Mg}{2\cos\frac{120°}{2}} = \frac{Mg}{2\cos 60°} = \frac{Mg}{2\times\frac{1}{2}} = Mg$

we get $T = Mg$ is maximum value of Tension for given values of $\theta = 120°$ only.

(1)



$\therefore \vec{A} = A\cos 30\,\hat{i} + A\sin 30\,\hat{j}$

$= 2 \times \frac{\sqrt{3}}{2}\hat{i} + 2\frac{1}{2}\hat{j}$

$= \sqrt{3}\,\hat{i} + \hat{j}$

$\vec{B} = -B\cos 30\,\hat{i} + B\sin 30\,\hat{j}$

$= -2\frac{\sqrt{3}}{2}\hat{i} + 2\frac{1}{2}\hat{j}$

$= -\sqrt{3}\,\hat{i} + \hat{j}$

$\therefore \vec{A} + \vec{B} = \sqrt{3}\hat{i} + \hat{j} - \sqrt{3}\hat{i} + \hat{j}$

$= 2\hat{j}$

(3) Given $\vec{A} = 3\hat{i} + 3\hat{j}\ m$; $\vec{B} = \hat{i} + 4\hat{j}\ m$; $\vec{C} = -2\hat{i} + 5\hat{j}\ m$.

$\therefore \vec{E} = -\vec{A} - \vec{B} + \vec{C}$

$= -(3\hat{i} + 3\hat{j}) - (\hat{i} + 4\hat{j}) + (-2\hat{i} + 5\hat{j})$

$\Rightarrow -3\hat{i} - 3\hat{j} - \hat{i} + 4\hat{j} - 2\hat{i} + 5\hat{j}$

$\vec{E} = -6\hat{i} + 6\hat{j}$

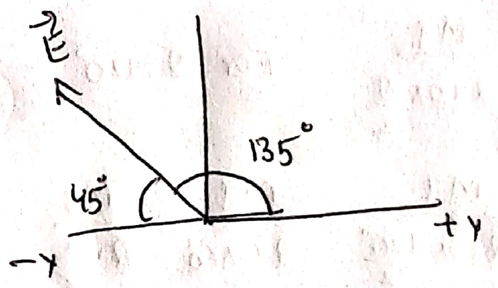
$|\vec{E}| = \cancel{\sqrt{(-8)^2 + (2)^2} = \sqrt{64+4} = \sqrt{68}}$

$|\vec{E}| = \sqrt{(-6)^2 + 6^2} = 6\sqrt{2}\ m$

Angle made by $\vec{E}$ with x-axis $\tan\theta = \frac{Y\ component}{X\ component}$

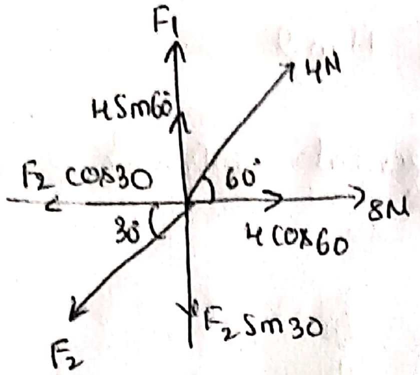
$\tan\theta = \frac{6}{-6} = -1$

$\theta = 135^\circ$



So $\vec{E}$ makes an angle 45° with $-x$-axis.

(4)



under equillibrium $F_{net} = 0$

Force along x-axis $= 0$

$\Rightarrow 8N + 4\cos 60 - F_2 \cos 30 = 0$

$\Rightarrow 8 + \cancel{4}^{2} \times \frac{1}{\cancel{2}} - F_2 \frac{\sqrt{3}}{2} = 0$

