

LIMITS OF TRIGONOMETRIC FUNCTIONS

(1)

(F⁺)

Class: IX . Mathematics

SOLUTIONS

TEACHING TASK

01. $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$
 $= \lim_{x \rightarrow 0} \frac{2 \sin^2 x}{x^2} = 2$

Ans: C

02. $\lim_{x \rightarrow \frac{\pi}{6}} \frac{2 - \sqrt{3} \cos x - \sin x}{(6x - \pi)^2} \left(\frac{0}{0}\right)$

By L-Hospital Rule

$= \lim_{x \rightarrow \frac{\pi}{6}} \frac{\sqrt{3} \sin x - \cos x}{2(6x - \pi) \cdot 6} \left(\frac{0}{0}\right)$

$= \lim_{x \rightarrow \frac{\pi}{6}} \frac{\sqrt{3} \cos x + \sin x}{2 \cdot 6 \cdot 6}$

$= \frac{\sqrt{3} \left(\frac{\sqrt{3}}{2}\right) + \frac{1}{2}}{72} = \frac{4}{2 \cdot 72} = \frac{1}{36}$

Ans: D

03. $\lim_{x \rightarrow 0} \frac{\tan 2x - x}{3x - \sin x} \left(\frac{0}{0}\right)$

By L-Hospital Rule

$= \lim_{x \rightarrow 0} \frac{\sec^2 2x \cdot 2 - 1}{3 - \cos x} = \frac{2-1}{3-1} = \frac{1}{2}$

Ans: A

03. General method

(2)

$$\lim_{x \rightarrow 0} \frac{\tan x - x}{3x - \sin x}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{\tan x}{x} - 1}{3 - \frac{\sin x}{x}} = \frac{2-1}{3-1} = \frac{1}{2}$$

Ans: A

04. $\lim_{x \rightarrow 2} \frac{\tan(x-2) [x^2 + (k-2)x - 2k]}{x^2 - 4x + 4} = 5$

$$\lim_{x \rightarrow 2} \frac{\tan(x-2) [x^2 + (k-2)x - 2k]}{(x-2)^2} = 5$$

$$= \lim_{x \rightarrow 2} \frac{x^2 + (k-2)x - 2k}{x-2} = 5$$

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

L-Hospital Rule

$$\Rightarrow \lim_{x \rightarrow 2} \frac{2x + (k-2)}{1} = 5$$

$$4 + k - 2 = 5$$

$$\Rightarrow k = 3$$

Ans: B

05. $\lim_{x \rightarrow 2} \frac{\sqrt{1 - \cos \{2(x-2)\}}}{x-2}$

$$= \lim_{x \rightarrow 2} \frac{\sqrt{2 \sin^2(x-2)}}{x-2}$$

$$= \sqrt{2} \cdot \lim_{x \rightarrow 2} \frac{|\sin(x-2)|}{x-2}$$

$$= \sqrt{2} \cdot \lim_{x \rightarrow 2} \frac{\pm \sin(x-2)}{(x-2)}$$

LHL $x \rightarrow 2^+$
 $x-2 > 0$

$$\sin(x-2) > 0$$

$$\therefore \text{LHL} = \sqrt{2}$$

RHL $x \rightarrow 2^-$
 $x-2 < 0$

$$\sin(x-2) < 0$$

$$\text{RHL} = -\sqrt{2}$$

$$\therefore \text{LHL} \neq \text{RHL}$$

Ans: B

06

We know

$$-1 \leq \sin\left(\frac{2}{x}\right) \leq 1$$

$$-x^2 \leq x^2 \sin\left(\frac{2}{x}\right) \leq x^2$$

$$\lim_{x \rightarrow 0} -x^2 = \lim_{x \rightarrow 0} x^2 = 0$$

$$\text{Hence, } \lim_{x \rightarrow 0} x^2 \sin\left(\frac{2}{x}\right) = 0$$

Ans: B

(3)

07.

