PROPERTIES OF TRIANGLES-I
WORK SHEET-7
TEACHING TASK

1. Given A:B:C = 2:3:5

We have $A = \frac{2}{10} \text{x} 180^0 = 36^0$

$B = \frac{3}{10} \text{x} 180^0 = 54^0$

$C = \frac{5}{10} \text{x} 180^0 = 90^0$

here, greatest angle = C = 90^0

Least angle = A = 36^0

We have $\frac{c}{\sin 90^0} = \frac{a}{\sin 36^0}$

$\Rightarrow \frac{c}{a} = \frac{\sin 90^0}{\sin 36^0}$

$\Rightarrow \frac{c}{a} = \frac{1}{\left(\frac{\sqrt{10-2\sqrt{5}}}{4}\right)}$

$\Rightarrow c : a = 4 : \sqrt{10-2\sqrt{5}}$

Ans : B

2. Given A+C = 2B

$\Rightarrow$ A+B+C = 3B

$\Rightarrow$ 180^0 = 3B

$\Rightarrow$ B = 60^0

$\Rightarrow$ A+C = 120^0

We have $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\Rightarrow \cos 60^0 = \frac{a^2 + c^2 - b^2}{2ac}$

$\Rightarrow \frac{1}{2} = \frac{a^2 + c^2 - b^2}{2ac}$

$\Rightarrow ac = a^2 + c^2 - b^2$

$\Rightarrow b^2 = a^2 - ac + c^2$

Now $\frac{a+c}{\sqrt{a^2 - ac + c^2}} = \frac{a+c}{\sqrt{b^2}} = \frac{a+c}{b}$

Now, $\dfrac{a+c}{b}=\dfrac{2R\sin A+2R\sin C}{2R\sin B}$

$$=\frac{\sin A+\sin C}{\sin B}$$

$$=\frac{2\sin\left(\frac{A+C}{2}\right).\cos\left(\frac{A-C}{2}\right)}{\sin B}$$

$$=\frac{2\sin\left(\frac{120^0}{2}\right).\cos\left(\frac{A-C}{2}\right)}{\sin 60^0}$$

$$=\frac{2\sin 60^0.\cos\left(\frac{A-C}{2}\right)}{\sin 60^0}$$

$$=2\cos\left(\frac{A-C}{2}\right)$$

Ans : A

3. Given $c^4-2(a^2+b^2)c^2+a^4+a^2b^2+b^4 = 0$

$\Rightarrow a^4+b^4+c^4+2a^2b^2-2c^2b^2-2a^2c^2 = a^2b^2$

$\Rightarrow \left(a^2+b^2-c^2\right)^2 = a^2b^2$

$\Rightarrow a^2+b^2-c^2 = ab$ .... (1)

Now, cosC $=\dfrac{a^2+b^2-c^2}{2ab}$

$=\dfrac{ab}{2ab}$ $\left(\because \textit{ from } (1)\right)$

$=\dfrac{1}{2}$

$\therefore\ c = 60^0$

Ans : C

4. Let a,b be two sides of a triangle.

Let a,b be the roots of $x^2-2\sqrt{3}x+2=0$

We have $a+b=2\sqrt{3}$

ab = 2

Now $\cos C=\dfrac{a^2+b^2-c^2}{2ab}$

$$\Rightarrow \cos\frac{\pi}{3}=\frac{(a+b)^2-2ab-c^2}{2ab}$$

$$\Rightarrow \frac{1}{2} = \frac{\left(2\sqrt{3}\right)^2 - 2(2) - c^2}{2(2)}$$

$$\Rightarrow 2 = 8 - c^2$$

$$\Rightarrow c^2 = 6$$

$$\Rightarrow c = \sqrt{6}$$

$\therefore$ The perimeter of the triangle

= a+b+c

$= 2\sqrt{3} + \sqrt{6}$

Ans : B

5. In $a\,\Delta\,ABC$

Given (a+b+c)(b+c-a) $= \lambda bc$

$$\Rightarrow (a+b+c)(a+b+c-2a) = \lambda bc$$

$$\Rightarrow (2s)(2s-2a) = \lambda bc$$

$$\Rightarrow 4s(s-a) = \lambda bc$$

$$\Rightarrow \frac{s(s-a)}{bc} = \frac{\lambda}{4}$$

$$\Rightarrow \sqrt{\frac{s(s-a)}{bc}} = \sqrt{\frac{\lambda}{4}}$$

$$\Rightarrow \cos\frac{A}{2} = \sqrt{\frac{\lambda}{4}}$$

We know $-1 \le \cos\frac{A}{2} \le 1$

$$\Rightarrow -1 \le \frac{\sqrt{\lambda}}{2} \le 1$$

$$\Rightarrow -2 \le \sqrt{\lambda} \le 2$$

$$\Rightarrow 0 \le \lambda^2 \le 4$$

λ can not be equal to either 0 or 4

$\therefore 0 < \lambda < 4$

Ans : C

6. Give $\tan\frac{A}{2}.Tan\frac{B}{2} + Tan\frac{B}{2}.Tan\frac{C}{2} = \frac{2}{3}$

$$\Rightarrow \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}.\sqrt{\frac{(s-a)(s-c)}{s(s-b)}} + \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}.\sqrt{\frac{(s-a)(s-b)}{s(s-c)}} = \frac{2}{3}$$

