

TRIANGLES

Ψ **Polygon:** A figure having more than two sides is called polygon.

§§ **Types of polygons:**

Ψ **Regular polygon:** If all angles are equal and all sides are equal, then it is called regular polygon.

Ψ **Irregular polygon:** If all angles are not equal and all sides are not equal, then it is called irregular polygon.

Ψ **Convex polygon :** If each angle of a polygon is less than 180° then it is called a Convex polygon.

Ψ **Concave polygon:** If at least one angle of a polygon is more than 180° then it is called a Concave polygon

Ψ **Names of polygon**

Name	Sides	Interior angle
Triangle	3	60°
Quadrilateral	4	90°
Pentagon	5	108°
Hexagon	6	120°
Heptagon	7	128.571°
Octagon	8	135°
Nanogon	9	140°
Decagon	10	144°

§§ **Triangles:**

Triangles are one of the fundamental figures used in Euclidean geometry. A triangle is a simple closed figure with three line segments.

Ψ **Sides:**

The line segments are called sides. namely **AB, BC, CA**

Ψ **Vertex:**

The point, where any two sides of a triangle meet, is called a vertex. namely **A,B,C**

Ψ **Interior angles:**

In $\triangle ABC$ the angles $\angle CAB, \angle ABC, \angle BCA$ or simply $\angle A, \angle B, \angle C$

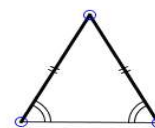
§§ **Classification Of Triangles**

There are two methods of classifying triangles: by the relative lengths of their sides and according to the size of their largest internal angle.

¶¶ **When looking at the lengths of sides:**

In an equilateral triangle all sides are of equal length.

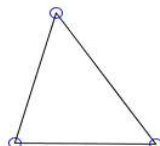
An equilateral triangle is also equiangular, ie, all its internal angles are equal - namely 60° .



In an isosceles triangle two sides are of equal length.

An isosceles triangle also has two equal internal angles that are opposite to the equal sides.

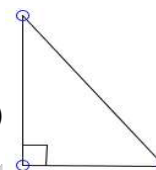
In a scalene triangle all sides have different lengths.



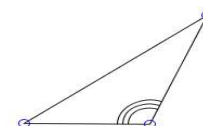
The internal angles in a scalene triangle are all different.

¶¶ When looking at the size of the largest internal angle:

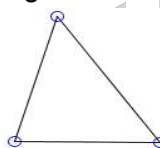
A right-angled triangle has one 90° internal angle (a right angle).



An obtuse triangle has one internal angle larger than 90° (an obtuse angle).



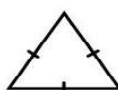
An acute triangle has internal angles that are all smaller than 90° (three acute angles).



Ψ Perimeter : The sum of the sides of a triangle is called the perimeter of the triangle.

Ψ Triangular inequality : In a triangle the sum of any two sides is more than the third side.

Classifying Triangles by their Sides



Equilateral - all sides congruent

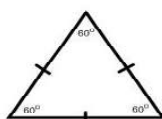


Isosceles - two sides congruent

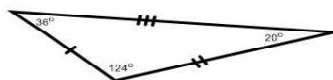


Scalene - no sides are congruent

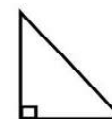
Classifying Triangles by their Angles



Acute - all angles acute



Obtuse - one obtuse angle



Right - one right angle

§§ Triangle Definitions

- ◆ A vertex refers to one of the points of the triangle, where two sides meet.
- ◆ Any one of the sides may be considered as the base of the triangle. The perpendicular distance from a base to the opposite vertex is called an altitude.

- ◆ A perpendicular bisector of a triangle is a straight line passing through the midpoint of a side and being perpendicular to it.
- ◆ The circumcenter is where three perpendicular bisectors meet in a single point; this point is the centre of the circumcircle, the circle passing through all three vertices.
- ◆ The line segment joining the midpoint of a side to the opposite vertex is called a median.
- ◆ The three medians intersecting point is called centroid. This is also the triangle's centre of gravity.
- ◆ The most famous mathematical theorem about triangles is Pythagoras Theorem. This states that, in a right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the other two sides.
- ◆ Two segments are congruent means they have the same length.
- ◆ The bisector of an angle is a ray containing the vertex of the angle and a point in the interior of the angle so that the two angles formed by the ray and the sides of the angle are congruent.
- ◆ Two angles are congruent means they have the same measure.

§§ **Properties of triangles:**

In $\triangle ABC$, the sides opposite to $\angle A, \angle B, \angle C$ are denoted by a, b, c respectively.

That is $a = BC, b = CA, c = AB$

We have,

(i) sum of the angles of a triangle is $180^\circ \therefore \angle A + \angle B + \angle C = 180^\circ$

(ii) The sum of any two sides of a triangle is greater than the third side.

i.e., (1) $a + b > c$ (2) $b + c > a$ (3) $c + a > b$

(iii) The difference of any two sides of a triangle is less than the third side.

i.e., (1) $|a - b| < c$ (2) $|b - c| < a$ (3) $|c - a| < b$

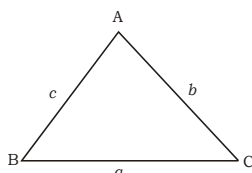
(iv) In a triangle, the side opposite to a bigger angle is bigger. For example, if $\angle B$ is bigger angle, then the side opposite to it is bigger.

(v) If two angles are equal, then their opposite sides are also equal.

Ex : If $\angle A = \angle B$, then $BC = CA$.

(vi) In a triangle, the angle opposite to a bigger side is bigger.

(vii) If two sides in a triangle are equal, then their opposite angles are equal.

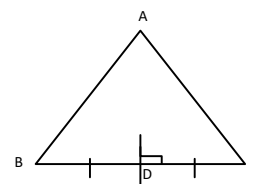
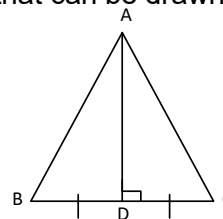


Ψ **Median :** A line segment which joins a vertex of a triangle to the mid point of the opposite side is called median. The number of such line segments that can be drawn in the triangle are three.

