

# FUNDAMENTALS OF LOGARITHMS

Class: VII, Mathematics

## SOLUTIONS

(F)

### TEACHING TASK

01.  $\log_4 x = 2 \Leftrightarrow x = 4^2 \Leftrightarrow x = 16$

Ans: A

02.  $\log_3 x = 9 \Leftrightarrow x = 3^9 \Leftrightarrow x = 81$

Ans: D

03.  $\log_2 a = 2 \Rightarrow a = 2^2 = 4$ .

$\therefore \log_4 4 = \log_4 4 = 1$

Ans: B

04. 10

Ans: B

05.  $3^{-2} = \frac{1}{9}$  opt: D  $\log_3 \left(\frac{1}{9}\right) = -2$

~~$x = 2 \log_3 3 =$~~   $\Rightarrow \frac{1}{9} = 3^{-2}$

Ans: D

06.  $\log_{\left(\frac{1}{a}\right)} a = b$

$\Rightarrow \left(\frac{1}{a}\right)^b = a$

Ans: B

07.  $\frac{1}{x} = \left(\frac{1}{y}\right)^{\frac{1}{2}}$

Ans: B

08.  $\log_{10} \frac{1}{1000} = x \Rightarrow \log_{10} 10^{-3} = -3 \times \log_{10} 10 = -3$

Ans: A

09.  $\log_{a^2 - b^2} (a^2 - b^2) = 1$

Ans: A

10.  $\log_{10} 0.001 = x \Rightarrow 0.001 = 10^x \quad \Rightarrow 10^{-4} = 10^x$   
 $\Rightarrow \frac{1}{1000} = 10^x \quad \Rightarrow x = -4$

Ans: D

11. Conceptual.

Ans: A, B, C, D

12.  $x^2 + 9 = 5^2 \Rightarrow x^2 + 9 = 25$

$\Rightarrow x^2 - 16 = 0 \Rightarrow x = \pm 4$

Ans: A, C

13.  $\log_5(x^2+1) = \log_5(x^2+1)$  (2)

option verification:  $x = 2$  or  $-2$

$\therefore \log_5 5 = \log_5 5$  (True)

Ans: B, D

14. Statement I: Conceptual (True)

Statement II: Conceptual (True)

Ans: A

15. Statement I:  $a^N = x \Rightarrow N = \log_a x$  (False)

Statement II: Conceptual (True)

Ans: D

16.  $\log_3(\log_2 x) = 1 \Rightarrow \log_2 x = 3 \Rightarrow x = 2^3 = 8$  Ans: A

17.  $\log_3[\log_3(\log_5 b)] = 0$   
 $\Rightarrow \log_3(\log_5 b) = 3^0 = 1$

$\log_5 b = 3^1 = 3$

$b = 5^3 = 125$

Ans: B

18.  $\log_{10}[\log_2(x^2-3x)] = 0$

$\Rightarrow \log_2(x^2-3x) = 10^0 = 1$

$\Rightarrow x^2-3x = 2^1 = 2$

$x^2-3x-2=0$

$x = \frac{3 \pm \sqrt{9+8}}{2}$

$= \frac{3 \pm \sqrt{17}}{2}$

Ans: B, D

19.  $\log_x \frac{1}{\sqrt[5]{2}} = \frac{-1}{5}$

$\Rightarrow \frac{1}{\sqrt[5]{2}} = x^{-1/5}$   
 $\Rightarrow 2^{-1/5} = x^{-1/5}$

$x = 2$

Ans: 2

20.  $\frac{1}{\sqrt[3]{2}} = x^{-1/3} \Rightarrow 2^{-1/3} = x^{-1/3} \Rightarrow x = 2$