$$\lim_{x \rightarrow \infty} \frac{2x^2 - \sin 3x}{x^2 + 10}$$

$$\lim_{x \rightarrow \infty} \frac{2 - \frac{\sin 3x}{x^2}}{1 + \frac{10}{x^2}}$$

$$= \lim_{x \rightarrow \infty} \frac{2 - \frac{1}{x} \cdot \left(\frac{\sin 3x}{x}\right)}{1 + \frac{10}{x^2}}$$

$$= \frac{2 - 0 \times \text{finite value}}{1 + 0} = 2$$

Ans: 2

08

$$\lim_{x \rightarrow 0} \frac{a^{\tan x} - a^{\sin x}}{\tan x - \sin x} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x \rightarrow 0} \frac{a^{\tan x} \cdot \log a \cdot \sec x - a^{\sin x} \cdot \log a \cdot \cos x}{\sec x - \cos x}$$

$$= \lim_{x \rightarrow 0} \frac{a^{\sin x} \left(a^{\tan x - \sin x} - 1 \right)}{(\tan x - \sin x)} = a^{\sin 0} \cdot \log a$$

$$= \log a$$

Ans: A



09.

$$\lim_{x \rightarrow \infty} \frac{x-1}{2} \cdot \tan\left(\frac{a}{2^x}\right)$$

$$= \lim_{x \rightarrow \infty} \frac{2^x}{2} \cdot \tan\left(\frac{a}{2^x}\right)$$

$$\Rightarrow \frac{1}{2} \cdot \lim_{x \rightarrow \infty} \frac{\tan\left(\frac{a}{2^x}\right)}{\left(\frac{1}{2^x}\right)}$$

$$= \frac{1}{2} \times a = \frac{a}{2}$$

(A)

Ans: A

10

$$\lim_{x \rightarrow 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$$

$$= \lim_{x \rightarrow 0} \frac{2 \sin^2 x (3 + 1)}{\frac{x \tan 4x}{x}} = \frac{2 \times 4}{4} = 2$$

Ans: C

JEE ADVANCED LEVEL

01.

$$\lim_{x \rightarrow 0} x^n \cdot \sin\left(\frac{1}{x^2}\right)$$

let $n = 2$

$$\lim_{x \rightarrow 0} x^2 \cdot \sin\left(\frac{1}{x^2}\right) = 0$$

let $n = -2$

$$\lim_{x \rightarrow 0} x^{-2} \cdot \sin\left(\frac{1}{x^2}\right)$$

$$= \lim_{x \rightarrow 0} \frac{\sin\left(\frac{1}{x^2}\right)}{\left(\frac{1}{x^2}\right)}$$

$$-1 \leq \sin\left(\frac{1}{x^2}\right) \leq 1$$

$$-\frac{1}{x^2} \leq \frac{\sin\left(\frac{1}{x^2}\right)}{x^2} \leq \frac{1}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{1}{x^2} = +\infty$$

$$\lim_{x \rightarrow 0} \frac{1}{x^2} = +\infty$$

$\therefore n < 0, \lim_{x \rightarrow 0} f(x)$

Does not exist
Ans: A, B

$$L = \lim_{a \rightarrow a} \frac{|2\sin a - 1|}{2\sin a - 1}$$

(5)

A) When $a = \frac{\pi}{6}$

$$\lim_{a \rightarrow \frac{\pi}{6}} \frac{|2\sin a - 1|}{2\sin a - 1}$$

$$\underline{\text{LHL}} \quad a \rightarrow \frac{\pi}{6}^-$$

$$\Rightarrow a < \frac{\pi}{6}$$

$$\Rightarrow \sin a < \sin \frac{\pi}{6}$$

$$\Rightarrow \sin a < \frac{1}{2}$$

$$\Rightarrow 2\sin a - 1 < 0$$

$$\therefore |2\sin a - 1| = -(2\sin a - 1)$$

$$\therefore \lim_{a \rightarrow \frac{\pi}{6}^-} \frac{-(2\sin a - 1)}{2\sin a - 1} = -1$$

$$\text{LHL} \neq \text{RHL}$$

$\therefore \lim_{a \rightarrow 0} f(x)$ Does not exist.

