RATIONALISATION OF SURDS

Class: . Mathematics

SOLUTIONS

TEACHING TASK

01. $(5+\sqrt{3})K$ = a rational no.

$\Rightarrow K = 5-\sqrt{3}$.

Since $(5+\sqrt{3})(5-\sqrt{3}) = 25-9 = 16$, a rational no.

Ans. A

02. $\frac{5+2\sqrt{2}}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}}$

$= \frac{15+10\sqrt{2}+6\sqrt{2}+8}{9-8} = 23+16\sqrt{2}$

Ans. D

03 $\sqrt{75}+\sqrt{32}$ R.F. $\sqrt{75}-\sqrt{32}$

i.e. $5\sqrt{3}-4\sqrt{2}$

Ans: C

04. $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \sqrt{5}-\sqrt{3}+$

04 $= \dfrac{(1+\sqrt{2})(\sqrt{5}-\sqrt{3}) + (1-\sqrt{2})(\sqrt{5}+\sqrt{3})}{5-3}$ (2)

$= \dfrac{\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}+\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}}{2}$

$= \dfrac{2\sqrt{5}-2\sqrt{6}}{2} = \sqrt{5}-\sqrt{6}$

$= 2.236 - 2.449$

$= -0.213$

Ans: B

05 $A+B = \dfrac{3\sqrt{2}-2}{3\sqrt{2}+2} + \dfrac{3\sqrt{2}+2}{3\sqrt{2}-2}$

$= \dfrac{(3\sqrt{2}-2)^2 + (3\sqrt{2}+2)^2}{(3\sqrt{2})^2 - (2)^2}$

$= \dfrac{2\left[(3\sqrt{2})^2 + (2)^2\right]}{18-4} = \dfrac{2\times 22}{14} = \dfrac{22}{7}$

Ans. D

06 $\dfrac{1}{\sqrt{3}-\sqrt{2}} \times \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \sqrt{3}+\sqrt{2} = 1.414 + 1.732$

$= 3.146$

Ans. B

07 $\dfrac{\sqrt{5}+\sqrt{6}}{\sqrt{5}-\sqrt{6}} \times \dfrac{\sqrt{5}+\sqrt{6}}{\sqrt{5}+\sqrt{6}} = \dfrac{5+6+2\sqrt{30}}{-1}$

$= -11 - 2\sqrt{30} = c + y\sqrt{d}$

$\therefore c = -11, \quad d = 30$

$2 \overline{)54}$
$3 \overline{)27}$
3

Ans: A

08 R.F. of $\sqrt[3]{2} - \sqrt[3]{3}$ is $2^{\frac{2}{3}} + \sqrt[3]{6} + 3^{\frac{2}{3}}$

Since $\left(2^{\frac{1}{3}} - 3^{\frac{1}{3}}\right)\left(2^{\frac{2}{3}} + 2^{\frac{1}{3}}.3^{\frac{1}{3}} + 3^{\frac{2}{3}}\right) = \left(2^{\frac{1}{3}}\right)^3 - \left(3^{\frac{1}{3}}\right)^3$

$= 2 - 3$

$= -1$, a rational no

Ans: A

09. The R.F. of $\sqrt{l^3} - \sqrt{m^5}$ is $-2\left(\sqrt{m^5} + \sqrt{l^3}\right)$

Since $\left(\sqrt{l^3} - \sqrt{m^5}\right) \times -2\left(\sqrt{m^5} + \sqrt{l^3}\right)$

$= -2\left[\left(\sqrt{l^3}\right)^2 - \left(\sqrt{m^5}\right)^2\right]$

$= -2\left[l^3 - m^5\right]$, which is a rational number

Ans. C

10. Statement I: $\dfrac{\sqrt[6]{2}}{\sqrt{3} \times \sqrt[3]{2}} = \dfrac{\sqrt[6]{2}}{\sqrt[6]{27} \cdot \sqrt[6]{4}}$

$= \sqrt[6]{\dfrac{2}{27 \times 4}} = \sqrt[6]{\dfrac{1}{54}}$ (False)

