T task

① Given $\vec{A}$ and $\vec{B}$ are unit vectors $|\vec{A}| = 1$; $|\vec{B}| = 1$

$\therefore \quad |\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

$= \sqrt{1^2 + 1^2 - 2 \times 1 \times 1 \times \cos\theta}$

$= \sqrt{2 - 2\cos\theta} = \cancel{2\sqrt{1-\cos\theta}} = \cancel{2}\sqrt{2(1-\cos\theta)}$

$= \sqrt{2(2\sin^2\frac{\theta}{2})} = 2\sin\frac{\theta}{2}$

②

Given

$|\vec{P} - \vec{Q}| = \vec{R}$ & $P - Q = R$

$\Rightarrow \sqrt{P^2 + Q^2 - 2PQ\cos\theta} = R$ & $(P-Q)^2 = R^2$

$\Rightarrow P^2 + Q^2 - 2PQ\cos\theta = R^2 \rightarrow ①$ & $P^2 + Q^2 - 2PQ \cancel{\cos\theta} = R^2 \rightarrow ②$

By equating ① & ②

$\Rightarrow P^2 + \cancel{Q^2} - 2PQ\cos\theta = P^2 + \cancel{Q^2} - 2PQ$

$\Rightarrow \quad \cancel{-2PQ}\cos\theta = \cancel{-2PQ}$

$\Rightarrow \quad \cos\theta = 1 \Rightarrow \theta = 0^\circ$

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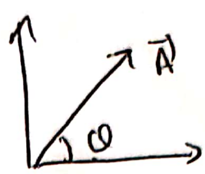
(28), (25) Let the vector be $\vec{A}$



Horizontal component $A_x = H$

$\Rightarrow |\vec{A}| \cos\theta = H$

Magnitude $\Rightarrow |\vec{A}| = \frac{H}{\cos\theta} = H \sec\theta$

Q. Vertical component $A_y = V$ In term of vertical component

$\Rightarrow |\vec{A}| \sin\theta = V$

$\Rightarrow |\vec{A}| = V \operatorname{cosec}\theta$

(Q 27)

According to given question $A_x = 3$ units

$A_y = 4$ units

$\therefore$ Magnitude of a vector $\vec{A} = \sqrt{A_x^2 + A_y^2} = \sqrt{3^2+4^2}$

$= 5$ units

(28)

A unit vector is the one whose magnitude is equal to '1'. So sum of squares of rectangular components are is equal to 1

(30)

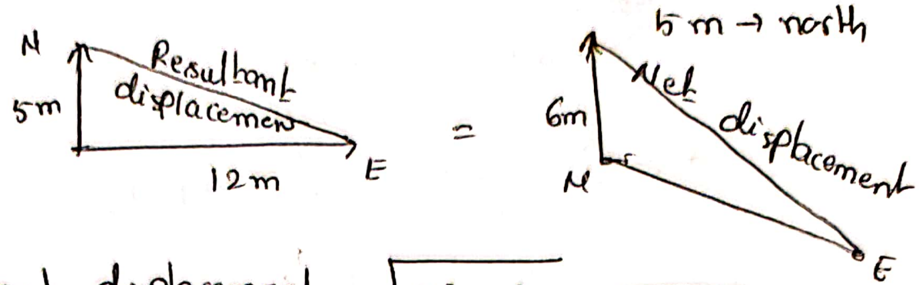
For the given vector $3\hat{i} - 4\hat{j}$ Horizontal component $= 3$

(31)

$l^2 + m^2 + n^2 = 1.$

(3)

Given displacement of the particle 12 m → East
5 m → north



∴ ⇒ Resultant displacement $= \sqrt{12^2+5^2} = \sqrt{144+25}$

$= 13m$

Net displacement $= \sqrt{13^2+6^2} = \sqrt{169+36} = 14.32m$

(4)

let the two vectors be $\vec{A}$ & $\vec{B}$

Given $|\vec{A}| = |\vec{B}| = P$.