$= 8 + 2 = \frac{\sqrt{3}}{2} F_2$

$\Rightarrow F_2 = \frac{20}{\sqrt{3}} N$

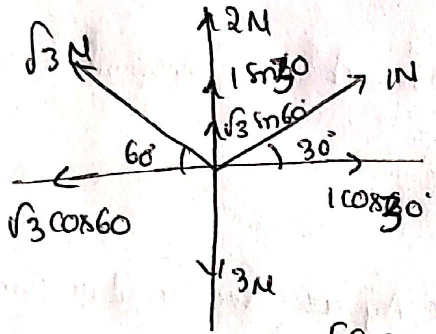
Force along y-axis $= 0$

$\Rightarrow F_1 + 4\sin 60 - F_2 \sin 30 = 0$

$\Rightarrow F_1 + \cancel{4}^{2} \frac{\sqrt{3}}{\cancel{2}} - \frac{\cancel{20}^{10}}{\sqrt{3}} \times \frac{1}{\cancel{2}} = 0$

$\Rightarrow F_1 + 2\sqrt{3} - \frac{10}{\sqrt{3}} = 0 \Rightarrow F_1 = \frac{10}{\sqrt{3}} - 2\sqrt{3} = \frac{4}{\sqrt{3}} N.$

(5)



Force along x-axis

$F_x = 1\cos 30 - \sqrt{3}\cos 60$

$= 1 \times \frac{\sqrt{3}}{2} - \sqrt{3}\left(\frac{1}{2}\right) = 0$

$F_x = 0$

Force along y-axis $\rightarrow F_y = 2 + 1\sin 30 + \sqrt{3}\sin 60 - 3$

$= 2 + \frac{1}{2} + \sqrt{3} \times \frac{\sqrt{3}}{2} - 3$

$= \frac{5}{2} + \frac{3}{2} - 3$

$= 4 - 3 = 1N.$

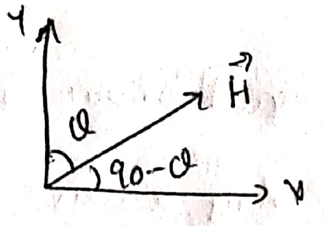
option A & C are correct.

(6)

The resultant of any two vectors $\vec{A}$ and $\vec{B}$ must be in between $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$

Here 17 is in between $10 + 8 = 18$ and $10 - 8 = 2$

(6) continuation



$\Rightarrow H_x = H\sin\theta$

↓

Horizontal component

$H_x = H\cos(90-\theta)$

(17), (18)

let the components of the vector are F_x and F_y

$\therefore \frac{F_x}{F_y} = \frac{3}{4}$ $\therefore$ that vector $\vec{F} = F_x\hat{i} + F_y\hat{j}$

$= 3\hat{i} + 4\hat{j}$.

magnitude of $\vec{F} = |\vec{F}| = \sqrt{F_x^2 + F_y^2} = \sqrt{3^2+4^2} = \sqrt{9+16} = 5$

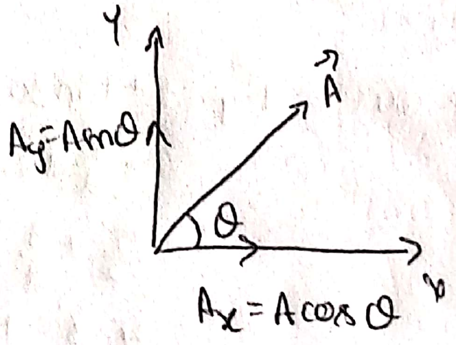
$\therefore$ unit vector of $\vec{F} = \frac{\vec{F}}{|\vec{F}|} = \frac{3\hat{i}+4\hat{j}}{5}$

(9)

Given vector $\vec{A} = 4\hat{i} + 3\hat{j}$ it's x-component = 4.

(10)

Given $\vec{A}$ is a vector



A_x & A_y are rectangular components

so Angle between them is $90°$ (or) $\frac{\pi}{2}$

$\frac{A_y}{A_x} = \frac{A\sin\theta}{A\cos\theta} = \tan\theta$

$\Rightarrow \theta = \tan^{-1}\left[\frac{A_y}{A_x}\right]$

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L Task CUQ's

(5) let the two vectors are $\hat{i} + 2\hat{j} - 3\hat{k} = \vec{A}$ and

$\vec{B} = 3\hat{i} + m\hat{j} - 9\hat{k}$

The condition for parallel is $\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}$

