$7^{th}$ class advanced

WS-1 Thasle

① Given $\lambda = \frac{h}{mv}$ $[h] = ML^2T^{-1}$ ; $[m] = M$ ; $[v] = LT^{-1}$

$\therefore [\lambda] = \frac{[h]}{[m][v]} = \frac{\cancel{M}L^{\cancel{2}}\cancel{T^{-1}}}{\cancel{M}L\cancel{T^{-1}}}$

$[\lambda] = L$

② Given $\tau = I\alpha$ $[I] = ML^2$ ; $[\alpha] = T^{-2}$

$\therefore [\tau] = [I][\alpha]$
$= ML^2T^{-2}$

③ Density = $\frac{\text{Mass}}{\text{volume}}$

$[\text{density}] = \frac{[\text{Mass}]}{[\text{volume}]}$ $[\text{volume}] = [lbh]$
$= [l][b][h]$
$= LLL = L^3$

$\Rightarrow [\text{density}] = \frac{M}{L^3} \Rightarrow [\text{density}] = ML^{-3}$

④ $[\text{Pressure}] = \frac{[F]}{[A]} = ML^{-1}T^{-2}$

$[\text{Power}] = \frac{[\text{work}]}{[\text{time}]} = \frac{ML^2T^{-2}}{T} = ML^2T^{-3}$

$[\text{Angle}] = M^0L^0T^0$

⑤ Given the power $= 200W$ also $L_2 = \frac{L}{2}$ : $m_2 = 2m$; $t_2 = 2t$

According to the relation $N_1U_1 = N_2U_2$

$\Rightarrow 200\, ML^2T^{-3} = N_2[M_2L_2^2T_2^{-3}]$

$\Rightarrow 200\, ML^2T^{-3} = N_2[2M(\frac{L}{2})^2(2T)^{-3}]$

$\Rightarrow 200\, \cancel{M}\cancel{L^2}\cancel{T^{-3}} = N_2\, 2\cancel{M}\frac{\cancel{L^2}}{2^{\cancel{2}}}\, 2^{-3}\, \cancel{T^{-3}}$

$\Rightarrow 200 = N_2\, \frac{1}{2} \times \frac{1}{2^3}$

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$\Rightarrow 200 = N_2 \frac{1}{16} \Rightarrow N_2 = 3200$ new units.

(6) Given $F = 2N$ $L = 4m$, $V = 6 m/s$

$\therefore$ From Dimensions of force $[F] = MLT^{-2}$

$\Rightarrow [F] = M LT^{-1} T^{-1}$

$[V] = LT^{-1}$

$\Rightarrow 2 = M \times \cancel{6}^{3} \times \frac{3}{\cancel{2}}$

$\Rightarrow 6 = 4 T^{-1}$

$\Rightarrow T^{-1} = \frac{6}{4} = \frac{3}{2}$

$\Rightarrow 2 = M \times 9$

$\Rightarrow M = \frac{2}{9}$ kg

(7) Given Force = F

Acceleration = A

Time = T

$[Energy] = ML^2T^{-2}$

$[F] = MLT^{-2}$ ; $[A] = LT^{-2}$ $[T] = T$.

Do options verification.

(A) $[FAT^{-2}] = [F][A][T^{2}]$

$\Rightarrow MLT^{-2} LT^{-2} T^{2} = ML^2 T^{0}$

[B] $[F^{-2} A^{-1} T] = [MLT^{-2}]^{-2} [LT^{-2}]^{-1} T$

$= M^{-2} L^{-2} T^{4} L^{-1} T^{2} T = M^{-2} L^{-3} T^{7}$

[D] $[F^{-1} A^{2} T^{0}] = [MLT^{-2}]^{-1} [LT^{-2}]^{2} T^{0}$

$\Rightarrow M^{-1} L^{-1} T^{2} L^{2} T^{-4} = M^{-1} L T^{-2}$

(8) Given volume $= V = [V] = L^3$

Area $= A = [A] = L^2$ ; $[velocity] = LT^{-1}$

$V \propto A^{\alpha} u^{\beta} t^{\gamma} \Rightarrow V = K A^{\alpha} u^{\beta} t^{\gamma}$ $[K] = M^0 L^0 T^0$