$\therefore |\vec{A}-\vec{B}| = \sqrt{A^2+B^2-2AB\cos\theta}$

$= \sqrt{P^2+P^2-2PP\cos\theta} = \sqrt{2P^2-2P^2\cos\theta}$

$= \sqrt{2P^2(1-\cos\theta)} = \sqrt{2P^2(2\sin^2\frac{\theta}{2})} = 2P\sin\frac{\theta}{2}$

(5)

let the two forces are F_1 & F_2

when F_1 & F_2 are acting in opp directions $F_1 - F_2 = 10$

when F_1 & F_2 are $\perp^{er}$ to each other then $\sqrt{F_1^2+F_2^2} = 60$

$\cancel{(F_1+F_2)^2} = (F_1-F_2)^2 = F_1^2+F_2^2-2F_1F_2$

$\Rightarrow 10^2 = (60)^2 - 2F_1F_2$

5th continuation

$$\Rightarrow 100 - \cancel{(600)}\, 3600 = -2F_1F_2$$

$$\Rightarrow 2F_1F_2 = 3500$$

$$\therefore (F_1+F_2)^2 = (F_1-F_2)^2 + 4F_1F_2$$

$$\Rightarrow (10)^2 + 2(3500)$$

$$(F_1+F_2)^2 \Rightarrow 7100$$

$$\Rightarrow F_1+F_2 = 10\sqrt{71} = 84.2$$

$$F_1 - F_2 = \quad = 10$$

$$2F_1 = 94.2$$

$$\Rightarrow F_1 = 47.1 \quad \text{and} \quad F_1 - F_2 = 10$$

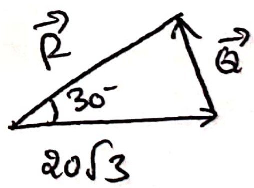
$$\Rightarrow F_2 = F_1 - 10$$

$$= 47.1 - 10$$

$$= 37.1$$

6. Let two vectors are $\vec{P}$ & $\vec{Q}$ and their resultant

is $\vec{R} = 20$ let $\vec{P} = 20\sqrt{3}$



$$\vec{R} = \vec{P} + \vec{Q}$$

$$\Rightarrow \vec{Q} = \vec{R} - \vec{P}$$

$$\Rightarrow |\vec{Q}| = \sqrt{R^2 + P^2 - 2RP\cos\theta}$$

$$= \sqrt{20^2 + (20\sqrt{3})^2 - 2(20)(20\sqrt{3})\cos 30}$$

$$= \sqrt{400 + 1200 - 400(2\sqrt{3})\frac{\sqrt{3}}{2}}$$

$$= \sqrt{1600 - 1200} = \sqrt{400} = 20.$$

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(7)

let Resultant $\vec{R} = 10m$; $\vec{P} = 6m$, $\vec{Q} = 8m$

$$\therefore \vec{R} = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$\Rightarrow 10 = \sqrt{6^2 + 8^2 + 2(6)(8)\cos\theta}$$

$$\Rightarrow 10^2 = 36 + 64 + 96\cos\theta$$

$$\Rightarrow 100 = 100 + 96\cos\theta$$

$\Rightarrow \cos\theta = 0 \Rightarrow \theta = 90^\circ$ that two vectors are Perpendicular to each other

(8)

let the two forces are $F_1 = 20N$; $F_2 = 20N$

$\theta = 120^\circ$

$$\text{Magnitude of Resultant} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{(20)^2 + (20)^2 + 2(20)(20)\cos 120^\circ}$$

$$= \sqrt{(20)^2 + (20)^2 + \cancel{2}(20)^2\left(-\frac{1}{\cancel{2}}\right)}$$

$$= \sqrt{(20)^2 + \cancel{(20)^2} - \cancel{(20)^2}}$$

$$= \sqrt{20^2} = 20N$$

Direction $\tan\alpha = \dfrac{F_2\sin\theta}{F_1 + F_2\cos\theta} = \dfrac{20\sin 120}{20 + 20\cos 120}$

$$= \frac{\cancel{20}\frac{\sqrt{3}}{2}}{\cancel{20}\left(1 + \left(-\frac{1}{2}\right)\right)} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

$\alpha = 60^\circ$

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(9)

let the two forces are F_1 & F_2

Given that $|\vec{F_1}| = |\vec{F_2}|$ and $\theta = 90°$

Resultant of two forces = 1414 N.