Ψ **Altitude:** The perpendicular drawn from any vertex of the triangle to the opposite side or its extension is called altitude. The number of such line segment that can be drawn in the triangle are 3.

Example :

Note: In an obtuse angled triangle, two altitudes fall on the extensions of the sides outside the triangle, and the third altitude falls inside the triangle. In an acute angled triangle all three altitudes lie within the triangle. In a right angled triangle the legs serve as altitudes.

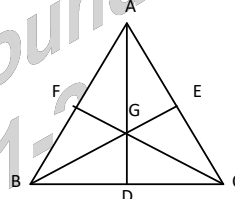


Ψ **Perpendicular bisectors:** The line passing through the mid point of the side and perpendicular to the same side is called perpendicular bisector. The number of such lines that can be drawn in the triangle are 3.

Ψ **Angular bisector:** An angular bisector of triangle is the line segment which divides any angle into two equal halves.

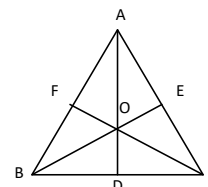
Ψ **Concurrent lines :** Three or more lines passing through the same point are called concurrent lines. That common point is called point of concurrence.

Ψ **Centroid :** The point of concurrence of the medians of a triangle is called centroid. It is denoted by 'G'

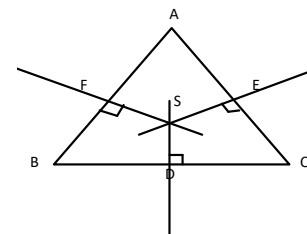


Note : 'G' divides AD in the ratio 2 : 1.

Ψ **Orthocentre :** The point of concurrence of the altitude of a triangle is called orthocenter. It is denoted by 'O' or 'H'.

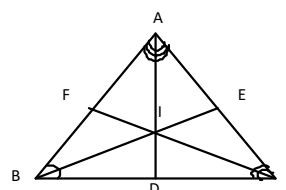


Ψ **Circumcentre :** The point of concurrence of perpendicular bisectors of the sides of a triangle is called circumcentre. It is denoted by 'S'.

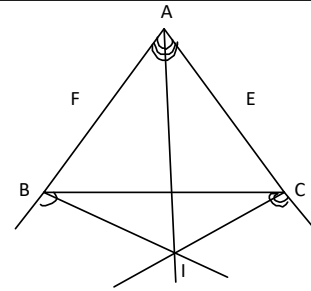


Ψ **Incenter :** The point of concurrent of internal angular bisectors of a triangle is called incenter.

It is denoted by I.



Ψ Excentre : The point of concurrence of internal bisector on one angle and the external bisectors of other two angles is called excentre and is denoted by I



TEACHING TASK

MCQ'S with single correct answers.

1. A closed polygon formed by three line segments is
 A) Rectangle B) Triangle C) Pentagon D) Hexagon
2. The no. of angles in a triangle is
 A) 4 B) 5 C) 3 D) 6
3. A Triangle having _____ no. of parts
 A) 2 B) 3 C) 4 D) 6
4. The sum of angles in a triangle is
 A) 120° B) 360° C) 180° D) 150°
5. If three sides of a triangle are equal then it is a _____ triangle
 A) Equilateral B) Isosceles C) scalene D) none
6. The sum of the three angles of a triangle is _____ right angles
 A) $\frac{1}{2}$ B) 1 C) 2 D) 4
7. A Triangle may contain ____
 A) Two obtuse angles B) Two right angles
 C) An obtuse angle and a right angle D) A right angle and two acute angles
8. In a right triangle the sum of the two acute angles is
 A) 180° B) 90° C) 60° D) >90°
9. In an isosceles triangle the vertical angle is 70° then each of the base angles is
 A) 35° B) 70° C) 55° D) 50°
10. In an isosceles triangle each of the base angles is x°. the vertical angles is
 A) (90 - x)° B) (90 - 2x)° C) (180 - x)° D) (180 - 2x)°
11. In $\triangle ABC$, $\angle A = 90^\circ$. The hypotenuse is _____
 A) AB B) BC C) AC D) none of these
12. The exterior angle of an equilateral triangle is

- A) 60° B) 90° C) 120° D) 150°
13. In $\triangle ABC$, BC is produced to D. If $\angle ACD = 130^\circ$ and $\angle A = 50^\circ$, then $\angle B =$
 A) 65° B) 70° C) 80° D) 90°
14. In $\triangle ABC$, $\angle A = 60^\circ$ and $AB = AC$. then $\triangle ABC$ is _____ triangle
 A) Equilateral B) Isosceles C) Obtuse D) Right
15. The point of concurrence of medians is called _____
 A) centroid B) orthocenter C) circumcenter D) incenter
16. Point of concurrence of altitudes is called _____
 A) centroid B) circumcenter C) orthocenter D) incenter
17. If one of the angles of a triangle is equal to the sum of the other two angles, then the angles are...
 A) $80^\circ, 50^\circ, 50^\circ$ B) $90^\circ, 45^\circ, 45^\circ$ C) $60^\circ, 60^\circ, 60^\circ$ D) $90^\circ, 60^\circ, 30^\circ$

MCQs with multi correct answers.

◆ This section contains multiple choice questions. Each question has 4 choices (A), (B), (C), (D), out of which ONE or MORE is correct. Choose the correct options

18. Sum of the three angles of a triangle is
 A) 180° B) 2 right angles C) 360° D) 90°
19. Sum of the exterior angles of an equilateral triangle is
 A) 60° B) 120° C) 360° D) one complete angle
20. If a, b, c are three sides of a $\triangle ABC$ and $a + b = 11\text{cm}$, $b + c = 13\text{cm}$ and $c + a = 12\text{cm}$ then its perimeter is
 A) 36 B) 18 C) Half of 36 D) none

Assertion and Reasoning type questions :

◆ This section contains certain number of questions. Each question contains Statement – 1 (Assertion) and Statement – 2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct Choose the correct option.