Ans: 2

21) a)  $N = a^x$

$\log_a N = \log_a a + \log_a N$   
 $= 1 + x$

b)  $4 = n^x$

$\log_n 4 = \log_n n^2$   
 $= \log_n n^2 + 1$   
 $= 2x + 1$

c)  $\log_n(x+1)$

$= a + 1$

d)  $\frac{1}{x}$

Ans: s, t, q, r

22. a)  $(a^2)^{\log b}$   
 $= a^{2 \log b}$   
 $= a^{\log b^2}$   
 $= (b^2)^{\log a}$   
 $\Rightarrow b^{2 \log a}$

b)  $(b^2)^{\log a}$   
 $= b^{2 \log a}$   
 $= b^{\log a^2}$   
 $= (a^2)^{\log b}$   
 $= a^{2 \log b}$

c)  $(a^2)^{\log b^2}$   
 $\Rightarrow a^{2 \log b^2}$   
 $= a^{\log b^4}$

d)  $(b^2)^{\log b^2}$   
 $= b^{2 \log b^2}$   
 $= b^{\log b^4}$

Ans: S, T, R, Q

23 a)  $\log_{0.001} 0.01 = \log_{(0.1)^3} (0.1)^2 = \frac{2}{3}$

b)  $\log_{0.01} 0.001 = \log_{(0.1)^2} (0.1)^3 = \frac{3}{2}$

c)  $\log_{0.0001} 0.001 = \log_{(0.1)^4} (0.1)^3 = \frac{3}{4}$

d)  $\log_{0.01} 0.0001 = \log_{(0.1)^2} (0.1)^4 = \frac{4}{2} = 2$

Ans: S, R, Q, P

LEARNERS TASK

01.  $2^4 = 16 \Rightarrow \log_2 16 = 4$       Ans: B
02.  $x = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$       Ans: B
03.  $5^2 = 25 \Rightarrow \log_5 25 = 2$       Ans: B
04.  $\log_{0.01} 0.01 = 1$       Ans: 1
05. Conceptual      Ans: C
06.  $7^{\log_7 6} = \frac{1}{6}$       Ans: B
07. Conceptual      Ans: D
08. Conceptual      Ans: B

09. Conceptual

Ans: C

10. Conceptual

Ans: D

JEE MAIN LEVEL QUESTIONS

01.  $\log \frac{(\sqrt[3]{abc})^3}{(abc)^{2/3}} = \frac{1}{(\frac{2}{3})} \log abc = \frac{3}{2}$

01.  $\log \frac{\sqrt[3]{ab \cdot bc \cdot ca}}{(abc)^{2/3}} = \log \frac{\sqrt[3]{a^2 b^2 c^2}}{(abc)^{2/3}}$   
 $= \log \frac{(abc)^{2/3}}{(abc)^{2/3}} = \frac{(\frac{2}{3})}{(\frac{2}{3})} = 1$

Ans: A

02.  $\log \left( \frac{64 \times 27}{256 \times 729} \right) = \log \left( \frac{1}{27 \times 2^2} \right)$

Ans: B

03.  $a = b^x, b = c^y, c = a^z$

$\Rightarrow \log_a a = \log_a b^x \cdot \log_b c^y \cdot \log_c a^z = xyz$   
 $\Rightarrow 1 = xyz$

Ans: A

04.  $\log_{0.01} 0.0001 = 2$

Ans: A

05.  $\log_{\frac{5}{3}} 5^{1.5} = \frac{1.5}{3} \times \log_5 5$   
 $= \frac{15}{30} = \frac{1}{2}$

Ans: B

06.  $\sqrt[7]{7} = 49^p \quad | \quad 2p = \frac{1}{7}$   
 $\Rightarrow 7^{\frac{1}{7}} = 7^{2p} \quad | \quad \Rightarrow p = \frac{1}{14}$

Ans: B

07.  $x = 10^a$   
 $\frac{10^{3a-2}}{10^2} = \frac{(10^a)^3}{10^2} = \frac{x^3}{100}$