$\therefore$ Resultant $= \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$\Rightarrow 1414 = \sqrt{F_1^2 + F_1^2 + 2F_1F_1\cos 90}$

$\Rightarrow 1414 = \sqrt{2F_1^2 + 2F_1^2(0)}$

$\Rightarrow 1414 = \sqrt{2F_1^2} = \sqrt{2}\, F_1$

$\Rightarrow F_1 = \frac{1414}{\sqrt{2}} = \frac{1414}{1.414} = 1000N$

(10)

let two forces are $F_1 = 1N$ and $F_2 = PN$.

Resultant $R = 1N$ and is $\perp^{er}$ to 1N i.e. $\alpha = 90°$

Direction of resultant $\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2\cos\theta}$

$\Rightarrow \tan 90° = \frac{P\sin\theta}{1 + P\cos\theta}$

$\Rightarrow \infty = \frac{P\sin\theta}{1 + P\cos\theta}$

$\Rightarrow 1 + P\cos\theta = \frac{P\sin\theta}{\infty} = 0$

$\Rightarrow \cos\theta = -\frac{1}{P} \rightarrow (1)$

$\therefore$ Magnitude of $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$\Rightarrow 1 = \sqrt{1 + P^2 + 2(1)(P)\left(-\frac{1}{P}\right)}$

$\Rightarrow 1 = 1 + P^2 - 2 \Rightarrow P = \sqrt{2}$

From (1)

$\cos\theta = -\frac{1}{\sqrt{2}}$

$\Rightarrow \theta = 135°$

(11)

Given $\vec{A} = 4\hat{i} - 2\hat{j} + 6\hat{k}$ & $\vec{B} = \hat{i} - 2\hat{j} - 3\hat{k}$

then $\vec{A}+\vec{B} = 4\hat{i} - 2\hat{j} + 6\hat{k} + \hat{i} - 2\hat{j} - 3\hat{k}$

$= 5\hat{i} - 4\hat{j} + 3\hat{k}$

$|\vec{A}+\vec{B}| = \sqrt{5^2 + (-4)^2 + 3^2} = \sqrt{25+16+9} = 5\sqrt{2}$

Angle made by the vector $\vec{A}+\vec{B}$ with x-axis

$\cos\alpha = \dfrac{x\text{-component}}{|\vec{A}+\vec{B}|} = \dfrac{5}{5\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

$\alpha = 45^\circ$ (or) $\cos\alpha = \dfrac{5}{\sqrt{50}}$ (or) $\sin\alpha = \dfrac{5}{\sqrt{50}}$

$\sec\alpha = \sec 45^\circ = \sqrt{2} \Rightarrow \alpha = \sec^{-1}(\sqrt{2})$

(or) $\operatorname{cosec}^{-1}(\sqrt{2})$

(15)

let $\vec{P} = 5N$; $\vec{Q} = 3N$ they are acting in same direction $(\theta = 0^\circ)$ their resultant $R = \sqrt{P^2+Q^2+2PQ\cos\theta}$

$R = \sqrt{P^2+Q^2+2PQ\cos 0^\circ} = \sqrt{P^2+Q^2+2PQ}$

$R = \sqrt{(P+Q)^2} = P+Q = 5+3 = 8N$

(16) let $\vec{P} = 20N$; $\vec{Q} = 25N$; $\vec{R} = 39N$.

$\therefore \vec{R} = \sqrt{P^2+Q^2+2PQ\cos\theta} \Rightarrow 39 = \sqrt{20^2+25^2+2(20)(25)\cos\theta}$

$\Rightarrow (39)^2 = 400+625+1000\cos\theta$

$\Rightarrow 1521 = 1025 + 1000\cos\theta$

$\Rightarrow 1000\cos\theta = 496$

$\Rightarrow \cos\theta = \dfrac{496}{1000} \simeq \dfrac{1}{2}$ ~~$\Rightarrow \theta = 60^\circ$~~

$\Rightarrow \theta \simeq 60^\circ$

(17) take solution from Thask (9)th question.

