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8. LCM AND HCF

TEACHING TASK

JEE MAINS LEVEL QUESTIONS
Multiple Choice Type:
1. 252 can be expressed as a product of primes as
   A) $2 \times 2 \times 3 \times 3 \times 7$ B) $2 \times 2 \times 2 \times 3 \times 7$
   C) $3 \times 3 \times 3 \times 3 \times 7$ D) $2 \times 3 \times 3 \times 3 \times 7$

Key: A

Solution:

252 as product of primes.

$252 = 2 \times 126 = 2 \times 2 \times 63 = 2^2 \times 3^2 \times 7$.

2. Which is the highest common factor of 36 and 84.
   A) 4   B) 6   C) 12   D) 18

Key: C

Solution:

HCF of 36 and 84

$36 = 2^2 \times 3^2$, $84 = 2^2 \times 3 \times 7$

common factors: $2^2 \times 3 = 4 \times 3 = 12$.

3. How many times is the HCF of 48, 36, 72 and 24 contained in their LCM?
   A) 12   B) 3   C) 24   D) 4

Key: A

Solution:

Compute HCF and LCM:

$48 = 2^4 \times 3$

$36 = 2^2 \times 3^2$

$72 = 2^3 \times 3^2$

$24 = 2^3 \times 3$

HCF = $2^{\min(4,2,3,3)} \times 3^{\min(1,2,2,1)} = 2^2 \times 3 = 4 \times 3 = 12$.

LCM = $2^{\max(4,2,3,3)} \times 3^{\max(1,2,2,1)} = 24 \times 3^2 = 16 \times 9 = 144$.

$144 \div 12 = 12$.

4. The number which is nearest to 457 and is exactly divisible by 11 is
   A) 459   B) 460   C) 462   D) 455

Key: C

Solution:

Number nearest to 457 divisible by 11

$11 \times 41 = 451$ (distance 6), $11 \times 42 = 462$ (distance 5)

nearest is 462.

5. If the LCM of two numbers and their product is 12, 24 respectively. Find HCF of that numbers.
   A) 12   B) 2   C) 3   D) 4

Key: B

Solution:
If LCM = 12 and product = 24, find HCF
Product = HCF × LCM
HCF = product ÷ LCM = 24 ÷ 12 = 2.

JEE ADVANCED LEVEL QUESTIONS

Multiple Correct type:

1.   Relation between LCM adn HCF of two numbers a & b is

   A) a x b = LCM x HCF

   B) $\dfrac{a}{b} = LCM \times HCF$

   C) Product of numbers = product of LCM & HCF

   D) $\dfrac{a}{b} = \dfrac{LCM}{HCF}$

Key: A & C (both state same relation in different wording).
Solution:
Relation between LCM and HCF of two numbers a & b
Correct identity: a×b=LCM(a,b)×HCF(a,b).

Statement Type:
   A) Both Statements are True.
   B) Both Statements are False.
   C) Statement - I is True, Statement - II is False.
   D) Statement - I is False, Statement - II is True.

2.   **Statement I**   :   LCM of 3, 5 is 15.
   Statement II   :   LCM of two primes is their product.

Key: A (Both True).
Solution:
Statement I: LCM of 3 and 5 is 15 — True.
Statement II: LCM of two primes is their product — True (primes have no common factor >1).

Comprehension Type :
   HCF of two or more numbers is the number which is the greatest coomon factor of them

3.   The HCF of 45 and 60 is.
   A) 5   B) 15   C) 45   D) 35

Key: B
Solution:
HCF of 45 and 60
45 = 3² × 5, 60 = 2² × 3 × 5
HCF = 3 × 5 = 15.

4.   If $P = a \times b \times c \times d$ and $Q = b \times c \times d \times 4$ then HCF of P,Q is
   A) bcd   B) bc   C) abc   D) 4abcd

Key: A
Solution:
The Highest Common Factor (HCF) is the product of the common prime factors.
P=a×b×c×d

Q=4×b×c×d=2²×b×c×d

The common factors are b,c,d. Thus, HCF = b×c×d=bcd.

Integer Type :

5.   The number that should be subtracted from 510 and 270 to get 24 as the GCD is .

Soluttion:

Let x be the number subtracted. Then numbers become 510-x and 270-x. Their difference is 240 (unchanged). If their GCD is 24 then both must be divisible by 24. So find x with 510 - x = 0 (mod 24).