Statement II: $\sqrt[3]{2} \times \sqrt[3]{2} = \left(\sqrt[3]{2}\right)^2$, which is not a rational no. (False)

Ans: B

11. Statement I:

$\left(a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}}\right)\left(a^{\frac{2}{3}} + b^{\frac{2}{3}} + c^{\frac{2}{3}} - a^{\frac{1}{3}}.b^{\frac{1}{3}} - b^{\frac{1}{3}}.c^{\frac{1}{3}} - c^{\frac{1}{3}}.a^{\frac{1}{3}}\right)$

$= \left(a^{\frac{1}{3}}\right)^3 + \left(b^{\frac{1}{3}}\right)^3 + \left(c^{\frac{1}{3}}\right)^3 - 3.a^{\frac{1}{3}}.b^{\frac{1}{3}}.c^{\frac{1}{3}} = a+b+c-$

11. Statement I:

$(a^{\frac{1}{3}}+b^{\frac{1}{3}}+c^{\frac{1}{3}})(a^{\frac{2}{3}}+b^{\frac{2}{3}}+c^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{1}{3}}-b^{\frac{1}{3}}c^{\frac{1}{3}}-c^{\frac{1}{3}}a^{\frac{1}{3}})$ (4)

$= (a^{\frac{1}{3}})^3+(b^{\frac{1}{3}})^3+(c^{\frac{1}{3}})^3 - 3\cdot a^{\frac{1}{3}}\cdot b^{\frac{1}{3}}\cdot c^{\frac{1}{3}}$

$= a+b+c-3\cdot\sqrt[3]{abc}$, which not a rational no.

(False)

Statement II: (Conceptual) False

Ans. B

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12. $\sqrt{2+\sqrt{6}} = \sqrt{x}+\sqrt{y}$

$a=2,\ b=6$

$x = \frac{a+\sqrt{a^2-b}}{2} = \frac{2+\sqrt{2^2-6}}{2} = \frac{2+\sqrt{-2}}{2}$, which is not a real no.

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13. $\sqrt{7+2\sqrt{12}} = \sqrt{7+\sqrt{48}} \Rightarrow a=7,\ b=48$

$l = \frac{a+\sqrt{a^2-b}}{2} = \frac{7+\sqrt{7^2-48}}{2} = \frac{7+1}{2} = 4$

$m = \frac{a-\sqrt{a^2-b}}{2} = \frac{7-\sqrt{7^2-48}}{2} = \frac{7-1}{2} = 3$

Ans: B

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14. $\sqrt{(x+y)^2+(x-y)^2}$

$= \sqrt{2(x^2+y^2)}$

$= \sqrt{2\left[(5+2\sqrt{6})^2+(5-2\sqrt{6})^2\right]}$

$= \sqrt{2\times 2[25+24]} = \sqrt{4\times 49} = 2\times 7 = 14$

15 $x = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$= \frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2} = \frac{5+3+2\sqrt{5}\cdot\sqrt{3}}{5-3}$

$= \frac{8+2\sqrt{15}}{2} = 4+\sqrt{15}$

$\frac{1}{x} = \frac{1}{4+\sqrt{15}} \times \frac{4-\sqrt{15}}{4-\sqrt{15}} = \frac{4-\sqrt{15}}{16-15} = 4-\sqrt{15}$

Now

$x^2+\frac{1}{x^2} = (4+\sqrt{15})^2 + (4-\sqrt{15})^2$

$= 2\left[4^2+(\sqrt{15})^2\right]$

$= 2\left[16+15\right] = 2\times 31 = 62$

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16 a) $x = 2+\sqrt{3} \Rightarrow \frac{1}{x} = 2-\sqrt{3} \Rightarrow x+\frac{1}{x} = 4$

b) $x = 2+\sqrt{5} \Rightarrow \frac{1}{x} = (2-\sqrt{5}) \Rightarrow x-\frac{1}{x} = 2\sqrt{5}$ 4

c) $x = 5+2\sqrt{6} \Rightarrow \frac{1}{x} = 5-2\sqrt{6} \Rightarrow x+\frac{1}{x} = 10$