(18)

let the two forces are F_1 and F_2.

Given $F_1 + F_2 = 7\,N$

$F_1 - F_2 = 1\,N$

Given $F_1 = 4x$

$F_2 = 3x$

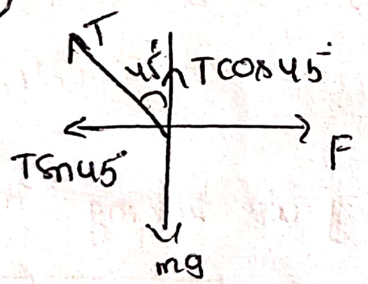
$\therefore$ By substituting F_1 & F_2 value in one of the above we get

$4x + 3x = 7$

$\Rightarrow 7x = 7$

$\Rightarrow x = 1$

(19)



From fig:- under equilibrium

$F = T\sin 45^\circ$

$Mg = T\cos 45^\circ$

$\Rightarrow \frac{F}{Mg} = \frac{\cancel{T}\sin 45^\circ}{\cancel{T}\cos 45^\circ} = \tan 45^\circ = 1$

(20)

See 9th [illegible] ws-3.

CUQ's

(6) let the two forces are F_1 & F_2

Given $F_1 + F_2 = 10\,N$

$F_1 - F_2 = 4\,N \Rightarrow 7 - F_2 = 4$

$2F_1 = 14\,N \Rightarrow F_2 = 3\,N$

The large force is $\Rightarrow F_1 = 7\,N$

(7) Given $\vec{P} + \vec{Q} = \vec{R}$ and $\vec{P} - \vec{Q} = \vec{S}$

$\Rightarrow P^2 + Q^2 + 2PQ\cos\theta = R^2$ $\quad P^2 + Q^2 - 2PQ\cos\theta = S^2$

$\therefore R^2 + S^2 = P^2 + Q^2 + \cancel{2PQ\cos\theta} + P^2 + Q^2 - \cancel{2PQ\cos\theta}$

$= 2P^2 + 2Q^2$

$= 2(P^2 + Q^2)$

(8) Given $F_1 = 3\hat{i} - 4\hat{j} + 2\hat{k}$; $F_2 = 2\hat{i} + 3\hat{j} - \hat{k}$, $F_3 = 2\hat{i} + 4\hat{j} - 5\hat{k}$

their resultant $F_1 + F_2 + F_3 = 3\hat{i} - \cancel{4\hat{j}} + 2\hat{k} + 2\hat{i} + 3\hat{j} - \hat{k} + 2\hat{i} + 4\hat{j} - 5\hat{k}$

$\Rightarrow 7\hat{i} + 3\hat{j} - 4\hat{k}$

(9) For magnitude of $|\vec{A} - \vec{B}|$ is maximum $\theta = 180^\circ$

$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta} = \sqrt{A^2 + B^2 - 2AB\cos 180^\circ}$

$= \sqrt{A^2 + B^2 - 2AB(-1)} = \sqrt{A^2 + B^2 + 2AB}$

$= \sqrt{(A+B)^2} = A + B$

(10)

Given $|\vec{a}-\vec{b}| = |\vec{a}| + |\vec{b}|$

$\Rightarrow \sqrt{a^2+b^2-2ab\cos\theta} = a+b$

$\Rightarrow a^2+b^2-2ab\cos\theta = (a+b)^2$

$\Rightarrow a^2 \cancel{+b^2} -2ab\cos\theta = a^2 \cancel{+b^2} + 2ab$

$\Rightarrow -\cancel{2ab}\cos\theta = \cancel{2ab}$

$\Rightarrow \cos\theta = -1 \Rightarrow \theta = 180^\circ$

Jee main level

(1)