$$\text{B) } \lim_{a \rightarrow \pi} \frac{|2\sin a - 1|}{2\sin a - 1}$$

$$= \frac{|2\sin \pi - 1|}{2\sin \pi - 1} = \frac{|-1|}{-1} = -1$$

$$\text{C) } \lim_{a \rightarrow \frac{\pi}{2}} \frac{|2\sin \frac{\pi}{2} - 1|}{2\sin \frac{\pi}{2} - 1}$$

$$= \frac{|2 \cdot 1 - 1|}{2 \cdot 1 - 1} = \frac{1}{1} = 1$$

$$\text{D) } \lim_{a \rightarrow 0} \frac{|2\sin 0 - 1|}{2\sin 0 - 1}$$

$$= \frac{|-1|}{-1} = -1$$

Ans: A, B, C

03.

$$\text{Assertion (A): } \lim_{x \rightarrow 0} \frac{\sqrt{1 - \cos 2x}}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\sqrt{2} \sin x}{x}$$

$$= \lim_{x \rightarrow 0} \frac{\sqrt{2} |\sin x|}{x}$$

$$\text{LHL} = \lim_{x \rightarrow 0^-} \frac{\sqrt{2} \cdot (-\sin x)}{x} = -\sqrt{2}$$

$$\text{RHL: } \lim_{x \rightarrow 0^+} \frac{\sqrt{2} \cdot \sin x}{x} = \sqrt{2}$$

$$\text{LHL} \neq \text{RHL}$$

$\therefore \lim_{x \rightarrow 0} f(x)$ does not exist
(True)

Reason:

$$\lim_{x \rightarrow 0} \frac{|x|}{x}$$

$$\text{LHL} \Rightarrow \lim_{x \rightarrow 0^-} \frac{-x}{x} = -1$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} \frac{x}{x} = 1$$

$\lim_{x \rightarrow 0} f(x)$ Does not exist

B

04

Assertion (A) $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$

(7)

$$= \lim_{x \rightarrow 0} \frac{\cos x - 1}{3x^2}$$

$$= \lim_{x \rightarrow 0} \frac{-\sin x}{6x} = -\frac{1}{6} \quad (\text{True})$$

Reason (R): $\lim_{x \rightarrow 0} \frac{e^{x^2} - \cos x}{x^2}$

$$= \lim_{x \rightarrow 0} \frac{(e^{x^2} - 1) + (1 - \cos x)}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{e^{x^2} - 1}{x^2} + \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$$

$$= 1 + \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{x}{2}}{x^2}$$

$$= 1 + 2 \cdot \left(\frac{1}{2}\right)^2 = 1 + \frac{1}{2} = \frac{3}{2} \quad (\text{True})$$