$\Rightarrow \frac{1}{3} = \frac{2}{m} = \frac{-3}{-9}$

$\therefore$ From $\frac{1}{3} = \frac{2}{m}$ we get $m = 6$

(6) let the vector be $\vec{A} = \hat{i} + \hat{k}$

$|\vec{A}| = \sqrt{1^2 + 1^2} = \sqrt{2}$ Here there is no $\hat{j}$ component i.e) y component $= 0$

$\therefore$ Directional cosines $\cos\alpha = \frac{x \text{ component}}{|\vec{A}|} = \frac{1}{\sqrt{2}}$

$\cos\beta = \frac{y\text{-component}}{|\vec{A}|} = \frac{0}{\sqrt{2}} = 0$

$\cos\gamma = \frac{z \text{ component}}{|\vec{A}|} = \frac{1}{\sqrt{2}}.$

$\therefore \cos\alpha, \cos\beta, \cos\gamma = \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$

(7) let the vector be $\vec{A} = \hat{i} + \hat{j}$

Angle made by the vector with x-axis is $\tan\theta = \frac{y\text{-component}}{x\text{-component}}$

$\Rightarrow \tan\theta = \frac{1}{1}$

$\Rightarrow \tan\theta = 1$

$\Rightarrow \theta = 45^\circ$

(8) Let the vector be $\vec{A} = \hat{i} + \hat{j} + \sqrt{2}\,\hat{k}$

$$|\vec{A}| = \sqrt{1^2 + 1^2 + (\sqrt{2})^2} = \sqrt{1+1+2} = \sqrt{4} = 2$$

Angle made by the vector with z-axis $\cos\gamma = \frac{A_z}{|\vec{A}|}$

$$\cos\gamma = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \gamma = 45^\circ$$

(9) Given that for a given vector $(\vec{A})$ it's horizontal and vertical components are equal.

ie $A\cos\theta = A\sin\theta$

$\Rightarrow \cos\theta = \sin\theta$ this is possible for $\theta = 45^\circ$

$\Rightarrow$

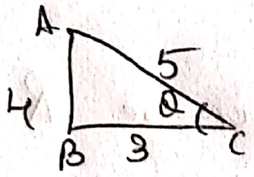
(10) Let the force $F = 50\,N$.

It's horizontal component $F_x = F\cos\theta = 30\,N$

$\Rightarrow 50\cos\theta = 30$

$\Rightarrow \cos\theta = \frac{30}{50} = \frac{3}{5}$

From Δ^{le} ABC



From Pythagorous theorem

$AC^2 = AB^2 + BC^2$

$\Rightarrow 5^2 = AB^2 + 3^2$

$\Rightarrow 25 = AB^2 + 9$

$\Rightarrow AB = 4$

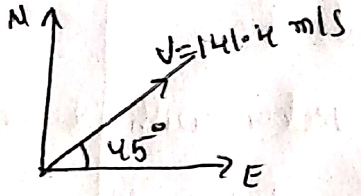
vertical component of the force $F_y = F\sin\theta$

$= \cancel{50}^{10} \times \frac{4}{\cancel{5}}$

$= 10 \times 4$

$= 40\,N$.

(11)



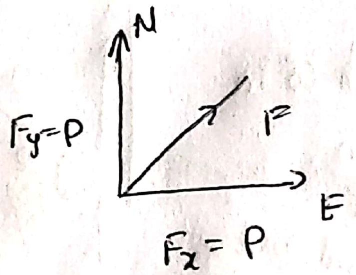
$\therefore$ North component of velocity $= v \sin\theta$

$= 141.4 \sin 45^\circ$

$= 141.4 \times \frac{1}{\sqrt{2}} = 141.4 \times \frac{1}{1.414}$

$= 1.00$ m/s

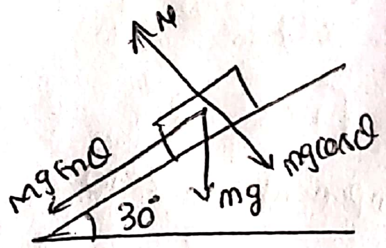
(12)