Given $|\vec{A}+\vec{B}| = |\vec{A}| + |\vec{B}|$

$\Rightarrow \sqrt{A^2+B^2+2AB\cos\theta} = A+B$

$\Rightarrow (A^2+B^2+2AB\cos\theta = (A+B)^2$

$\Rightarrow A^2 \cancel{+B^2} + 2AB\cos\theta = A^2 \cancel{+B^2} + 2AB$

$\Rightarrow \cancel{2AB}\cos\theta = \cancel{2AB}$

$\Rightarrow \cos\theta = 1 \Rightarrow \theta = 0^\circ$

(2) let the forces are F_1 & F_2

Given $\frac{F_1+F_2}{F_1-F_2} = \frac{4}{3} \Rightarrow 3(F_1+F_2) = 4(F_1-F_2)$

$\Rightarrow 3F_1 + 3F_2 = 4F_1 - 4F_2$

$\Rightarrow 4F_2 + 3F_2 = 4F_1 - 3F_1$

$\Rightarrow 7F_2 = F_1$

$\Rightarrow \frac{F_1}{F_2} = \frac{7}{1}$

(3)

let the Given vector is $\vec{A} = \hat{j} + \hat{k}$

Magnitude of $\vec{A} = |\vec{A}| = \sqrt{1^2+1^2} = \sqrt{2}$

Angle made by the vector with Y-axis is

$$\cos\beta = \frac{\text{Y-component}}{|\vec{A}|} = \frac{1}{\sqrt{2}}$$

$\Rightarrow \beta = 45^\circ$

(4)

let the two forces are $2P$ and $\sqrt{2}P$

Their resultant $R = \sqrt{10}\,P$.

$\therefore$ $R = \sqrt{P^2+Q^2+2PQ\cos\theta}$

$\Rightarrow R^2 = P^2+Q^2+2PQ\cos\theta$

$\Rightarrow (\sqrt{10}P)^2 = (2P)^2+(\sqrt{2}P)^2+2(2P)(\sqrt{2}P)\cos\theta$

$\Rightarrow 10P^2 = 4P^2+2P^2+4\sqrt{2}P^2\cos\theta$

$\Rightarrow 10\cancel{P^2} = 6\cancel{P^2}+4\sqrt{2}\cancel{P^2}\cos\theta$

$\Rightarrow 10-6 = 4\sqrt{2}\cos\theta \Rightarrow \cancel{4}\sqrt{2}\cos\theta = \cancel{4} \Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$

(5) let the two forces are $Q = 500$ gm wt, $P = 250$ gm wt.

Given the resultant is Perpendicular to any one of the two forces let it be P Then $\tan\alpha = \frac{Q\sin\theta}{P+Q\cos\theta} \Rightarrow \tan 90 = \frac{Q\sin\theta}{P+Q\cos\theta}$

$\Rightarrow \infty = \frac{Q\sin\theta}{P+Q\cos\theta}$

$\Rightarrow P+Q\cos\theta = 0 \Rightarrow \cos\theta = -\frac{P}{Q} = \frac{-250}{500} = -\frac{1}{2}$

$\Rightarrow \theta = 120^\circ$

(6)

Let the two forces are $\vec{P}$ and $\vec{Q}$

Given $\frac{\vec{P}}{\vec{Q}} = \frac{3}{5} = \frac{3x}{5x}$, their resultant $R = 35N$

Angle between them $\theta = 60^\circ$

$\therefore$ the magnitude of ~~each~~ Resultant force $R = \sqrt{P^2+Q^2+2PQ\cos\theta}$

$\Rightarrow R^2 = P^2+Q^2+2PQ\cos\theta$

$\Rightarrow 35^2 = (3x)^2 + (5x)^2 + 2(3x)(5x)\cos 60$

$\Rightarrow (35)^2 = 9x^2+25x^2+\cancel{2}\times 15x^2 \times \frac{1}{\cancel{2}}$

$\Rightarrow (35)^2 = 49x^2$

$\Rightarrow x^2 = \frac{(35)^2}{49} \Rightarrow x = \frac{35}{7} = 5$

$\therefore P = 3x = 15N$ and $Q = 5\times 5 = 25N$.