$\Rightarrow [V] = [K][A]^{\alpha} [u]^{\beta} [t]^{\gamma}$

$\Rightarrow M^0 L^3 T^0 = M^0 L^0 T^0 (L^2)^{\alpha} (LT^{-1})^{\beta} (T)^{\gamma}$

$\Rightarrow M^0 L^3 T^0 = M^0 L^0 T^0 L^{2\alpha} L^{\beta} T^{-\beta} T^{\gamma}$

$$M^0 L^3 T^0 = M^0 L^{2\alpha+\beta} T^{-\beta+\gamma}$$

compare the powers on both sides

$\Rightarrow 2\alpha+\beta = 3 \qquad \gamma-\beta=0 \Rightarrow \gamma=\beta$

~~If $\alpha=0$ then $\beta=3$~~

$\gamma=\beta \qquad \alpha = \frac{3-\beta}{2}$

$\alpha \;\#\; \beta=\gamma$

(9) unit of length = 10 cm

mass = 10 gm $\qquad N_1 = 1\,N$

time = 0.1 sec.

From the relation $N_1 u_1 = N_2 u_2$

$\Rightarrow 1 \times kg\, m\, sec^{-2} = N_2\,(10 gm)(10 cm)(0.1 sec)^{-2}$

$\Rightarrow$ ~~1 kgm~~ $\Rightarrow 1\times 10^3 gm \times 10^2 cm\, sec^{-2} = N_2\, 10^4\, gm\, cm\, sec^{-2}$

$\Rightarrow 10^5\, gm\, cm\, sec^{-2} = \cancel{N_2\, 10^4\, gm\, cm\, sec^{-2}}$

$\Rightarrow \cancel{10}$

$$\frac{N_1}{N_2} = \frac{u_2}{u_1} \Rightarrow \frac{1N}{1\text{ New unit}} = \frac{kg\; m\; sec^{-2}}{10 gm \times 10 cm \times (0.1)^{-2} sec^{-2}}$$

$$\Rightarrow \frac{1N}{1\text{ New unit}} = \frac{1000\, gm \times 100\, cm \times sec^{-2}}{100\, gm\, cm \times 10^2\, sec^{-2}}$$

$$\Rightarrow \frac{1N}{1\text{ new unit}} = 10 \Rightarrow 1 \text{ New unit} = \frac{1}{10} = 0.1\,N$$

(10) Given escape velocity $u = K g^a R^b$

$[u] = M^0 L T^{-1}$ ; $[k] = M^0 L^0 T^0$ : $[g] = LT^{-2}$ ; $[R] = L$

$\therefore [u] = [k][g]^a [R]^b$

$\Rightarrow M^0 L T^{-1} = M^0 L^0 T^0 (LT^{-2})^a (L)^b$

$\Rightarrow M^0 L T^{-1} = M^0 L^0 T^0 L^a T^{-2a} L^b$

$\Rightarrow M^0 L T^{-1} = M^0 L^{a+b} T^{-2a}$

compare the powers on both sides.

$a+b=1$ ; $-2a=-1$

$\Rightarrow b=1-a$ ; $a=\frac{1}{2}$

$\Rightarrow b=1-\frac{1}{2}=\frac{1}{2}$ $\therefore v\propto g^{\frac{1}{2}}R^{\frac{1}{2}} \Rightarrow v\propto (gR)^{\frac{1}{2}}$

$\Rightarrow v\propto\sqrt{gR}$

(11) Assertion A: $s=ut+\frac{1}{2}at^2$ is correct

Because $[s]=L$ ; $[ut]=[u][t]=LT^{-1}T$

$[ut]=L$

$[at^2]=[a][t]^2$ For Both L.H.S and

$=LT^{-2}T^2$ R.H.S dimensions are

$[at^2]=L$ same.

(15) Given centripetal force $F=k\,m^a v^b r^c$.

$[F]=MLT^{-2}$ ; $[k]=M^0L^0T^0$ $[m]=M$ ; $[v]=LT^{-1}$

$[r]=L$ From the relation

$\therefore [F]=[k][m]^a[v]^b[r]^c$

$\Rightarrow MLT^{-2}=M^0L^0T^0M^a(LT^{-1})^b(L)^c$

$\Rightarrow MLT^{-2}=M^0L^0T^0M^aL^bT^{-b}L^c$

$\Rightarrow MLT^{-2}=M^aL^{a+c}T^{-b}$

compare the powers on both sides

$a=1$ ; $a+c=1$ $-b=-2$

$\Rightarrow c=1-a$ $b=2$

$\Rightarrow c=1-2$

$c=-1$.