Compute 510 mod 24: 24×21 = 504 → 510 = 6 (mod 24). So x = 6 (mod 24). Smallest positive x = 6. Check: 510-6 = 504 and 270-6 = 264. gcd(504,264) = 24 (since 504 = 24×21, 264 = 24×11 and gcd(21,11)=1).

Matrix Matching Type :

6.   **Column I**   **Column II**

a) The product of 2 numbers   p) $\dfrac{HCF}{LCM}$

b) The two prime numbers differ   q) co-prime
      by 2 are

c) The numbers having HCF as 1 are   r) HCF x LCM

d) (1, 3), (5, 7), (11, 13) are examples are   s) twin primes

Key: a→r, b→s, c→q, d→p.

LEARNERS TASK

CONCEPTUAL UNDERSTANDING QUESTIONS (CUQ's)

Multiple Choice Type :

1.   Prime factorisation of 100 is

A) 10^2 B) $5^2 \times 4$ C) $2^2 \times 5^2$ D) 25×4

Key: C

Solution:

Prime factorisation of 100

$100 = 10 \times 10 = (2 \times 5) \times (2 \times 5) = 2^2 \times 5^2$.

2.   If two numbers are relatively prime their LCM is

A) 1   B) their product   C) greater of two   D) smaller of two

Key: B (their product).

Solution:

Relatively prime ⇒ no common factor > 1, so LCM = product.

3.   If A and B are two positive numbers, then relation between LCM and HCF is

A) HCF x LCM = product of A and B

B) HCF x LCM = $\dfrac{A}{B}$

C) $A \times B = \dfrac{HCF}{LCM}$

D) $\dfrac{LCM}{HCF} = \dfrac{A}{B}$

Key: A

Solution:

HCF(A,B)×LCM(A,B)=A×B.

4.   HCF of given fractions =

A) HCF of numerators $\times$ LCM of denominators

B) $\dfrac{\text{HCF of numerators}}{\text{LCM of denominators}}$

C) $\dfrac{\text{LCM of denominators}}{\text{HCF of numerators}}$

D) $\dfrac{\text{LCM of numerators}}{\text{HCF of denominators}}$

Key: A (HCF of numerators / LCM of denominators).

Solution:

If fractions are $\dfrac{p_1}{q_1}, \dfrac{p_2}{q_2}$, ...then HCF $\left(\dfrac{p_1}{q_1}, \dfrac{p_2}{q_2}, ...\right) = \dfrac{HCF(p_1, p_2, ...)}{LCM(q_1, q_2, ...)}$.

5.   LCM of given fractions =

A) LCM of numerators $\times$ HCF of denominators

B) $\dfrac{\text{LCM of numerators}}{\text{HCF of denominators}}$

C) $\dfrac{\text{HCF of denominators}}{\text{LCM of numerators}}$

D) $\dfrac{\text{LCM of denominators}}{\text{HCF of numerators}}$

Key: A (LCM of numerators / HCF of denominators).

LCM $\left(\dfrac{p_1}{q_1}, \dfrac{p_2}{q_2}, ...\right) = \dfrac{LCM(p_1, p_2, ...)}{HCF(q_1, q_2, ...)}$.

JEE MAINS LEVEL QUESTIONS

Multiple Choice Type :

1.   The length and breadth of a room is 325 cm and 2.25m respectively. Find the largest size of the square tile in meters which can cover the floor of the room exactly.

A) 0.25m   B) 0.30m   C) 0.20m   D) 0.35m

Key: A

Solution:

Room: length 325 cm and breadth 2.25 m. Largest square tile size (in meters).

Convert breadth to cm: 2.25 m = 225 cm. So sides are 325 cm and 225 cm. Largest square tile side = GCD(325,225).

Compute gcd: 325 - 225 = 100; gcd(225,100) = gcd(100,25) = 25 cm = 0.25 m.

2.   Write the greatest 6-digited number having atleast three different digits.

A) 888897   B) 999978   C) 999987   D) 999897

Key: C

Solution:

Greatest 6-digit number having at least three different digits.

We want the largest 6-digit number with at least 3 distinct digits. The largest with =3 distinct digits is 999987 (digits: 9,9,9,9,8,7) — that's option C and is larger than the other options listed.

3. A milk man has 20 litres of milk in one container and 30000 millilitres of milk in another container. Determine the capacity of the biggest container which the milk man can use to measure milk from either container an exact number of times.