Ans: A

05

Assertion (A): $\lim_{x \rightarrow 0} \frac{\sqrt{1 - \cos 2x}}{x}$

$$= \lim_{x \rightarrow 0} \frac{\sqrt{2 \sin^2 x}}{x}$$

$$= \sqrt{2} \cdot \lim_{x \rightarrow 0} \frac{|\sin x|}{x}$$

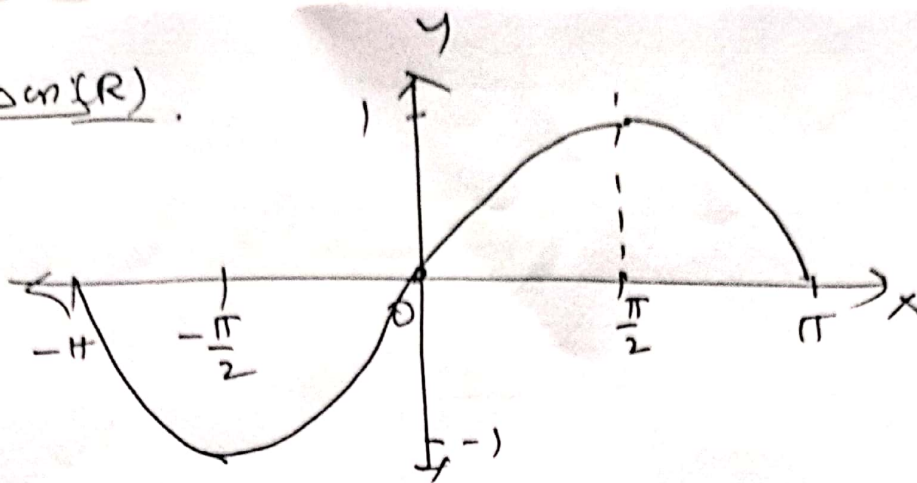
$$\therefore \text{LHL} \Rightarrow \sqrt{2} \cdot \lim_{x \rightarrow 0^-} \frac{-\sin x}{x} = -\sqrt{2}$$

$$\text{RHL} \Rightarrow \sqrt{2} \cdot \lim_{x \rightarrow 0^+} \frac{\sin x}{x} = \sqrt{2}$$

LHL \neq RHL \therefore Limit does not exist

Reason (R).

(8)



$$\text{clearly } 0 < x < \frac{\pi}{2} \Rightarrow \sin x > 0 \Rightarrow |\sin x| = \sin x$$

$$-\frac{\pi}{2} < x < 0 \Rightarrow \sin x < 0 \Rightarrow |\sin x| = -\sin x$$

(True)

Ans: B

66 (i) $\lim_{x \rightarrow 0} \frac{\sin x^\circ}{x}$

$$= \lim_{x \rightarrow 0} \frac{\sin \frac{\pi x}{180}}{x} = \frac{\pi}{180}$$

(ii) $\lim_{x \rightarrow 0} \frac{\tan 2x - x}{3x - \sin x} = \lim_{x \rightarrow 0} \frac{\frac{\tan 2x}{x} - 1}{3 - \frac{\sin x}{x}} = \frac{2-1}{3-1} = \frac{1}{2}$

(iii) $\lim_{x \rightarrow 0} \frac{\sin(\pi \cos^2 x)}{x^2}$

$$= \lim_{x \rightarrow 0} \frac{\sin(\pi(1 - \sin^2 x))}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{\sin(\pi - \pi \sin^2 x)}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{\sin(\pi \sin^2 x)}{x^2}$$

$$= \lim_{x \rightarrow 0} \frac{\sin(\pi \sin^2 x)}{(\pi \sin^2 x)} \times \frac{\pi \sin^2 x}{x^2}$$

$$= 1 \times \pi$$

$$= \pi$$

6

$$\begin{aligned} \text{(iv) } & \lim_{x \rightarrow \infty} 2^{x-1} \tan\left(\frac{9}{2^x}\right) \\ &= \frac{1}{2} \lim_{x \rightarrow \infty} 2^x \cdot \tan\left(\frac{9}{2^x}\right) \\ &= \frac{1}{2} \lim_{x \rightarrow \infty} \frac{\tan\left(\frac{9}{2^x}\right)}{\left(\frac{1}{2^x}\right)} \end{aligned}$$

Let $\frac{1}{2^x} = k$
 As $x \rightarrow \infty$, $k \rightarrow 0$

$$\lim_{k \rightarrow 0} \frac{\tan k}{k} = \frac{9}{2}$$

Ans: D, A, B, C

7 (i) $\lim_{x \rightarrow 0} \frac{\sqrt{1+\tan x} - \sqrt{1-\tan x}}{\sin x} + \frac{\sec^2 x}{2\sqrt{1-\tan x}}$

C.S.D.R

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{2\sqrt{1+\tan x}} \times \sec^2 x}{\sin x} + \frac{\sec^2 x}{2\sqrt{1-\tan x}}$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