(17) $v \propto g^p h^q$ $[v] = LT^{-1}$; $[g] = LT^{-2}$; $[h] = L$

$\therefore$ By using dimensional analysis

$$v = k\, g^p h^q \qquad [k] = M^0 L^0 T^0$$

$$\Rightarrow [v] = [k]\,[g]^p [h]^q$$

$$\Rightarrow LT^{-1} = M^0 L^0 T^0 (LT^{-2})^p (L)^q$$

$$\Rightarrow LT^{-1} = M^0 L^0 T^0 L^p T^{-2p} L^q$$

$$\Rightarrow LT^{-1} = L^{p+q} T^{-2p}$$ compare the powers on both sides.

$\therefore p+q = 1$ ; $-2p = -1$

$\Rightarrow q = 1-p$ $\qquad \Rightarrow p = \frac{1}{2}$

$= 1 - \frac{1}{2} \Rightarrow q = \frac{1}{2}$

(18) Given Relation $p = \frac{At}{B} + \frac{B}{c-x}$.

In this c and x are subtracting this is possible only when c & x are having same dimensions.

$[c] = [x] = L$.

(19) Given $\frac{U_1}{U_2} = \frac{2}{3}$ $\therefore \frac{M_1}{M_2} = \frac{L_1}{L_2} = \frac{T_1}{T_2} = \frac{2}{3}$.

$[\text{Angular momentum}] = ML^2T^{-1}$

$$\therefore \frac{[\text{Angular momentum}]_1}{[\text{Angular momentum}]_2} = \left[\frac{M_1}{M_2}\right]\left[\frac{L_1}{L_2}\right]^2\left[\frac{T_1}{T_2}\right]^{-1}$$

$$= \left[\frac{2}{3}\right]\left[\frac{2}{3}\right]^2\left[\frac{2}{3}\right]^{-1}$$

$$= \left[\cancel{\frac{2}{3}}\right]\left[\frac{4}{9}\right]\left[\cancel{\frac{3}{2}}\right]$$

$$= \frac{4}{9}$$

LTask

SAQ's

① Given $dp$ = change in Pressure $\Rightarrow [dp] = ML^{-1}T^{-2}$

$dx$ = distance $\Rightarrow [dx] = L$

$$\therefore \left[\frac{dp}{dx}\right] = \frac{ML^{-1}T^{-2}}{L} = ML^{-1}T^{-2}L^{-1} = ML^{-2}T^{-2}.$$

② ⓐ $[\text{Pressure}] = ML^{-1}T^{-2}$

ⓑ $[\text{Latent heat}] = M^0L^2T^{-2}$

ⓒ $[\text{universal gravitational constant}] = [G] = M^{-1}L^3T^{-2}$

ⓓ $[\text{surface tension}] = [T] = ML^0T^{-2}$

③ Given that $L_2 = 2L$ ; $m_2 = 2M$ ; $T_2 = 2T$

$\therefore$ The unit of force $F = M_2L_2T_2^{-2}$

$= [2M][2L][2T]^{-2}$

$\Rightarrow \cancel{2^2}ML\cancel{2^2}T^{-2}$

$= MLT^{-2}$

$\therefore$ No change in force.

④ Given that $M_2 = 1$ kg ; $L_2 = 1$ m ; $T_2 = 1$ min $= 60$ sec.

$\therefore$ The unit of Pressure $P = M_2L_2^{-1}T_2^{-2}$

$= 1 \; 1^{-1} \; (60)^{-2}$

$\Rightarrow \frac{1}{(60)^2} = \frac{1}{3600}$ N/m$^2$

⑤ $[\text{Force}] = [F] = ML^{0}T^{-2}$

$[\text{velocity}] = [v] = LT^{-1}$ ; $[s] = L$.