- A) Both A and R are correct and R is correct explanation of A
 B) Both A and R are correct and R is not correct explanation of n
 C) A is correct and R is incorrect D) A is incorrect and R is correct
21. **A:** In Triangle ABC, if $\angle A = 40^\circ$ and $\angle B = 40^\circ$ then $AC = BC$
R : In a triangle, the sides opposite to equal angles are equal.

Comprehension type.

◆ This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which ONLY ONE is correct. Choose the correct option.

In triangle ABC, $\angle B = 2\angle C$ and D is a point on BC such that AD bisects $\angle BAC$ and $AC = BC$

22. $\angle BAC =$
 A) 36° B) 72° C) 108° D) 64°
23. $\angle ADC =$
 A) 36° B) 72° C) 108° D) 54°
24. $\angle ABC =$
 A) 36° B) 72° C) 108° D) 64°

Matching

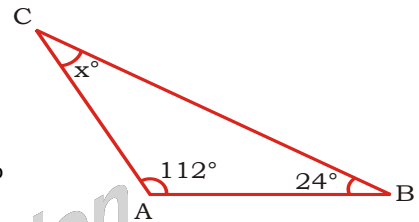
25. In the following $\triangle ABC$, $\angle A = 112^\circ$, $\angle B = 24^\circ$,

Column-I

- a) $x^\circ = -$
 b) $\angle A + \angle C =$
 c) $\angle C - \angle B =$
 d) $2\angle A - (\angle B + \angle C) =$

Column-II

- p) 156°
 q) $4(x - 5)^\circ$
 r) 44°
 s) 20°
 t) 136°

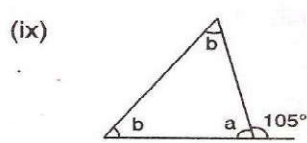
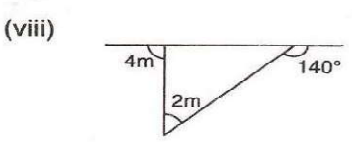
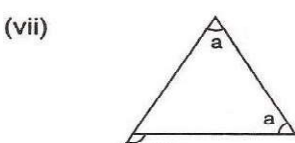
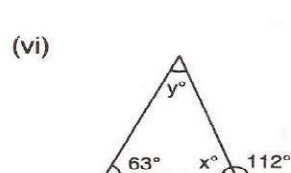
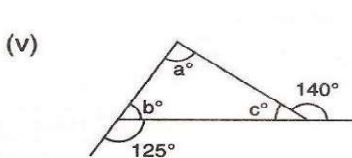
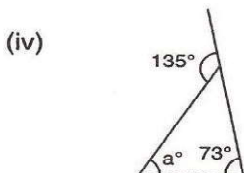
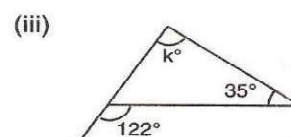
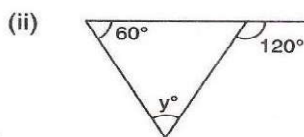
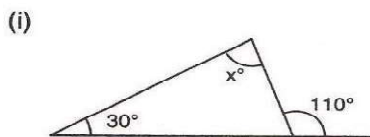


- A) a-r, b-p, c-s, d-p
 C) a-r, b-s, c-p, d-p

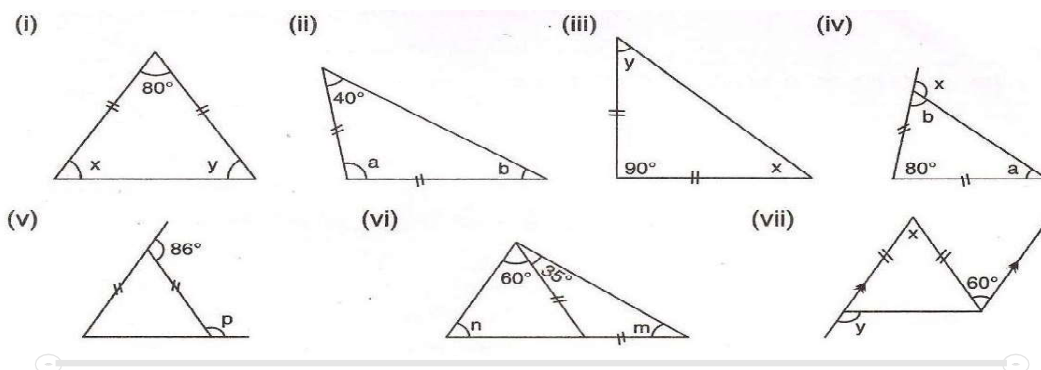
- B) a-r, b-p, c-s, d-t
 D) a-t, b-p, c-s, d-p

Solve the following

26. Three angles of triangle are $(2x+20)^\circ$, $(x+30)^\circ$ and $(2x-10)^\circ$ then find the angles.
27. Calculate the angles of a triangle, if they are in the ratio 4:5:6.
28. One angle of triangle is 60° . The other two angles are in the ratio of 5:7. Find two angles.
29. Find the unknown marked angles in the given figures.



30. One base angle of an isosceles triangle is 65° find its angle of vertex?
 31. If one base angle of an isosceles triangle is double of the vertical angle, find all its angles?
 32. Find the unknown angles in the given figures.