Ans: B

Ans: B

08.  $x = 10^a$

(5)

$$10^{3a-2} = \frac{10^{3a}}{100} = \frac{(10^a)^3}{100} = \frac{x^3}{100}$$

Ans: C

09.  $x = 10^k$

$$10^{2k-3} = \frac{(10^k)^2}{10^3} = \frac{x^2}{1000}$$

Ans: ~~A~~

10.  $\log_{100} m + \log_{100} n = n + m$

$$\Rightarrow \log_{100} mn = m + n$$

Ans: D

11.  $1296 = 6^{x+2}$

$$\Rightarrow 6^4 = 6^{x+2}$$

$$\Rightarrow x = 2$$

$$\Rightarrow (x-2)^n = 10$$

$$\begin{array}{r} 6 \overline{) 1296} \\ \underline{6} \phantom{00} \\ 6 \phantom{00} \\ \underline{6} \phantom{00} \\ 0 \phantom{00} \end{array}$$

12.  $(m+n)^2 = mn \Rightarrow m+n = \pm \sqrt{mn}$

$$\Rightarrow \frac{m+n}{\sqrt{mn}} = \pm 1$$

$$\sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}} = \frac{m+n}{\sqrt{mn}} = \frac{\pm \sqrt{mn}}{\sqrt{mn}} = \pm 1$$

Ans: C

13.  $a^{\log_b \sqrt{x}} = b$

$$\log_a \sqrt{x} = 1$$

$$\Rightarrow b^{\log_a \sqrt{x}} = b^1$$

$$\Rightarrow a = \sqrt{x}$$

Now  $a + \sqrt{x} = \sqrt{x} + \sqrt{x} = 2\sqrt{x}$

$$\Rightarrow a + \sqrt{x} = a + a = 2a$$

Ans: D

14.  $(x^2)^{\log_{y^2} z^2} = x^2$

$$\log_{y^2} z^2 = 1$$

$$\Rightarrow (x^2)^{\log_{y^2} z^2} = x^2$$

$$z^2 = y^2 \Rightarrow z = y$$

$$\begin{aligned} x^2 + y^2 + ay \\ \Rightarrow y^2 + y^2 + y^2 \\ = 3y^2 \end{aligned}$$

Ans: B

15

$$x^y = \left(\frac{y}{x}\right)^{\log_y z}$$

$$\Rightarrow x = x^{y \cdot \log_y z}$$

$$\Rightarrow z = y \cdot \log_y z$$

$$\Rightarrow \frac{z}{y} = \log_y z$$

$$\log_y z^2 = 2 \cdot \log_y z$$

$$= \frac{2z}{y}$$

(6)

Ans: B

### ADVANCED LEVEL

01.  $(a+b)^2 = a+b$

$$\Rightarrow a+b = 1$$

if  $a=b \Rightarrow a+a=1$

$$\Rightarrow a = \frac{1}{2} \text{ \& } b = \frac{1}{2}$$

$$a^3 + b^3 = \frac{1}{8} + \frac{1}{8} = \frac{1}{4} \text{ also } \sqrt[3]{\frac{1}{64}}$$

Ans: B, D

02.  $x^3 + y^3 = 1$

$$\Rightarrow (x+y)(x^2 - xy + y^2) = 1$$

i.e.  $x+y$  \&  $x^2 - xy + y^2$  are reciprocals

Ans: A, C, D

03

$$x = 2^3; \quad y = 3^5$$

$$x = 8, \quad y = 243$$

$$A) x = \frac{y-3}{15} = \frac{243-3}{15}$$

$$= \frac{240}{15} = 16$$

$$A) x = \frac{y-8}{15}$$

$$= \frac{243-8}{15}$$

$$= \frac{235}{15} = \frac{47}{3}$$

Ans: —

04

Statement II: Conceptual True

Ans: —

05 Statement I:

$$\sqrt[5]{a^x} = y$$

$$\log a^{x/5} = \log y$$

$$\Rightarrow a^{x/5} = y$$

$$\Rightarrow \frac{x}{5} \cdot \log a = \log y$$

$$\Rightarrow x \cdot \log a = 5 \log y$$

(True)

Ans: A

Statement II: Conceptual (True)

06

$$\log \sqrt[3]{5} = \log 5^{1/3} = \frac{(\frac{1}{3})}{(\frac{1}{5})} \log 5 = \frac{1}{3}$$