Then the ratio $\left[\frac{Fs}{V^2}\right] = \frac{[F][s]}{[v]^2}$

$$= \frac{MLT^{-2}L}{(LT^{-1})^2} = \frac{M\cancel{L^2}\cancel{T^{-2}}}{\cancel{L^2}\cancel{T^{-2}}} = ML^0T^0$$

(6) (a) $[Force] = MLT^{-2}$ ; $[mass] = M$ ; $[velocity] = LT^{-1}$

$[density] = ML^{-3}$    $[Pressure] = ML^{-1}T^{-2}$

$[volume] = L^3$    $[Energy] = ML^2T^{-2}$

$[Area] = L^2$    $[radius] = L$

$\therefore$ (a) $\frac{mass \times (velocity)^2}{radius} = \frac{M \times (LT^{-1})^2}{L} = \frac{M\cancel{L^2}T^{-2}}{\cancel{L}} = MLT^{-2}$

$= Force$

so 'a' is correct

(b) $\frac{Energy}{Volume} = \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2} = Pressure$. it is correct

(c) $Area \times (velocity)^2 \times density = L^2 \times (LT^{-1})^2 \times ML^{-3}$

$= L^2 \times L^2T^{-2} \times ML^{-3}$

$= MLT^{-2} = Force$ correct

(7) Given that speed $v \propto \sigma^a \rho^b \lambda^c$

$\Rightarrow V = k\sigma^a \rho^b \lambda^c$

$\Rightarrow M^0LT^{-1} = k(MT^{-2})^a(ML^{-3})^b(L)^c$

$\Rightarrow M^0LT^{-1} = k\, M^aT^{-2a}M^bL^{-3b}L^{3c}$

$\Rightarrow M^0LT^{-1} = k\, M^{a+b}L^{3c-3b}T^{-2a}$

$[speed] = M^0LT^{-1}$

$[Surface\ Tension] = ML^0T^{-2}$

$[density] = ML^{-3}$

$[wavelength] = L$

compare powers on both sides we get

$a+b=0$

$\Rightarrow a=-b$

$\Rightarrow b = -\frac{1}{2}$

$3c-3b=1$

$\Rightarrow 3(c)-3(-\frac{1}{2})=1$

$\Rightarrow 3(c+\frac{1}{2})=1$

$-2a=-1 \Rightarrow a=\frac{1}{2}$

$\Rightarrow c+\frac{1}{2}=\frac{1}{3}$

$\Rightarrow c=-\frac{1}{6}$

substitute a, b, c values in v

$$v = k\,\sigma^{1/2}\rho^{-\frac{1}{2}}\lambda^{-\frac{1}{6}}$$

$$\Rightarrow v \propto \frac{\sigma^{\frac{1}{2}}}{\rho^{\frac{1}{2}}\lambda^{\frac{1}{6}}} \Rightarrow v \propto \frac{\sigma^{\frac{1}{2}}}{\rho^{\frac{1}{2}}(\lambda^{\frac{1}{3}})^{\frac{1}{2}}}$$

$$\Rightarrow v \propto \left[\frac{\sigma}{\rho\,\lambda^{\frac{1}{3}}}\right]^{\frac{1}{2}} \Rightarrow v \propto \sqrt{\frac{\sigma}{\rho\,\lambda^{\frac{1}{3}}}}$$

(8) Given force, length and time are choosen as fundamental quantities. $[Force] = MLT^{-2}$

Do option verification. $[length] = L$ ; $[time] = T$

(a) $FLT^{-2} = MLT^{-2}LT^{-2} = ML^2T^{-4}$ It is not dimension for mass

(b) $FL^{-1}T^2 = M\cancel{L}\cancel{T^{-2}}\cancel{L^{-1}}\cancel{T^2} = M$ which is dimensions of mass so answere (b) correct.

(10) $[F] = MLT^{-2}$ ; $[mass] = M$ ; $[time] = T$

$\therefore$ $[FM^{-1}T] = [F][M]^{-1}[T]$

$$\Rightarrow ML\cancel{T^{-2}}\cancel{M^{-1}}\cancel{T}$$

$= LT^{-1}$ = ~~vel~~ dimensions of velocity.