LEARNER'S TASK

◆ ◆ ◆ **BEGINNERS (Level - I)** ◆ ◆ ◆

- I. **MCQs with single correct answer type.**
1. Which of the following is the set of measures of the sides of a triangle
 A) 8cm, 4cm, 20cm B) 9cm, 17cm, 25cm
 C) 11cm, 16cm, 28cm D) None of these
 2. In which of the following cases a right triangle cannot be constructed
 A) 12cm, 5cm, 13cm B) 8cm, 6cm, 10cm
 C) 5cm, 9cm, 11cm D) None of these
 3. Each angle in an equilateral is
 A) 180° B) 90° C) 60° D) 360°
 4. In an isosceles triangle one of its angles is equal to 40° . Then the greatest angle is
 A) 60° B) 80° C) 90° D) 70°
 5. What do you call the triangle whose two of its angles are 40° and 70°
 A) Equilateral B) Obtuse C) Isosceles D) Acute
 6. Which of the following cannot be the ratio of angles of a right-angled triangle
 A) 1:2:3 B) 1:1:2 C) 1:3:6 D) 1:5:6
 7. In $\triangle ABC$ and $\triangle PQR$, it is given that $AB=AC$, $\angle C = \angle P$ and $\angle B = \angle Q$ then two angles are
 A) Isosceles but not congruent B) Isosceles and congruent
 C) congruent but not Isosceles D) neither congruent nor congruent
 8. In an isosceles triangle, vertical angle is x° , the base angle is

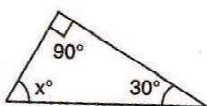
- A) x^0 B) $(90 - x)^0$ C) $(180 - x)^0$ D) $(90 - \frac{x}{2})^0$
9. ABC is a triangle in which BC=10cm and AC=13cm.If AD is the perpendicular bisector of BC, then find the length of AD
 A)12cm B)13cm C)10cm D)5cm
10. In $\triangle ABC$ If $\angle A > \angle B + \angle C$, then it is an _____ triangle
 A)acute B)obtuse C)right D)isosceles
11. The point of concurrence of perpendicular bisectors of sides in a triangle is called
 A) centroid B) circumcenter C) orthocenter D) incenter
12. The point of concurrence of internal angular bisectors in a triangle is called
 A) centroid B) circumcenter C) incenter D) orthocenter

◆ ■ ◆ **ACHIEVERS (Level - II)** ◆ ■ ◆

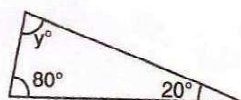
Solve the following

1. State if the triangles are possible with the following angles.
 (i) $20^0, 70^0$ and 90^0 (ii) $40^0, 130^0$ and 20^0
 (iii) $60^0, 60^0$ and 50^0 (iv) $125^0, 40^0$ and 15^0
2. State if the triangles are possible with the given sides of triangle.
 (i)3cm,4cm,5cm (ii)6cm,6cm, 12cm
 (iii)9cm,7cm,6cm (iv)4cm,7cm,6cm
3. If the angles of a triangle are equal, find its angles.
4. Calculate the unknown marked angles in each figure.

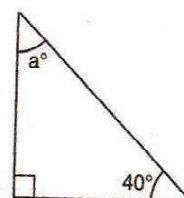
(i)



(ii)

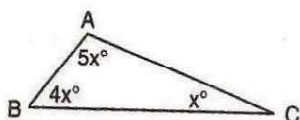


(iii)

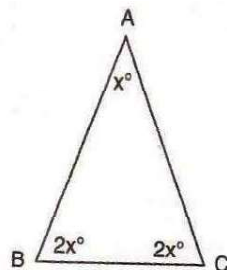


5. Find the value of each angle in the given figures

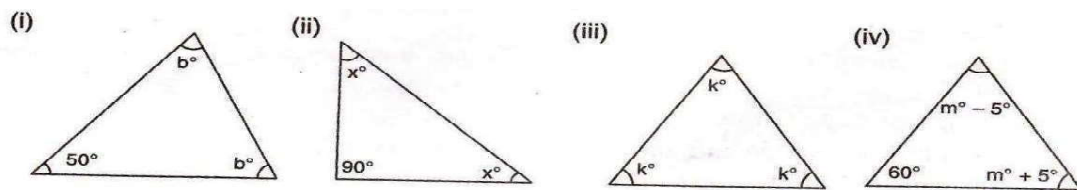
(i)



(ii)

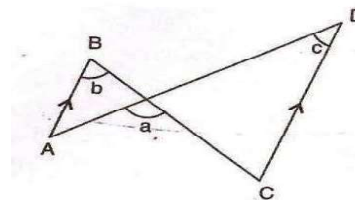


6. Find the unknown marked angles in the given figures



7. In the given figure, show that: $\angle a = \angle b + \angle c$

- i) If $\angle b = 60^\circ$ and $\angle c = 50^\circ$, find $\angle a$
- ii) If $\angle a = 100^\circ$ and $\angle b = 55^\circ$, find $\angle c$
- iii) If $\angle a = 108^\circ$ and $\angle c = 48^\circ$, find $\angle b$



EXPLORERS (Level - III)



MCQ'S with multi correct answer type.

◆ This section contains multiple choice questions. Each question has 4 choices (A), (B), (C), (D), out of which **ONE or MORE** is correct. Choose the correct options

1. A triangle having $90^\circ, 45^\circ, 45^\circ$ angles is called a
 - A) acute angled triangle
 - B) equilateral triangle
 - C) obtuse angled triangle
 - D) right angled isosceles triangle
2. If $AB = 6\text{cm}, BC = 8\text{cm}, AC = 10\text{cm}$ are the measures of the sides of the triangle ABC, then the largest angle is
 - A) $\angle A$
 - B) $\angle B$
 - C) $\angle C$
 - D) both $\angle A$ & $\angle B$
3. If in $\triangle ABC, \angle A = 60^\circ, \angle B = 100^\circ, \angle C = 20^\circ$, then the smallest side is
 - A) BC
 - B) CA
 - C) AB
 - D) None
4. If one of the angles of a \triangle is equal to the sum of the other two angles, then the angles are
 - A) $80^\circ, 50^\circ, 50^\circ$
 - B) $90^\circ, 45^\circ, 45^\circ$
 - C) $60^\circ, 60^\circ, 60^\circ$
 - D) $100^\circ, 20^\circ, 80^\circ$

Assertion and Reasoning type questions :

◆ This section contains certain number of questions. Each question contains Statement – 1 (Assertion) and Statement – 2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct Choose the correct option.

- A) Both A and R are correct
 - B) Both A and R are False
 - C) A is correct and R is incorrect
 - D) A is incorrect and R is correct
5. **A :** In $\triangle XYZ$, if $\angle X = 60^\circ, \angle Y = 80^\circ$, then smallest side is XY
R : In a triangle, the side opposite to greatest angle is smallest side.