Ans: B

07

$$\log \sqrt[5]{36} = 5 \Rightarrow \log 6^{1/3} = 5$$

$$\Rightarrow \log 36^{1/6} = 5 \Rightarrow \frac{x}{3} = 5 \Rightarrow x = 15$$

Ans: C

08

$$\log_{0.001} 0.01 = \frac{x}{y}$$

$$\Rightarrow \log_{(0.1)^3} (0.1)^2 = \frac{2}{3} = \frac{x}{y} \Rightarrow 2y = 3x$$

Ans: B

09

$$\log_{10} x = p \Rightarrow x = 10^p$$

$$10^{p+1} = 10^p \times 10^1 = 10x$$

Ans: B

10

$$10^{p+3} = 10^3 \times 10^3 = 1000x$$

Ans: B

11.

$$10^{p-2} = \frac{10^p}{10^2} = \frac{x}{100}$$

Ans: B

12.

$$\log x^{1/30} = \frac{1}{30} \log x$$

Ans: B

13.

$$\log_{10} y^{1/7} = 0$$

$$y = 1$$

$$\Rightarrow y^{1/7} = 1$$

Ans: A



14.  $\sqrt{x+y} = 2^0 = 1$   $\left| \right.$   $\sqrt[3]{x-y} = 2$  (8)  
 $x+y = 1 \rightarrow \textcircled{1}$   $\Rightarrow x-y = 8 \rightarrow \textcircled{2}$   
 Solving  $\textcircled{1}$  &  $\textcircled{2}$   $x = \frac{9}{2}$  &  $y = -\frac{7}{2}$

$\therefore \frac{x}{y} = \frac{(\frac{9}{2})}{(-\frac{7}{2})} = -\frac{9}{7}$  Ans:

15.  $x^3 = y^3$  Ans:  $2y^3$   
 Now  $x^2 + y^3 = y^3 + y^3 = 2y^3$

16.  $a^2 = 2^2 = 4$   $\left| \right.$   $\log b^2 = a^2$   $\left| \right.$   $b = \pm 2$   
 $\Rightarrow a = \pm 2$   $\Rightarrow b^2 = 4$

17 a)  $a^m = a^n \Rightarrow m = n$   
 p)  $\log_a a^m = m \log_a a = m = n$  (True)  
 b)  $\log_a a^{\frac{1}{n}} = \frac{1}{n} \Rightarrow \frac{1}{n} = \frac{1}{m} \Rightarrow m = n$  (True)  
 r)  $\log_a a^n = n = m$  (True)  
 b)  $a^{\frac{1}{m}} = a^{\frac{1}{n}} \Rightarrow m = n$  (all above options are true)  
 c)  $m^n = (\frac{1}{n}) = n^{-1} \Rightarrow a \neq \phi$   
 $\log_m (\frac{1}{n}) = \log_m n^{-1} = -\log_m n = \log_m n^{-1} = a$   
 d)  $\log_{(\frac{1}{n})} (\frac{1}{n}) = \log_{(\frac{1}{n})} (\frac{1}{n})^1 = a$  Ans:  
 Ans: a  $\rightarrow$  p, q, r, b  $\rightarrow$  p, q, r, c  $\rightarrow$  t, d  $\rightarrow$  s

18 a)  $\log \sqrt[3]{x} = a \Rightarrow \frac{1}{3} = a \Rightarrow \sqrt{x} = \frac{1}{\sqrt{3}}$   
 b)  $\log \sqrt[4]{x} = a \Rightarrow \frac{1}{4} = a \Rightarrow \sqrt[3]{x} = \sqrt[3]{\frac{1}{4}}$   
 c)  $\log \sqrt[3]{a} = m \Rightarrow m = \frac{1}{3} \Rightarrow m^2 = \frac{1}{9}$   
 d)  $a = \frac{1}{10} \Rightarrow a^2 = \frac{1}{100}$   $\Rightarrow$  THE ENDE  
 Ans: r, t, s, p