(7)

Let the two forces are $\vec{P}$ and $\vec{Q}$

The greatest resultant $\vec{P} + \vec{Q} = 29$

least resultant $P - Q = 5$

$2P = 34 \Rightarrow P = 17$ kg wt

From $P-Q=5 \Rightarrow Q = P-5 = 17-5 = 12$ kg wt

Given magnitude of each force is increased by 3 kg wt

$\therefore P' = 17+3 = 20$ kg wt ; $Q' = 15$ kg wt

$\therefore$ Given P' and Q' are perpendicular each other

Their resultant $R = \sqrt{P'^2+Q'^2} = \sqrt{(20)^2+(15)^2}$

$= \sqrt{400+225}$

$= \sqrt{625} = 25$ kg wt

(8)

let the displacements are $x = 300\,m$ & $y = 400\,m$

they are acting at right angles to each other.

Their resultant $= \sqrt{x^2 + y^2 + 2xy \cos\theta}$

$= \sqrt{(300)^2 + (400)^2 + 2xy \cos 90}$

$= \sqrt{9 \times 100^2 + 16 \times 100^2 + 2xy\,(0)}$

$= 100\sqrt{9+16} = 500\,m$

(9)

The two forces acting on the particle are . P and Q

Given $\vec{P} = 3N$, $\vec{Q} = 4N$.

The net force acting on the particle in in between

$\vec{P} + \vec{Q}$ and $\vec{P} - \vec{Q}$ $\Rightarrow$ $3+4$ to $3 \sim 4$

$\Rightarrow$ $7N.$ to $1\,N$

(10)

let the two forces are $\vec{P} = 8N$; $\vec{Q} = 15N$

Their resultant $\vec{R} = 17N.$

According to Parallelogram law of vectors

$\vec{R} = \sqrt{P^2 + Q^2 + 2PQ \cos\theta}$

$\Rightarrow 17 = \sqrt{8^2 + 15^2 + 2 \times 8 \times 15 \cos\theta}$

$\Rightarrow 17^2 = 64 + 225 + 240 \cos\theta$

$\Rightarrow \cancel{289} = \cancel{289} + 240 \cos\theta$

$\Rightarrow 240 \cos\theta = 0 \Rightarrow \cos\theta = 0$

$\Rightarrow \theta = 90^\circ$

(11)

let the two forces are $|\vec{P}| = |\vec{Q}| = 10\,N$ & $\theta = 60^\circ$

The magnitude of Resultant $R = \sqrt{P^2+Q^2+2PQ\cos\theta}$

$\Rightarrow R = \sqrt{10^2+10^2+2(10)(10)\cos 60^\circ}$

$= \sqrt{100+100+\cancel{2}\times 100\times \frac{1}{\cancel{2}}}$

$= \sqrt{300} = 10\sqrt{3}\,N.$

The direction of resultant $\tan\alpha = \dfrac{Q\sin\theta}{P+Q\cos\theta}$

$\Rightarrow \tan\alpha = \dfrac{10\sin 60}{10+10\cos 60} = \dfrac{\cancel{10}\,\frac{\sqrt{3}}{2}}{\cancel{10}\left(1+\frac{1}{2}\right)}$

$\Rightarrow \tan\alpha = \dfrac{\frac{\sqrt{3}}{\cancel{2}}}{\frac{3}{\cancel{2}}} = \dfrac{\sqrt{3}}{3} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \alpha = 30^\circ$

(12)

let the two forces are $F_1 = x+y$; $F_2 = x-y$.

Resultant $R = \sqrt{2(x^2+y^2)}$

According to Parallelogram law of vector

$R = \sqrt{F_1^2+F_2^2+2F_1F_2\cos\theta}$

$\Rightarrow \sqrt{2(x^2+y^2)} = \sqrt{(x+y)^2+(x-y)^2+2(x+y)(x-y)\cos\theta}$

squaring on both sides. we get

$\Rightarrow 2(x^2+y^2) = x^2+y^2+\cancel{2xy}+x^2+y^2-\cancel{2xy}+2(x^2-y^2)\cos\theta$

$\Rightarrow \cancel{2x^2}+\cancel{2y^2} = \cancel{2x^2}+\cancel{2y^2}+2(x^2-y^2)\cos\theta$

$\Rightarrow 2(x^2-y^2)\cos\theta = 0$

$\Rightarrow \cos\theta = 0 \Rightarrow \theta = 90^\circ$

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(13)

let the two vectors are P and Q

Given $|\vec{P}| = 8$, $|\vec{Q}| = 6$; Their resultant $R = 10$.