A) 10l   B) 20l   C) 15l   D) 25l

Key: A

Solution:

Milkman: 20 L and 30000 mL (30 L). Biggest container which measures both exactly = GCD(20,30) = 10 L.

4. Find the greatest number which divides 171 and 251 leaving remainder 3 and 6 respectively.

A) 9   B) 10   C) 7   D) 5

Key: C

Solution:

If divisor d leaves remainders 3 and 6 then d divides 171-3 = 168 and 251-6 = 245.

So d = gcd(168,245). Compute gcd:

245 - 168 = 77; 168 - 2×77 = 14; 77 - 5×14 = 7; 14 - 2×7 = 0 $\Rightarrow$ gcd = 7.

5. Find the LCM of $\dfrac{36}{225}, \dfrac{48}{150}, \dfrac{72}{65}$

A) $\dfrac{72}{225}$ B) $\dfrac{36}{65}$ C) $\dfrac{144}{5}$ D) $\dfrac{288}{5}$

Key: C

Solution:

LCM of fractions = $\dfrac{LCM\ of\ numerators}{HCF\ of\ deno\min ators}$

Numerators: 36, 48, 72

LCM = $2^4 \times 3^2 = 144$

Denominators: 225, 150, 65

HCF = 5 (only common prime factor)

Thus,

LCM = $\dfrac{144}{5}$

JEE ADVANCED LEVEL QUESTIONS

Multiple Correct Type:

1. Which of the following is true

   A) the LCM of two even numbers is their product

   B) the LCM of two numbers is always greather than either of the numbers

   C) the LCM of two prime numbers is always their product

   D) if 'a' is a factor of 'b' then LCM of a and b is equal to b

Key: C and D

Statement Type:

   A) Both Statements are True.

   B) Both Statements are False.

2.   **Statement I** : Product of two numbers = product of their HCF and LCM

      Statement II : If the product and HCF of two numbers are 4107 and 37 respectively, then the greater number is 111.

Key: A (Both True).

Solution:

Statement I: This is standard identity: for integers x,y,

$x \cdot y = HCF(x,y) \times LCM(x,y)$.

So Statement I is True.

Statement II: Let the two numbers be 37m and 37n with gcd(m,n) = 1.

Their product = 37^2 mn.

Given product = 4107. Compute 37^2 = 1369.

$$mn = \frac{4107}{1369} = 3.$$

Since m,n are coprime positive integers with product 3, they must be 1 and 3. Hence numbers are 37 and 37×3=111. The greater is 111. Statement II is True.

Comprehension Type :

$$\text{HCF of fractions} = \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$$

3.   The HCF of $\dfrac{9}{10}, \dfrac{12}{25}, \dfrac{18}{35}$ and $\dfrac{21}{40}$ is

   A) $\dfrac{3}{5}$ B) $\dfrac{252}{5}$ C) $\dfrac{3}{1400}$ D) $\dfrac{63}{700}$

Key: C

Solution:

$\text{HCF of fractions} = \dfrac{HCF\ of\ numerators}{LCM\ of\ deno\min ators}$

Numerators: 9, 12, 18, 21 → HCF = 3

Denominators: 10, 25, 35, 40 → LCM = $2^3 \times 5^2 \times 7 = 1400$

Thus,

$HCF = \dfrac{3}{1400}$

4.   The HCF of $\dfrac{8}{9}$ and $\dfrac{12}{27}$ is

   A) $\dfrac{4}{27}$ B) $\dfrac{5}{27}$ C) $\dfrac{5}{28}$ D) $\dfrac{3}{28}$

Key: A

Solution:

$\text{HCF of fractions} = \dfrac{HCF\ of\ numerators}{LCM\ of\ deno\min ators}$

Numerators: 8 and 12 → HCF = 4

Denominators: 9 and 27 → LCM = 27
Thus,

$$\text{HCF} = \frac{4}{27}$$

Integer Type :
5.   HCF of 36 and 84 is.
Solution:
Prime factors:
$36=2^2×3^2$
$84=2^2×3×7$
Common factors: $2^2×3=4×3=12$
Matrix Matching Type :
6.   **Column I**   **Column II**
a) Common factors of 18 and 24.   p) 3
b) HCF of 55 and 121 is   q) 6
c) LCM of 12, 24 is   r) 19
d) Common factors of 38 and 57 is   s) 11
                                     t) 24

Key: a–q, b–s, c–t, d–r