Comprehension Type:

◆ This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct. Choose the correct option.

In a triangle ABC, the angles are $\angle A = 2x - 10$, $\angle B = x - 10$, $\angle C = x$

6. Which of the following is a right angle?
 A) $\angle A$ B) $\angle B$ C) $\angle C$ D) both 1 & 2
7. Which angles are acute?
 A) $\angle A$, $\angle C$ B) $\angle B$, $\angle A$ C) $\angle C$, $\angle B$ D) both 2 & 3
8. Which of the following is the largest side?
 A) \overline{AB} B) \overline{BC} C) \overline{CA} D) both 1 & 2

Matching

◆ This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column-I** have to be matched with statements (p, q, r, s) in **Column-II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A-p, A-s, B-r, B-r, C-p, C-q and D-s, then the correct bubbled 4*4 matrix should be as follows:

- | COLUMN-I | COLUMN-II |
|---------------------------|---------------------------|
| 9. i) 8cm, 3cm, 5cm | a) Right angled triangle |
| ii) 6cm, 6cm, 6cm | b) Scalane triangle |
| iii) 8cm, 7cm, 8cm | c) Equilateral triangle |
| iv) 3cm, 4cm, 5cm | d) Isosceles triangle |
| A) i-a, ii-d, iii-b, iv-c | B) i-b, ii-c, iii-d, iv-a |
| C) i-c, ii-a, iii-b, iv-d | D) i-a, ii-b, iii-c, iv-d |



ΦΦ TEACHING TASK :

- 1.B 2.C 3.D 4.C 5.A 6.C 7.D 8.B 9.C 10.D 11.B 12.C
 13.C 14.A 15.A 16.C 17.B 18.A,B 19.C,D 20.B,C 21.A 22.B
 23.C 24.B 25.A 26) $76^\circ, 58^\circ, 46^\circ$ 27) $48^\circ, 60^\circ, 72^\circ$ 28) $50^\circ, 70^\circ$
 29) i) 80° ii) 60° iii) 95° iv) 62° v) $85^\circ, 55^\circ$ vi) $68^\circ, 49^\circ$ viii) 25° ix) $75^\circ, 52.5^\circ$
 30) 50° 31) $90^\circ, 45^\circ, 45^\circ$ 32) i) $50^\circ, 50^\circ$ ii) $100^\circ, 40^\circ$ iii) $45^\circ, 45^\circ$
 iv) $50^\circ, 50^\circ, 100^\circ$ v) 137° vi) $35^\circ, 50^\circ$ vii) $60^\circ, 120^\circ$

ΦΦ LEARNER'S TASK :

□ BEGINNERS :

- 1.B 2.C 3.C 4.D 5.D 6.D 7.B 8.C 9.A 10.B 11.B 12.C

□ ACHIEVERS :

- 1) i, iii, iv 2) i, iii, iv 3) $60^\circ, 60^\circ, 60^\circ$ 4) i) 60° ii) 80° iii) 50°
 5) i) $90^\circ, 72^\circ, 18^\circ$ ii) $36^\circ, 72^\circ, 72^\circ$ 6) i) 65° ii) 45° iii) 60° iv) $60^\circ, 65^\circ, 55^\circ$
 7) i) 1100 ii) 450 iii) 600

□ EXPLORERS :

- 1.D 2.B 3.C 4.B 5.C 6.A 7.C 8.B 9.B

§§ Construction of Triangles

Generally, triangles are constructed under the following cases.

- (i) When the lengths of three sides are given
- (ii) When the lengths of two sides and the included angle are given.
- (iii) When two angles and the included side is given

Ψ For an isosceles triangle:

- (iv) When base and one of the base angles are given
- (v) When one of the equal sides and the vertex angle are given

Ψ For an equilateral triangle:

- (vi) The equal side of triangle is given

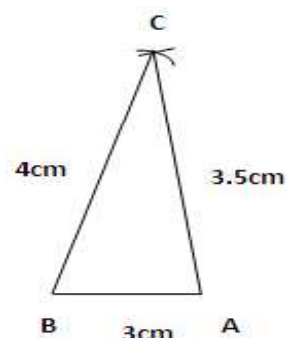
Ψ For a right triangle:

- (vii) When the lengths of the sides, containing the right angle, are given
- (viii) When the lengths of one side and the hypotenuse are given.

√ Example1:

Construct $\triangle ABC$, such that $AB=3\text{cm}$, $BC=4\text{cm}$, $CA=3.5\text{cm}$.

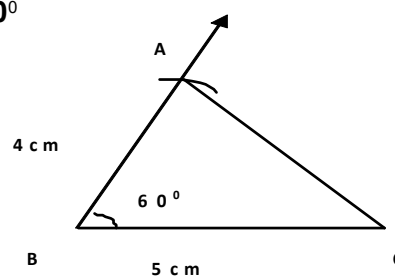
- Steps:**
1. Draw a line segment $AB=3\text{cm}$.
 2. With A as centre, draw an arc of radius 3.5cm and with B as centre draw another arc with radius 4cm .
Let these arcs meet at C.
 3. Join BC and AC.
Then, triangle ABC so obtained is the required triangle.



√ Example2:

Construct $\triangle ABC$, such that $AB=4\text{cm}$, $BC=5\text{cm}$, $\angle B=60^\circ$

- Steps:**
1. Draw a line segment $BC=5\text{cm}$.
 2. At B, draw an angle $\angle PBC=60^\circ$
 3. With B as centre, draw an arc of 4cm radius, which cuts PB at A. Join AC.