According to parallelogram law of vectors

$$R = \sqrt{P^2+Q^2+2PQ\cos\theta}$$

$$\Rightarrow 10 = \sqrt{8^2+6^2+2\times8\times6\cos\theta}$$

$$\Rightarrow 10^2 = 64+36+96\cos\theta$$

$$\Rightarrow 100 = 100+96\cos\theta \Rightarrow 96\cos\theta = 0$$

$$\Rightarrow \cos\theta = 0 \Rightarrow \theta = 90^\circ$$

(14)

let the two vectors are P and Q

There resultant is R. Given $|\vec{P}| = |\vec{Q}| = |\vec{R}|$

According to parallelogram law of vectors

$$R^2 = P^2+Q^2+2PQ\cos\theta$$

$$\Rightarrow \cancel{P^2} = \cancel{P^2} + P^2 + 2PP\cos\theta$$

$$\Rightarrow 0 = P^2 + 2P^2\cos\theta \Rightarrow 2\cancel{P^2}\cos\theta = -\cancel{P^2}$$

$$\Rightarrow 2\cos\theta = -1$$

$$\Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = 120^\circ$$

(15)

Given $|\vec{F_1}| = |\vec{F_2}| = f$

According to parallelogram law of vector

$$R^2 = P^2+Q^2+2PQ\cos\theta$$

$$\Rightarrow R^2 = f^2+f^2+2ff\cos\theta$$

$$\Rightarrow R^2 = 2f^2+2f^2\cos\theta$$

$$\Rightarrow R^2 = 2f^2(1+\cos\theta) = 2f^2(2\cos^2\frac{\theta}{2})$$

$$\Rightarrow R^2 = 4f^2\cos^2\frac{\theta}{2}$$

$$\Rightarrow R = 2f\cos\frac{\theta}{2}$$

(16)

Given $|\vec{A}| = |\vec{B}|$ and

$|\vec{A}+\vec{B}| = n\,|\vec{A}-\vec{B}|$

$\Rightarrow \sqrt{A^2+B^2+2AB\cos\theta} = n\sqrt{A^2+B^2-2AB\cos\theta}$

on squaring both sides

$\Rightarrow \left(\sqrt{A^2+B^2+2AB\cos\theta}\right)^2 = n^2\left[\sqrt{A^2+B^2-2AB\cos\theta}\right]^2$

$\Rightarrow A^2+B^2+2AB\cos\theta = n^2\left(A^2+B^2-2AB\cos\theta\right]$

Since $|\vec{A}| = |\vec{B}|$

$\therefore A^2+A^2+2AA\cos\theta = n^2\left(A^2+A^2-2AA\cos\theta\right)$

$\Rightarrow 2A^2+2A^2\cos\theta = n^2\left(2A^2-2A^2\cos\theta\right)$

$\Rightarrow \cancel{2A^2}\left(1+\cos\theta\right) = n^2\,\cancel{2A^2}\left(1-\cos\theta\right)$

$\Rightarrow \cancel{2}\cos^2\frac{\theta}{2} = n^2\,\cancel{2}\sin^2\frac{\theta}{2}$

$\Rightarrow \cos\frac{\theta}{2} = n\sin\frac{\theta}{2} \Rightarrow \frac{1}{n} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}$

$\Rightarrow \tan\frac{\theta}{2} = \frac{1}{n} \Rightarrow \frac{\theta}{2} = \tan^{-1}\left(\frac{1}{n}\right)$

$\Rightarrow \theta = 2\tan^{-1}\left(\frac{1}{n}\right)$

(19)

Let the two forces are $\vec{P} = 7N$ & $\vec{Q} = 5N$.

The resultant of two forces (or) vectors lies inbetween $\vec{P} \sim \vec{Q}$ and $P+Q$

$\Rightarrow 7 \sim 5$ and $7+5$

$\Rightarrow$ 2N and 12N, so 3N lies in between 2N and 12N.