$\therefore \quad F = \sqrt{F_x^2 + F_y^2}$

$F = \sqrt{P^2 + P^2} = \sqrt{2}\, P$

(13)



Given mass $m = 100$ kg ; $g = 10$ m/s^2

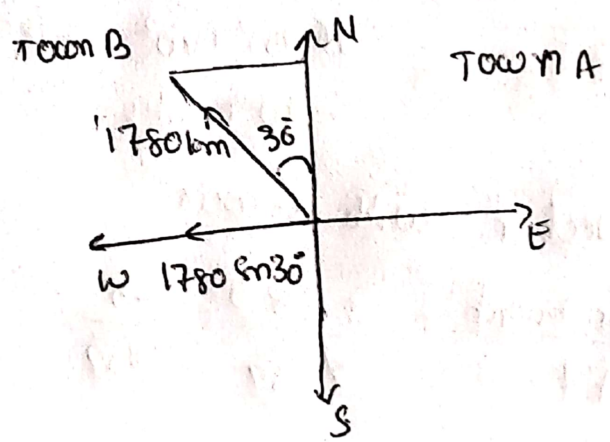
The component of force parallel to slope $= mg \sin\theta$

$= 100 \times 10 \times \sin 30$

$= 100 \times 10 \times \frac{1}{2}$

$= 500$ N.

(14) According to given data



The west of A from B

$= 1780 \sin 30$

$= 1780 \times \frac{1}{2}$

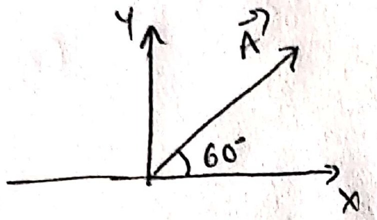
$= 890$ km

(15)

let the vector be $\vec{A} = \hat{i} + \sqrt{3}\,\hat{j}$

The angle made by the vector with ~~x-axis is~~

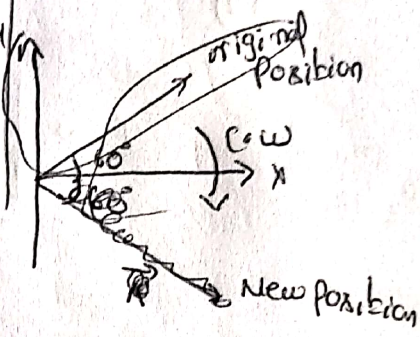
x-axis is $\tan\alpha = \dfrac{y\text{-component}}{x\text{-component}} = \dfrac{\sqrt{3}}{1}$



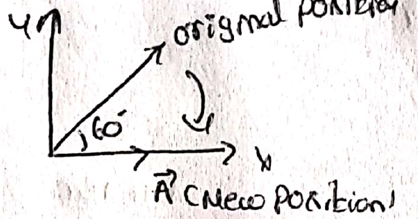
$\Rightarrow \alpha = 60°$

After Rotating through 60° in cw direction

The position of $\vec{A}$



$\therefore$



Here $\vec{A}$ falls on x-axis

ie $\theta = 0$ $\therefore |\vec{A}| = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = 2$

$\vec{A} = 2\hat{i}$ y-component is zero.

(16)

let the vector be $\vec{A}$

Given $A_x = -25$ units ; $A_y = 40$ units

$\therefore$ The magnitude of the vector $|\vec{A}| = \sqrt{A_x^2 + A_y^2}$

$= \sqrt{(-25)^2 + 40^2} = 5\sqrt{89}$ units

$=$ ~~50 units~~

Angle made by the vector with x-axis

is $\cos\alpha = \dfrac{x\text{ component}}{|\vec{A}|} = \dfrac{-\cancel{25}\,5}{\cancel{5}\sqrt{89}} = \dfrac{-5}{\sqrt{89}}$

(17)

Given vertical component of vector = Horizontal component

$\Rightarrow A\sin\theta = A\cos\theta$

$\Rightarrow \sin\theta = \cos\theta \rightarrow$ this is Possible if $\theta = 45^\circ$