(ii) $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2} = \frac{2 \sin^2 x}{x^2}$

$$= \lim_{x \rightarrow 0} \frac{2 \sin^2 x}{x^2} = 2 \left(\frac{\pi}{180}\right)^2$$

$$\begin{aligned} \text{(iii)} \quad & \lim_{x \rightarrow 0} \frac{1 - \cos x}{x \log(1+x)} \\ &= \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{x}{2}}{x^2 \log(1+x)} = 2 \left(\frac{1}{2}\right)^2 = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad & \lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{2(x - \sin x)} \\ &= \frac{1}{2} \lim_{x \rightarrow 0} \frac{e^{\sin x} (e^x - e^{-x})}{(x - \sin x)} \\ &= \frac{1}{2} \cdot \end{aligned}$$

Ans: B, A, C, C

08

$$\begin{aligned} -1 &\leq \cos\left(\frac{2}{x}\right) \leq 1 \\ -x^3 &\leq x^3 \cos\left(\frac{2}{x}\right) \leq x^3 \\ \lim_{x \rightarrow 0} -x^3 &= \lim_{x \rightarrow 0} x^3 = 0 \\ \therefore \lim_{x \rightarrow 0} x^3 \cos\left(\frac{2}{x}\right) &= 0. \end{aligned}$$

Ans: A

09

$$\begin{aligned} 0 &\leq \sin^2 x \leq 1 \\ x &\leq x + \sin^2 x \leq 3 \\ 2x^2 &\leq x^2 (2 + \sin^2 x) \leq 3x^2 \\ \frac{2x^2}{x+100} &\leq \frac{x^2 (2 + \sin^2 x)}{x+100} \leq \frac{3x^2}{x+100} \\ \left(\frac{2}{x} + \frac{100}{x^2}\right) &\leq \frac{x^2 (2 + \sin^2 x)}{x+100} \leq \left(\frac{3}{x} + \frac{100}{x^2}\right) \end{aligned}$$

Ans: C

$$\lim_{x \rightarrow \infty} \frac{2x^2}{x+100} = \infty$$

$$\lim_{x \rightarrow \infty} \frac{3x^2}{x+100} = \infty$$

10. $-1 \leq \sin\left(\frac{x}{x}\right) \leq 1$
 $-x^2 \leq x^2 \sin\left(\frac{x}{x}\right) \leq x^2$

$\therefore \lim_{x \rightarrow 0} -x^2 = \lim_{x \rightarrow 0} x^2 = 0$

$\therefore \lim_{x \rightarrow 0} x^2 \sin\left(\frac{x}{x}\right) = 0$

Ans: A

LEARNERS TASK

Ques

01. $\lim_{(x-a) \rightarrow 0} \frac{\tan(x-a)}{(x-a)} = 1$

Ans: A

02. 3

Ans: E

03. $\frac{9}{10}$

Ans: D

04. $\lim_{x \rightarrow 0} \frac{1 - \cos 4x}{1 - \cos 6x} = \frac{2^2}{3^2} = \frac{4}{9}$
 $= \lim_{x \rightarrow 0} \frac{8 \sin^2 2x}{2 \sin^2 3x}$

Ans: B

05. 1

Ans: B

06. ~~Same~~ Conceptual

Ans: C

07. Conceptual

Ans: C

08. 1

Ans: D

09. Conceptual

Ans: C

10. Conceptual.

Ans: A

JEE MAINS LEVEL

(12)

01. $\lim_{x \rightarrow 0} \frac{\sin 7x + \sin 5x}{\cos 5x - \tan 2x} = \frac{7+5}{5-2} = \frac{12}{3} = 4$

Ans: B

02. $\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{2x - \sin(x-2)}$

$= \lim_{x \rightarrow 2} \frac{(x-2)(x+1)}{2 - \cos(x-2)}$

$= \frac{2 \cdot 2 - 1}{2 - 1} = \frac{3}{1} = 3$

Ans: D

03. $\lim_{x \rightarrow 0} \frac{\sec x - \sec bx}{x^2}$

$\lim_{x \rightarrow 0} \frac{\sec ax \cdot \tan ax \cdot a - \sec bx \cdot \tan bx \cdot b}{2x}$