Their resultant 3N is possible

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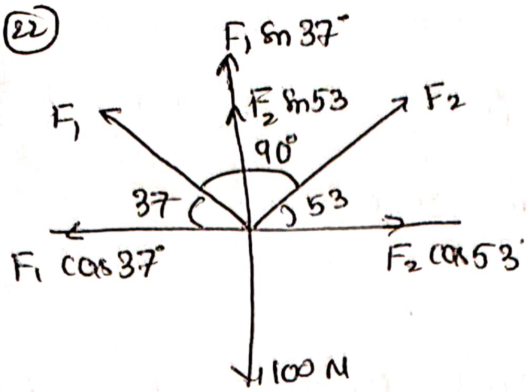
(20), (21), (22)



under equilibrium

Along x-axis

$F_2 \cos 53 = F_1 \cos 37$

$\Rightarrow F_2 \times \frac{3}{\cancel{5}} = F_1 \times \frac{4}{\cancel{5}}$

$\Rightarrow F_2 = \frac{4}{3} F_1$

Along y-axis

$F_1 \sin 37° + F_2 \sin 53 = 100$

$\Rightarrow F_1 \times \frac{3}{5} + \frac{4F_1}{3} \times \frac{4}{5} = 100$

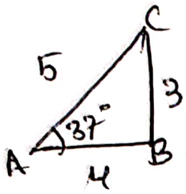
$\Rightarrow \frac{9F_1 + 16F_1}{15} = 100$

$\Rightarrow \frac{\cancel{25}F_1}{15} = \cancel{100}\ 4$

$\Rightarrow F_1 = 60\,N.$

$\therefore F_2 = \frac{4}{\cancel{3}} \times \cancel{60}^{20} = 80\,N.$

Given $\tan 37° = \frac{3}{4}$; $\tan 53 = \frac{4}{3}$



$AC^2 = AB^2 + BC^2$

$= 4^2 + 3^2$

$= 16 + 9$

$\Rightarrow AC^2 = 25$

$\Rightarrow AC = 5\,N$

$\cos 53° = \frac{3}{5}$

$\sin 53 = \frac{4}{5}$

$\cos 37° = \frac{4}{5}$; $\sin 37° = \frac{3}{5}$

Angle between F_1 & F_2 is 90°

(23)

let two forces are F_1 & F_2

Given $F_1 = F_2 = 15N$ and $\theta = 120°$; Resultant $= 5x$

According to Parallelogram law of vectors

$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos\theta}$

$\Rightarrow 5x = \sqrt{(15)^2 + (15)^2 + 2(15)(15)\cos 120°}$

$\Rightarrow 5x = \sqrt{(15)^2 + (15)^2 + \cancel{2}(15)^2 \left(\frac{-1}{\cancel{2}}\right)}$

$\Rightarrow 5x = \sqrt{(15)^2 + \cancel{(15)^2} - \cancel{(15)^2}}$

$\Rightarrow 5x = \sqrt{15^2}$

$\Rightarrow 5x = 15$

$\Rightarrow x = 3$

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(24)

let the two forces are $F_1 = 10N$; $F_2 = 5N$.

Here the force is minimum when angle between them is $180°$

Then the resultant $R = F_1 - F_2 = 10 - 5 = 5N$

(25)

let the two forces are $\vec{P}$ and $\vec{Q}$

Given $\vec{Q} = 2\vec{P}$

Given resultant is perpendicular to smaller force ie $\alpha = 90°$

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$$\Rightarrow \tan 90° = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$$\Rightarrow P + Q\cos\theta = \frac{Q\sin\theta}{\tan 90°}$$

$$\Rightarrow P + Q\cos\theta = 0$$

$$\Rightarrow \cos\theta = -\frac{P}{Q} = -\frac{P}{2P} = -\frac{1}{2}$$

$$\Rightarrow \theta = 120°$$

Given $\theta = \frac{2\pi}{n}$

$$\Rightarrow \frac{2\pi}{n} = \frac{2\pi}{3}$$

$$\Rightarrow \underline{n = 3}.$$

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