LEARNER'S TASK

Solve the following

1. Construct $\triangle ABC$ such that
 - a) $AB=6\text{cm}$, $BC=4\text{cm}$ and $CA=5.5\text{cm}$
 - b) $CB=6.5\text{cm}$, $CA=4.2\text{cm}$ and $BA=5.1\text{cm}$

- c)BC=4cm,AC=5cm and AB=3.5cm
2. Construct $\triangle ABC$ such that
 a)AB=7cm,BC=5cm and $\angle B=60^\circ$ b)CB=6cm,CA=5.7cm and $\angle C=75^\circ$
 c)AB=6.5cm,AC=5.8cm and $\angle A=45^\circ$
3. Construct $\triangle PQR$ such that
 a)PQ=7cm, $\angle Q=60^\circ$ and $\angle P=45^\circ$. Measure $\angle R$
 b)QR=4.4cm, $\angle R=30^\circ$ and $\angle Q=75^\circ$. Measure PQ, PR
 c)PR=5.8cm, $\angle P=60^\circ$ and $\angle R=45^\circ$. Measure $\angle Q$ and verify it by calculations
4. Construct an isosceles $\triangle ABC$ such that
 a)base BC=4cm and base angle= 30°
 b)base AB=6.2cm and base angle= 45°
 c)base AC=5cm and base angle= 75° . Measure other two angles
5. Construct an equilateral $\triangle ABC$ such that
 a)AB=5cm. Draw the perpendicular bisectors of BC and AC. Let P be the point of intersection of these two bisectors. Measure PA, PB, PC
 b)Each side is 6cm.
6. Construct a right angle triangle
 a)AB=4.5cm, AC=3.5cm and $\angle A=90^\circ$
 b)c=4cm, a=5cm (hypotenuse) and $\angle A=90^\circ$

§§ **THE PYTHAGOREAN THEOREM**

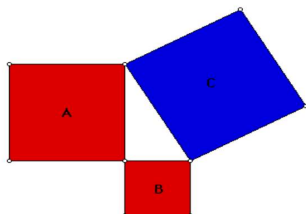
The Pythagorean Theorem was one of the earliest theorems known to ancient civilizations. This famous theorem is named for the Greek mathematician and philosopher, Pythagoras. Pythagoras founded the Pythagorean School of Mathematics in Cortona, a Greek seaport in Southern Italy. He is credited with many contributions to mathematics although some of them may have actually been the work of his students.

The Pythagorean Theorem is Pythagoras' most famous mathematical contribution. According to legend, Pythagoras was so happy when he discovered the theorem that he offered a sacrifice of oxen. The later discovery that the square root of 2 is irrational and therefore, cannot be expressed as a ratio of two integers, greatly troubled Pythagoras and his followers. They were devout in their belief that any two lengths were integral multiples of some unit length. Many attempts were made to suppress the knowledge that the square root of 2 is irrational. It is even said that the man who divulged the secret was drowned at sea.

The Pythagorean Theorem is a statement about triangles containing a right angle.

The Pythagorean Theorem states that:

"The area of the square built upon the hypotenuse of a right triangle is equal to the sum of the areas of the squares upon the remaining sides."

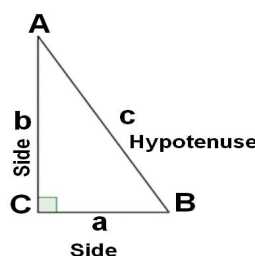


According to the Pythagorean Theorem, the sum of the areas of the two red squares, squares A and B, is equal to the area of the blue square, square C.

Area of square A = a^2 , Area of square B = b^2 , Area of Square C = c^2

Thus, the Pythagorean Theorem stated algebraically is: $a^2 + b^2 = c^2$

For a right triangle with sides of lengths a, b, and c, where c is the length of the hypotenuse.



Pythagoras studied right triangles, and the relationships between the legs and the hypotenuse of a right triangle, before deriving his theory.

Therefore, the square on c is equal to the sum of the squares on a and b.

§§ **The Pythagorean Problem:**

Find all right triangles whose sides are of integral length, thus finding all solutions in the positive integers of the Pythagorean equation: $x^2 + y^2 = z^2$

The three integers (x, y, z) that satisfy this equation is called a Pythagorean triple.

Some Pythagorean Triples: If a, b, c are three sides of a right angled triangle, then

a	3	5	15	7	21	35	9	45	63
b	4	12	8	24	20	12	40	28	16
c	5	13	17	25	29	37	41	53	65
a	11	33	55	77	99	13	39	65	91
b	60	56	48	36	20	84	80	72	60
c	61	65	73	85	101	85	89	97	109

Euclid's formula is a fundamental formula for generating Pythagorean triples given an arbitrary pair of integers m and n with $m > n > 0$. The formula states that the integers

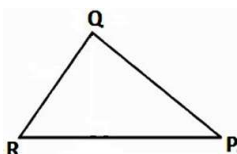
$$a = m^2 - n^2, b = 2mn, c = m^2 + n^2$$

If $n=1, a = m^2 - 1, b = 2m, c = m^2 + 1.$

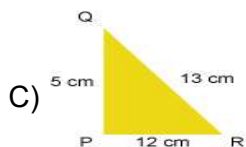
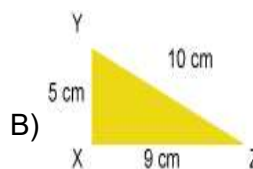
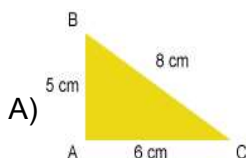
TEACHING TASK

I. MCQ's with single correct answers

1. What is the length of the hypotenuse, if two sides of right angled triangle are 15 m and 8 m?
 A) 16 m B) 17 m C) 18 m D) 19 m
2. Find the length of the third side, if base is 16 cm and hypotenuse are 20 cm ?
 A) 22m B) 12 m C) 20 m D) 8 m
3. Find the base if hypotenuse is 10m and height are 8 m?
 A) 6m B) 10 m C) 12 m D) 20 m
4. Triveni is decorating a ballroom ceiling with garland. If the rectangular ceiling is 15 metres by 8 metres, how much garland will Carey need to reach from one corner of the ceiling to the opposite corner?
 A) 10 m B) 12 m C) 17 m D) 15 m
5. In $\triangle PQR$, $PQ=6\sqrt{3}$ cm, $PR = 12$ cm and $QR = 6$ cm. The angle Q is:___

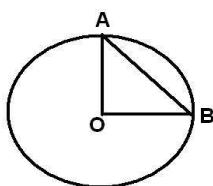


- A) 120° B) 60° C) 90° D) 45°
6. Which of the following triangles is right-angled?