$= \frac{1}{2} \cdot \lim_{x \rightarrow 0} \left[\sec ax \cdot \frac{\tan ax}{x} - \sec bx \cdot \frac{\tan bx}{x} \right]$

$= \frac{1}{2} (a^2 - b^2)$

Ans: A

04. $\lim_{x \rightarrow 5} \frac{\sin^2(x-5) \cdot \tan(x-5)}{(x+5)(x-5) \cdot (x-5)}$

$= \lim_{x \rightarrow 5} \frac{\sin^2(x-5)}{(x-5)^2} \cdot \lim_{x \rightarrow 0} \frac{\tan(x-5)}{x+5}$

$= 1 \times 0$

$= 0$

Ans: C

05. $\lim_{x \rightarrow \frac{\pi}{4}} \frac{1 - \cot^2 x}{2 - \cot x - \cot^2 x} \left(\frac{0}{0}\right)$

$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{-3 \cot^2 x \cdot (-\operatorname{cosec}^2 x)}{-(-\operatorname{cosec}^2 x) - 3 \cot^2 x \cdot (-\operatorname{cosec}^2 x)}$

$= \lim_{x \rightarrow \frac{\pi}{4}} \frac{3 \cot^2 x \cdot \operatorname{cosec}^2 x}{\operatorname{cosec}^2 x + 3 \cot^2 x \cdot \operatorname{cosec}^2 x}$

$= \frac{3 \cdot (\sqrt{2})^2}{(\sqrt{2})^2 + 3(\sqrt{2})^2} = \frac{6}{4} = \frac{3}{2}$

Ans: B

06. $\lim_{x \rightarrow \infty} \frac{x-1}{2} \cdot \tan\left(\frac{a}{2x}\right) = \frac{a}{2}$

Ans: A

07. $\lim_{x \rightarrow \infty} x \left(\tan^{-1}\left(\frac{x+1}{x+u}\right) - \frac{\pi}{4} \right)$

$= \lim_{x \rightarrow \infty} x \left(\tan^{-1}\left(\frac{x+1}{x+u}\right) - \tan^{-1}(1) \right)$

$= \lim_{x \rightarrow \infty} x \left(\tan^{-1}\left(\frac{\frac{x+1}{x}}{\frac{x+u}{x}}\right) - \frac{\pi}{4} \right)$

$= \lim_{x \rightarrow \infty} x \left(\tan^{-1}\left(\frac{-3}{2x+5}\right) \right)$

$\Rightarrow \lim_{x \rightarrow \infty} \frac{\tan^{-1}\left(\frac{-3}{2x+5}\right)}{\left(\frac{-3}{2x+5}\right)} \times \lim_{x \rightarrow \infty} \frac{-3x}{2x+5}$

$= 1 \times \frac{-3}{2} = -\frac{3}{2}$

Ans: C

$$\lim_{x \rightarrow 1} \frac{a^{x-1} - 1}{\sin \pi x} \left(\frac{0}{0} \right)$$

$$= \lim_{x \rightarrow 1} \frac{a^{x-1} \cdot \log(a)}{\cos \pi x \cdot \pi}$$

$$= -\frac{\log a}{\pi}$$

Ans: C

$$\lim_{x \rightarrow 2} \frac{\sin(e^{x-2} - 1)}{\log(x-1)} \left(\frac{0}{0} \right)$$

$$= \lim_{x \rightarrow 2} \frac{\cos(e^{x-2} - 1) \cdot e^{x-2}}{\left(\frac{1}{x-1} \right)}$$

Ans: A

$$\lim_{x \rightarrow 0} \frac{a^{\tan x} - a^{\sin x}}{\tan x - \sin x} = \ln a$$

Ans: A

JEE ADVANCED

$$\lim_{x \rightarrow 0} \frac{\sin x - 2 \sin 3x + \sin 5x}{x} = -2 \cdot 3 + 5 = 0$$

opt: A $\neq 0$

$$\lim_{x \rightarrow \frac{\pi}{2}} \cos x = \cos \frac{\pi}{2} = 0$$

opt: B

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{x^3 - 2x}{\cos x} = \frac{0^3 - 2 \cdot 0}{0} = \frac{0}{0}$$

opt: C

Ans: A, B, C

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sec^2 x \cdot 2}{1}$$

02.