(18)

Given velocity $v = 50$ m/s

one of it's component $V_x = 30$ m/s

we know that magnitude of any vector

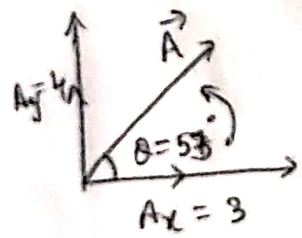
$$|\vec{v}| = \sqrt{(x\,component)^2 + (y\,component)^2}$$

$$\Rightarrow 50 = \sqrt{30^2 + V_y^2}$$

$$\Rightarrow 50^2 = 900 + V_y^2$$

$$\Rightarrow V_y^2 = 2500 - 900 = 1600 \Rightarrow V_y = 40 \text{ m/s}$$

(19) let the Given vector be $\vec{A} = 3\hat{i} + 4\hat{j}$



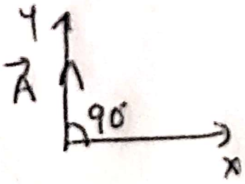
Angle made by the vector with x-axis

$\tan\theta = \dfrac{A_y}{A_x} = \dfrac{4}{3}$; $|\vec{A}| = \sqrt{3^2 + 4^2}$

$\Rightarrow \theta = 53^\circ$ $= 5$.

After turned through an angle 37° in ACW direction

then the vector falls on Y-axis because $53 + 37 = 90^\circ$



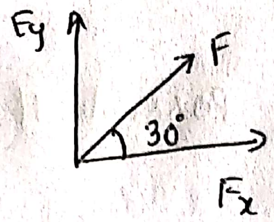
NO x-component for the vector.

only y-component $= |\vec{A}| \sin 90^\circ$

$\Rightarrow 5 \times 1 = 5\hat{j} \rightarrow$ it is on Y-axis

(20)

let $F = 10\,N$



$F_x = F\cos\theta$; $F_y = F\sin\theta$

$\Rightarrow F_x = 10\cos 30°$; $= 10\sin 30$

$= 10\frac{\sqrt{3}}{2}$; $= 10\,\frac{1}{2}$

$= 5\sqrt{3}\,N$; $= 5\,N.$

(21)

If any vector $\vec{A}$ is along horizontal direction then $\theta = 0°$

$\therefore$ vertical component $= |\vec{A}|\sin\theta$

$= |\vec{A}|\sin 0 = 0$

(22)

Given vector $\vec{A} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$

$|\vec{A}| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{2}+\frac{1}{2}} = 1$

unit vector means whose magnitude $= 1$

so given vector is a unit vector

(24)

Given force $= F$

It x-component $F_x = X$

$\therefore F^2 = F_x^2 + F_y^2$

$\Rightarrow F^2 - F_x^2 = F_y^2$

$\Rightarrow F_y = \sqrt{F^2 - F_x^2} = \sqrt{F^2 - X^2}$

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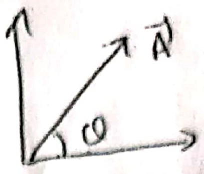
(28), (25)

let the vector be $\vec{A}$



Horizontal component $A_x = H$

$\Rightarrow |\vec{A}| \cos\theta = H$

Magnitude $\Rightarrow |\vec{A}| = \frac{H}{\cos\theta} = H \sec\theta$

Q. vertical component $A_y = Y$ in term of vertical component

$\Rightarrow |\vec{A}| \sin\theta = Y$

$\Rightarrow |\vec{A}| = y \operatorname{cosec}\theta$

(Q 27)

According to given question $A_x = 3$ units

$A_y = 4$ units

$\therefore$ Magnitude of a vector $\vec{A} = \sqrt{A_x^2 + A_y^2} = \sqrt{3^2 + 4^2}$

$= 5$ units

(28)

A unit vector is the one whose magnitude is equal to '1'. So sum of squares of rectangular components are is equal to 1

(30)

For the given vector $3\hat{i} - 4\hat{j}$ Horizontal component $= 3$

(31)

$l^2 + m^2 + n^2 = 1.$