(16) Given velocity $\propto g^a \lambda^b \rho^c$ $[velocity] = LT^{-1}M^0$

$\therefore$ By using dimensional analysis [acceleration due to gravity] $= LT^{-2}$

$v = k\,g^a\lambda^b\rho^c$

$[v] = [k][g]^a[\lambda]^b[\rho]^c$ $[wavelength] = L$

$[density] = ML^{-3}$

$\Rightarrow M^0LT^{-1} = M^0L^0T^0(LT^{-2})^a(L)^b(ML^{-3})^c$ $[k] = M^0L^0T^0$

$\Rightarrow M^0LT^{-1} = M^0L^0T^0L^aT^{-2a}L^bM^cL^{-3c}$

$\Rightarrow M^0LT^{-1} = M^cL^{a+b-3c}T^{-2a}$ compare the Powers on both sides

$\Rightarrow c = 0$ ; $a+b-3c = 1$ $-2a = -1 \Rightarrow a = \frac{1}{2} \rightarrow$ Ans for (i)

Ans for (iii) $\Rightarrow \cancel{a}+\cancel{b}-\cancel{3c}$ $\frac{1}{2}+b-3(0) = 1$

$\Rightarrow \frac{1}{2}+b = 1 \Rightarrow b = 1-\frac{1}{2} \Rightarrow b = \frac{1}{2} \rightarrow$ Ans for (ii)

(17) Given unit of force $F = 1000N$ Pressure $= 40\ Pa$

$\Rightarrow MLT^{-2} = 1000N \rightarrow (1)$ ; $ML^{-1}T^{-2} = 40\ Pa \rightarrow (2)$

By doing $\frac{(1)}{(2)}$ we get $\frac{MLT^{-2}}{ML^{-1}T^{-2}} = \frac{1000}{40} \Rightarrow \frac{L}{L^{-1}} = 25$

$\Rightarrow L^2 = 25 \Rightarrow L = \sqrt{25}$

$\Rightarrow L = 5$ units

(18) Given $x = F\sqrt{d}$ $[Force] = MLT^{-2}$

$[density] = ML^{-3}$

$\therefore [x] = [F][d]^{1/2}$

$= MLT^{-2}[ML^{-3}]^{1/2}$

$\Rightarrow MLT^{-2}M^{\frac{1}{2}}L^{\frac{-3}{2}}$

$[x] = M^{1+\frac{1}{2}}L^{1-\frac{3}{2}}T^{-2}$

$= M^{\frac{3}{2}}L^{-\frac{1}{2}}T^{-2}$

(19) Given Relation in Time Period $\propto p^a d^b E^c$

$[Time\ Period] = T$ ; $[Pressure] = [p] = ML^{-1}T^{-2}$

$[density] = [d] = ML^{-3}$ ; $[Energy] = [E] = ML^2T^{-2}$

$\therefore$ By dimensional analysis

$[T] \propto [p]^a [d]^b [E]^c$

$\Rightarrow M^0L^0T \propto (ML^{-1}T^{-2})^a(ML^{-3})^b(ML^2T^{-2})^c$

$\Rightarrow M^0L^0T^1 \propto M^aL^{-a}T^{-2a}M^bL^{-3b}M^cL^{2c}T^{-2c}$

$\Rightarrow M^0L^0T^1 \propto M^{a+b+c}L^{-a-3b+2c}T^{-2a-2c}$

compare Powers on both sides

$a+b+c=0$
$\Rightarrow 3a+3b+3c=0 \rightarrow (1)$ ; $-a-3b+2c=0 \rightarrow (2)$ ; $-2a-2c=1 \rightarrow (3)$

By solving (1) & (2) we get

$$\begin{array}{r} 3a+3b+3c=0 \\ -a-3b+2c=0 \\ \hline 2a+5c=0 \rightarrow (4) \end{array}$$

From (4) & (3)

$$\begin{array}{r} 2a+5c=0 \\ -2a-2c=1 \\ \hline 3c=1 \end{array}$$

$\Rightarrow \boxed{c = 1/3}$

$\therefore -2a - \frac{2}{3} = 1 \Rightarrow -2a = \frac{5}{3}$

$\Rightarrow \boxed{a = -\frac{5}{6}}$

From (1)

$-\frac{5}{6} + b + \frac{1}{3} = 0$

$\Rightarrow \boxed{b = \frac{1}{2}}$