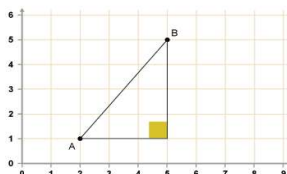
D) All of these

7. A right angled triangle inscribed in a circle, OA and OB are radii and $OA = OB = 4$ cm. Find length of AB?



- A) $2\sqrt{16}$ B) $2\sqrt{8}$ C) $4\sqrt{2}$ D) $2\sqrt{2}$

8. Find the value of AB, from the given figure.



- A) 3 units B) 4 units C) 5 units D) 7units
9. Which of the following is not a pythagorean triplet?
 A) (16, 63, 65) B) (22, 120, 122) C) (14, 48, 50) D) (8,16, 17)
10. Which of the following is a pythagorean triplet?
 A) (5, 8, 10) B) (8, 16, 17) C) (10, 24, 26) D) (1, 2, 3)
11. (5, 12, 13) is a _____
 A) Pythagoren Triplet B) Relatively Primes
 C) Primes D) Both (A) and (B)

MCQ'S with more than one correct answers

◆ This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D), out of which **ONE or MORE** is correct. Choose the correct options

12. Which of the following given lengths can be sides of a right triangle?
 A) 6, 8, 10 B) 4, 4, 8
 C) 10, 24, 26 D. All of these
13. Brad is about to ride a straight water slide. The launching platform is at the top of a tower that is 12 m tall. The splash pool at the end of the slide is 16 m from the base of the tower. How long is the water slide itself?
 A) 20 m B) 4 X 5 m C) 2 X 10 cm D) 30 m
14. A right triangle has a hypotenuse of length 17. If one side has a length of 15, what is the length of the third side?
 A) 2^3 B) 5 C) 8 D) 4×2
15. Determine if the lengths represent the sides of an acute, right, or obtuse triangle if a triangle is possible.
 A) 7, 24, 25 B) 5, 12, 13 C) 6, 8, 9 D) 3, 4, 5

Assertion and Reasoning

◆ This section contains certain number of questions. Each question contains Statement – 1 (Assertion) and Statement – 2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct Choose the correct option.

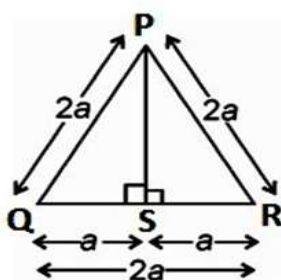
- A) Both A and R are correct
 B) Both A and R are False
 C) A is correct and R is incorrect D) A is incorrect and R is correct

16. A: In a right angle triangle, the sides are 13 cm, 84 cm, 85 cm
 R: a fundamental formula for generating Pythagorean triples given an arbitrary pair of integers m and n with $m > 0$. The formula states that the integers $a = m^2 - 1, b = 2m, c = m^2 + 1$

Comprehension Type

◆ This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct. Choose the correct option.

PQR is an equilateral triangle of side $2a$.



17. Find its altitude(Height) of triangle PQR
 A) \sqrt{a} B) $a\sqrt{3}$ C) $a\sqrt{5}$ D) $a\sqrt{2}$
18. The sides of triangle PSR are
 A) $(a, 2a, a\sqrt{3})$ B) $(a, 2a, \sqrt{3})$ C) $(a, 2a, a\sqrt{2})$ D) $(a, 2a, \sqrt{2})$
19. If $a=2$ cm, then perimeter of triangle PQR _____ cm,
 A) 10 cm B) 12 cm C) 15 cm D) 20 cm

Match the following

◆ This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column-I** have to be matched with statements (p, q, r, s) in **Column-II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A-p, A-s, B-r, B-r; C-p, C-q and D-s, then the correct bubbled 4×4 matrix should be as follows:

Find the third pythagorean triplet

20. Column - I

- i) 10, 24
- ii) 80, 82
- iii) 5, 13
- iv) 8, 15

- A) i-a, ii-b, iii-c, iv-d
- C) i-c, ii-b, iii-a, iv-d

Column - II

- a) 17
- b) 12
- c) 18
- d) 26
- e) 15
- B) i-a, ii-b, iii-e, iv-d
- D) i-d, ii-c, iii-b, iv-a

Solve the following

21. A mouse has made holes in opposite corners of a rectangular kitchen. The kitchen is 3 metres by 4 metres. If the mouse runs straight from one hole to the other, how far will it run?
22. The formula for finding one side of a right triangle when the other two sides are known is $a^2 + b^2 = c^2$, where a and b represent the lengths of the legs of the triangle and c represents the length of the hypotenuse. Suppose the hypotenuse of a right triangle measures 15 cm and one

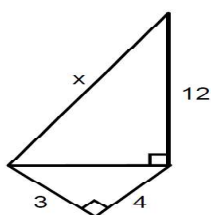
- leg measures 9 cm. What is the length of the other leg?
23. PQR is an isosceles right-angled triangle at point R. Prove that $PQ^2 = 2PR^2$
24. A ladder of 10 m length reaches a window of 8m above the ground. Find the distance of the foot of the ladder from the base of the wall.
25. Each set of measurements below represents the sides lengths of a triangle. Verify which triangles are right angled triangles?
- a) 3 cm, 4 cm, 6 cm b) 7 cm, 24 cm, 25 cm c) 6 cm, 8 cm, 10 cm