$\Rightarrow \left(x+\frac{1}{x}\right)^2 = 100$.

d) $x = 6-\sqrt{35} \Rightarrow \frac{1}{x} = 6+\sqrt{35} \Rightarrow x+\frac{1}{x} = 12$

$x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\cdot x\cdot\frac{1}{x}\left(x+\frac{1}{x}\right)$

$= (12)^3 - 3(12) = 1692$

Ans : γ, γ, P, Q

LEARNER'S TASK

CUQ's

01 R.F. of $\sqrt{2}$ is $\sqrt{2}$, since $\sqrt{2}\cdot\sqrt{2}=2$ Ans: A

02 R.F of $3\sqrt{3}$ is $\sqrt{3}$, since $3\sqrt{3}\times\sqrt{3}=9$ Ans: A

03 R.F. of $\sqrt[3]{4}$ is $\sqrt[3]{4^2}$

since $\sqrt[3]{4\times4^2}=\sqrt[3]{4^3}=4$. Ans: B

04 $(\sqrt{5}-\sqrt{6})\times(\sqrt{5}+\sqrt{6}) = 5-6=-1$ Ans: C

05 $(4-\sqrt{5})\times(4+\sqrt{5})=16-5=11$, a rational no.

$(4-\sqrt{5})+(4+\sqrt{5})=8$, a rational no. Ans: C

06 $\sqrt[n]{a^x}\times\sqrt[n]{a^{n-x}}=\sqrt[n]{a^x\times a^{n-x}}=\sqrt[n]{a^n}=a$.

Ans: C

07 $\frac{\sqrt{2}+1}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}=\frac{2+1+2\sqrt{2}}{2-1}=3+2\sqrt{2}$ Ans: A

08 $\frac{\sqrt{x}}{\sqrt{x}-\sqrt{y}}\times\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}}=\frac{x+\sqrt{xy}}{x-y}$ Ans: B

09 $\sqrt[m]{a^n}\times\sqrt[m]{a^{m-n}}=\sqrt[m]{a^n\times a^{m-n}}=\sqrt[m]{a^m}=a$

Ans: B

10 $\sqrt[3]{25}=\sqrt[3]{5^2}\times\sqrt[3]{5}=\sqrt[3]{5^3}=5$.

Ans: C

JEE MAINS LEVEL

01 $4\sqrt{3} \times 2\sqrt{3} = 8 \times 3 = 24$

Ans: C

02 $$\frac{7+3\sqrt{5}}{3+\sqrt{5}} + \frac{7-3\sqrt{5}}{3-\sqrt{5}}$$

$$= \frac{(7+3\sqrt{5})(3-\sqrt{5}) + (7-3\sqrt{5})(3+\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$$

$$= \frac{21 - 7\sqrt{5} + 9\sqrt{5} - 15 + 21 + 7\sqrt{5} - 9\sqrt{5} - 15}{9-5}$$

Ans: D

$$= \frac{12}{4} = 3$$

03 opt: D is wrong statement

Ans: D

04 $$a = \frac{5-\sqrt{21}}{2} \Rightarrow \frac{1}{a} = \frac{2}{5-\sqrt{21}} \times \frac{5+\sqrt{21}}{5+\sqrt{21}}$$

$$= \frac{2(5+\sqrt{21})}{25-21} = \frac{2(5+\sqrt{21})}{4}$$

$$= \frac{5+\sqrt{21}}{2}$$

Now $$a^2 + \frac{1}{a^2} = \left(\frac{5-\sqrt{21}}{2}\right)^2 + \left(\frac{5+\sqrt{21}}{2}\right)^2$$

$$= \frac{2(5^2 + (\sqrt{21})^2)}{4} = \frac{2(25+21)}{4} = 23$$

$$= \frac{2 \times 46}{4} = 23$$

Ans: C

05 $\frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$ (08)