15

$\lim_{x \rightarrow 0} \frac{\sin 2x}{x} = 2$

Ans: A, D

opt'D $\Rightarrow 2$

Assertion (A): $\lim_{x \rightarrow 0} \frac{\sin 3x}{\tan^{-1} 6x} = \frac{3}{6} = \frac{1}{2}$ (True)

Ans: A

Reason (R): Conceptual (True)

Assertion (A): Conceptual (True)

Ans: B

Reason (R): Conceptual (True)

(i) $\lim_{x \rightarrow 0} \frac{|\sin x|}{x}$ = Does not exist

(ii) $\lim_{x \rightarrow 0} \frac{\sin x^0}{x} = \frac{\pi}{180}$

(iii) $\lim_{x \rightarrow 0} \frac{|x|}{x}$ = Does not exist

(iv) $\lim_{x \rightarrow a} \sin^{-1} x = \sin^{-1} a \Leftrightarrow -1 \leq a \leq 1$

Ans: F, A, F, B

(i) $\lim_{x \rightarrow 0} \frac{\sin^2 x}{\sin^4 x} = \frac{(2)^2}{(4)^2} = \frac{4}{16} = \frac{1}{4}$

(ii) $\lim_{x \rightarrow \pi} \frac{\sin x}{x - \pi} = \lim_{(x - \pi) \rightarrow 0} \frac{-\sin(x - \pi)}{(x - \pi)} = -1$

(iii) $\lim_{x \rightarrow 0} \frac{\sin x}{\sqrt{x+1} - \sqrt{1-x}} = x \frac{\sqrt{x+1} + \sqrt{1-x}}{\sqrt{x+1} + \sqrt{1-x}}$

(16)

$$= \lim_{x \rightarrow 0} \frac{\sin x}{2x} \cdot (\sqrt{x+1} + \sqrt{1-x})$$

$$= \frac{1}{2} \cdot 2 = 1$$

(iv) $\lim_{x \rightarrow 0} |\cos x| = 1$.

Ans: D, A, C, C

07. $\lim_{x \rightarrow 0} \frac{\sin^{-1} 5x + \sin^{-1} x}{\sin^{-1} 3x + \sin^{-1} 3x} = \frac{5+1}{1+3} = \frac{6}{4} = \frac{3}{2}$ Ans: D

08. $\lim_{x \rightarrow 0} \frac{\sin^{-1} 3x + \tan 4x}{\tan 3x + \tan x} = \frac{3+4}{3+1} = \frac{7}{4}$ Ans: A

09. $\lim_{x \rightarrow 0} \frac{\sin^{-1} 3x - \tan 4x}{\tan 3x + \sin^{-1} x} = \frac{3-4}{3+1} = \frac{-1}{4}$

09. $\lim_{x \rightarrow 0} \frac{\sin^{-1} 3x - \tan 4x}{\tan 3x + \sin^{-1} x} \left(\frac{0}{0}\right)$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{\sqrt{1-9x^2}} \cdot 3 - \sec^2 4x \cdot 4}{\sec^2 3x \cdot 3 + \frac{1}{\sqrt{1-x^2}}}$$

$$= \frac{3-4}{3+1} = \frac{-1}{4}$$

Ans: —

10. $\lim_{x \rightarrow 0} \frac{\sin 3x + \sin 7x}{\tan 3x + \tan 7x} = \frac{3+7}{3+7} = \frac{10}{10} = 1$

Ans: C

⇒ THE END ←