◆ ◆ ◆ **BEGINNERS (Level - I)** ◆ ◆ ◆

MCQ's with single correct answers

1. Find the value of x from the given figure?



- A) 169 B) 144 C) 12 D) 13
2. Which of the following is a right angled triangle?
- A) 15, 112, 113 B) 1, 2, 3 c) 7, 8, 9 D) 10, 11, 12
3. In $\triangle ABC$, $\angle B=90^\circ$, $a = 3$ cm $c = 4$ cm, Find the length of the 3rd side ?
- A) 1 cm B) 2 cm C) 3 cm D) 5 cm
4. A room is 6 m long, 5 m wide and 3 m high. Find the distance from the corner of the floor to the opposite corner of the ceiling.
- A) $\sqrt{70}$ B) $\sqrt{80}$ c) $\sqrt{75}$ D) $\sqrt{85}$
5. A sloped mountain road is 13 km long. It covers a horizontal distance of 12 km. What is the change in elevation of the road?
- A) 5 km B) 6 km C) 7 km D) 8km
6. A cat is stranded in a tree. You lean a 10 m ladder against the tree. It is 8 m from the base of the tree. How far up the tree does the ladder reach?
- A) 6 m B) 26 m C) 7 m D) 20 m
7. Which of the following is a pythagoren triplet?
- A) 11, 60, 61 B) 7, 24, 25 C) 18, 80, 82 D) All of these
8. Which of the following is not a pythagoren triplet?
- A) 8, 15, 17 B) 4, 3, 5 C) 6, 8, 10 D) 5, 6, 8

◆ ■ ■ ◆ **ACHIEVERS (Level - II)** ◆ ■ ■ ◆

Solve the following

1. A sailboat has a right angle sail that is 12 ft wide and 18 ft on the hypotenuse. How high is the sail? What is the area of the sail? Round to the nearest tenth.
2. Find the length of the hypotenuse of a right triangle with legs of 3 inches and 5 inches.
3. A triangle has side length of 5 cm, $\sqrt{96}$ cm and 11 cm.
 - a) Is this triangle a right triangle?
 - b) Do these side lengths form a Pythagorean triple? Explain.
4. Jovi is laying a foundation for a garage with dimensions 8 m by 6 m. Jovi measures a diagonal. How long should the diagonal be?

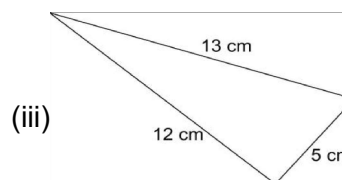
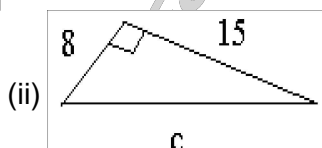
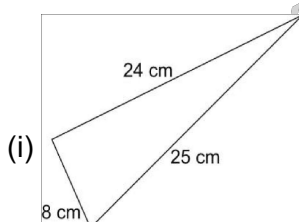
◀ ■ ■ ■ ▶ **EXPLORERS (Level - III)** ▶ ■ ■ ■ ◀

MCQ'S with more than one correct answers

◆ This section contains multiple choice questions. Each question has 4 choices (A), (B), (C),(D), out of which **ONE or MORE** is correct. Choose the correct options

1. Which sets of numbers below are Pythagorean triples?

A) 20, 21, 29 B) 11, 34, 35 C) 20, 101, 99 D) 30, 34, 16
2. Which of the triangles below appears to be a right triangle?



- A) Only (i) B) Only (ii) C) Only (iii) D) All of these
3. Two numbers in a Pythagorean triple are 77 and 85. Find the third number.

A) 2×18 B) 6^2 C) 4×9 D) 12×3

Assertion and Reasoning type questions:

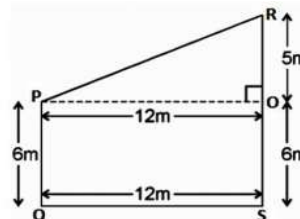
◆ This section contains certain number of questions. Each question contains Statement – 1 (Assertion) and Statement – 2 (Reason). Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct Choose the correct option.

- A) Both A and R are correct
 - B) Both A and R are False
 - C) A is correct and R is incorrect
 - D) A is incorrect and R is correct
4. **A:** The sides of right angled triangle are 3 cm, 4 cm, 5 cm
R: The Pythagorean Theorem stated algebraically is: $a^2 + b^2 = c^2$. If $\triangle ABC$ is a right angled triangle

Comprehension type :

◆ This section contains paragraph. Based upon each paragraph multiple choice questions have to be answered. Each question has 4 choices (A) , (B) ,(C) and (D) out of which **ONLY ONE** is correct. Choose the correct option.

Two planes of heights 6 m and 11m respectively stand on a plane ground. If the distance between the feet of the poles is 12 m,



5. What is the distance between their tops.
 A) 11m B) 12 m C) 10 m D) 13 m
6. The total height of SR is _____
 A) 10m B) 12 m C) 11 m D) 25 m
7. The difference of heights of the planes is _____
 A) 12m B) 6 m C) 5 m D) 15 m

Match the following

◆ This section contains Matrix-Match Type questions. Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in **Column-I** have to be matched with statements (p, q, r, s) in **Column-II**. The answers to these questions have to be appropriately bubbled as illustrated in the following example.

If the correct matches are A-p,A-s,B-r,B-r,C-p,C-q and D-s,then the correct bubbled 4*4 matrix should be as follows:

8. Find the length of the side not given when the hypotenuse is c and the legs are a and b.

Column - I

- i) a= 10, b = 24
- ii) a = 6, c = 10
- iii)b= 18, c = 30
- iv)a= 5, b = 12

- A) i-s, ii-q, iii-r, iv-t
- C) i-s, ii-p, iii-q, iv-r

Column - II

- p) b=8
- q) a=24
- r) c=13
- s) c=26
- t) a=26
- u) b=9

- B) i-q, ii-t, iii-r, iv-b
- D) i-s, ii-q, iii-r, iv-u



ΦΦ TEACHING TASK :

- | | | | | | | | | | | | |
|--------|----------|-----|--------|------|------|-----|------|------|--------|------|--------|
| 1.B | 2-B | 3-A | 4-C | 5-C | 6-C | 7-C | 8-C | 9-A | 10 - C | 11-D | 12.A,C |
| 13.B,C | 14.A,C,D | | 15.B,D | 16.A | 17.B | | 18.A | 19.B | | 20.D | |

ΦΦ LEARNER'S TASK :

- BEGINNERS : 1-D 2.A 3.D 4.A 5.A 6.A 7.D 8.D
- EXPLORERS: 1.C,D 2.C 3.A,B,C 4.A 5.D 6.C 7.C 8.C