$= \frac{(\sqrt{2}-1)^2}{(\sqrt{2})^2-(1)^2} = \frac{3-2\sqrt{2}}{2-1} = 3-2\sqrt{2} = a+b\sqrt{2}$

here $a=3$, $b=-2$

Ans: B

06 $x+y = \frac{5+\sqrt{6}}{5-\sqrt{6}} + \frac{5-\sqrt{6}}{5+\sqrt{6}}$

$= \frac{(5+\sqrt{6})^2+(5-\sqrt{6})^2}{(5)^2-(\sqrt{6})^2}$

$= \frac{2[5^2+(\sqrt{6})^2]}{25-6}$

$= \frac{2[25+6]}{19} = \frac{62}{19}$

$x-y = \frac{5+\sqrt{6}}{5-\sqrt{6}} - \frac{5-\sqrt{6}}{5+\sqrt{6}}$

$= \frac{(5+\sqrt{6})^2-(5-\sqrt{6})^2}{(5-\sqrt{6})(5+\sqrt{6})}$

$= \frac{4 \cdot 5 \cdot \sqrt{6}}{19}$

$= \frac{20\sqrt{6}}{19}$

Now $(x+y) \div (x-y) = \frac{62}{19} \times \frac{19}{20\sqrt{6}} = \frac{31}{10\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{31\sqrt{6}}{60}$

Ans: D

07. $\frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5} = \frac{2.2360}{5} = 0.4472$ Ans. B

08 $\frac{2\sqrt{5}}{5\sqrt{3}-3\sqrt{5}} \times \frac{(5\sqrt{3}+3\sqrt{5})}{(5\sqrt{3}+3\sqrt{5})} = \frac{10\sqrt{15}+30}{30} = \frac{\sqrt{15}+3}{3}$

Ans: C

09 $\sqrt{6} \times 2\sqrt{6} = 12$

$\sqrt{6} \times -3\sqrt{6} = -18$

$\sqrt{6} \times 4\sqrt{6} = 24$, All are R.F.s

Ans. A, B, C

10. ~~$\sqrt[6]{1331} = \sqrt[6]{3 \times 11^2}$~~

10. $\sqrt[6]{1331} = \sqrt[6]{11^3} \times \sqrt[6]{11^3} = \sqrt[6]{11^6} = 11$ (89)

Hence R.F of $\sqrt[6]{1331}$ itself

Ans: D

11. Statement I:

$(x^3 - \sqrt[3]{y})(x^6 + x^3\sqrt[3]{y} + y^{\frac{2}{3}})$

$= (a-b)(a^2 + ab + b^2) = a^3 - b^3$

$= (x^3)^3 - (\sqrt[3]{y})^3$

$= x^9 - y$ a rational number

(True)

Statement II:

R.F. of $\sqrt{3} - \sqrt{2}$ is $\sqrt{3} + \sqrt{2}$

R.F. of $\sqrt{7} + \sqrt{5}$ is $\sqrt{7} - \sqrt{5}$

sum $= \sqrt{3} + \sqrt{2} + \sqrt{7} - \sqrt{5}$ (True)

Ans: A

12 $\sqrt[11]{12^7 \times 5^{10}} \times \sqrt[11]{12^4 \times 5} = \sqrt[11]{12^{11} \times 5^{11}}$ $= 12 \times 5$
$= 60.$

Ans: D

13. $\frac{10}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$

$= 5 \times 1.414$

$= 7.070$

Integral part = 7

Ans: 7

(10)

a) $\sqrt{75} + \sqrt{72}$

$= 5\sqrt{5} + 6\sqrt{2}$ $\therefore RF = 5\sqrt{5} - 6\sqrt{2}$

b) $\sqrt{98} - \sqrt{50}$

$= 7\sqrt{2} - 5\sqrt{2}$ $\therefore RF =$ ~~$\sqrt{2}$ 50~~ $2\sqrt{2}$

$= 2\sqrt{2}$

c) $\sqrt{192} - \sqrt{162}$

$= 8\sqrt{3} - 9\sqrt{2}$ $\therefore RF = 8\sqrt{3} + 9\sqrt{2}$

d) $\sqrt{175} - \sqrt{245}$

$= 5\sqrt{11} - 7\sqrt{5}$ $R.F = 5\sqrt{11} + 7\sqrt{5}$

$\Rightarrow$ THE END $\in$