$$\Rightarrow \frac{s-c}{s}+\frac{s-a}{s}=\frac{2}{3}$$

$$\Rightarrow \frac{2s-a-c}{s}=\frac{2}{3}$$

$$\Rightarrow a+b+c-a-c=\frac{2s}{3}$$

$$\Rightarrow 3b=a+b+c$$

$$\Rightarrow 2b=a+c$$

Ans : B

7. Let ΔABC be an equilateral tringle with side 1 unit

We have $\alpha=\beta=\gamma=\frac{\sqrt{3}}{2}$

$$\Delta=\frac{\sqrt{3}}{4}$$

Now $\frac{1}{\alpha}+\frac{1}{\beta}-\frac{1}{\gamma}-\frac{2ab}{(a+b+c)\Delta}.\cos^2\frac{c}{2}$

$$=\frac{2}{\sqrt{3}}+\frac{2}{\sqrt{3}}-\frac{2}{\sqrt{3}}-\frac{2\text{x}1\text{x}1}{(1+1+1)\,\text{x}\,\frac{\sqrt{3}}{4}}.\cos^2\left(\frac{60^0}{2}\right)$$

$$=\frac{2}{\sqrt{3}}-\frac{2}{3}\,\text{x}\,\frac{4}{\sqrt{3}}\,\text{x}\,\frac{3}{4}$$

$$=\frac{2}{\sqrt{3}}-\frac{2}{\sqrt{3}}=0$$

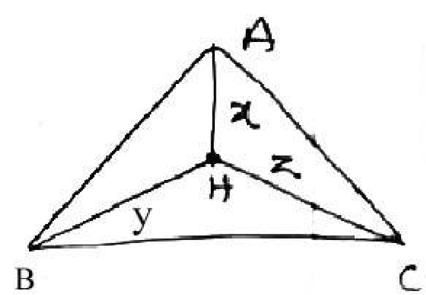
Ans : A

8. Given H is the orthountre of ΔABC we know, the distance of H from the vertex A is 2RsinA

AH = x = 2RcosA

y = 2RcosB

z = 2RcosC



Now $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=\frac{a}{2R\cos A}+\frac{b}{2R\cos B}+\frac{c}{2R\cos C}$

We know, $\frac{a}{2R}=\sin A,\ \frac{b}{2R}=\sin B,\ \frac{c}{2R}=\sin c$

$$=\frac{\sin A}{\cos A}+\frac{\sin B}{\cos B}+\frac{\sin C}{\cos C}$$

$= TanA + TanB + TanC$

In ΔABC, we know that

TanA + TanB + TanC = TanA.TanB.TanC

i.e $\Sigma \tan A = \pi \tan A$

$$\Rightarrow \Sigma \frac{\sin A}{\cos A} = \pi \frac{\sin A}{\cos A}$$

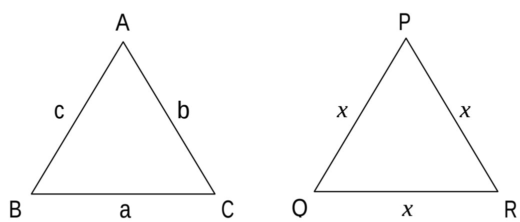
$$\Rightarrow \Sigma \frac{a}{2R\cos A} = \pi \frac{a}{2R\cos A}$$

$$\Rightarrow \Sigma \frac{a}{x} = \pi \frac{a}{x}$$

$$\Rightarrow \frac{a}{x} + \frac{b}{y} + \frac{c}{z} = \frac{a}{x}.\frac{b}{y}.\frac{c}{z}$$

Ans : A

9.



Given a,b,c are in A.P

$$\Rightarrow 2b = a + c$$

Perimeter of $\Delta ABC = a + b + c$

= a + c + b

= 2b + b

= 3b

Given, primeter of $\Delta ABC =$ perimeter of the equilateral ΔPQR

$$\therefore 3b = 3x$$

$$\Rightarrow b = x \qquad \text{.......... (i)}$$

Now, Area of $\Delta ABC = \frac{3}{5}$ x Area of ΔPQR

$$= \frac{3}{5} \text{x} \frac{\sqrt{3}}{4} x^2$$

$$= \frac{3\sqrt{3}}{20} b^2 \qquad \text{from (i)}$$

Ans : B

10. Given in ΔABC, a, b and A are given we have $\cos A = \frac{b^2+c^2-a^2}{2bc}$

$$\Rightarrow b^2+c^2-a^2 = 2bc\cos A$$

Also, we know that c_1+c_2 = 2bcosA

$c_1+c_2 = b^2-a^2$

We have, $c_1 - c_2 = \sqrt{(c_1+c_2)^2 - 4c_1c_2}$

$$= \sqrt{(2b\cos A)^2 - 4(b^2-a^2)}$$

$$= \sqrt{4b^2\cos^2 A - 4b^2 + 4a^2}$$

$$= \sqrt{4b^2 - 4b^2(1-\cos^2 A)}$$

$$= \sqrt{4a^2 - 4b^2\sin^2 A}$$

$$= \sqrt{4a^2 - (2b\cos A)^2 . Tan^2 A}$$

$$= \sqrt{4a^2 - (c_1+c_2)^2 . Tan^2 A}$$

Ans : A

Multiple correct answer type

11. Given the equation $c^2x^2 - c(a+b)x + ab = 0$.

Given sinA and sinB are the roots of the above equation,

We have $\sin A + \sin B = \frac{c(a+b)}{c^2}$

$$\Rightarrow \sin A + \sin B = \frac{a+b}{c} \qquad \text{........ (i)}$$

$$\Rightarrow \sin A + \sin B = \frac{2R\sin A + 2R\sin B}{2R\sin C}$$

$$\Rightarrow \sin A + \sin B = \frac{\sin A + \sin B}{\sin C}$$

$$\Rightarrow \sin c = 1$$

$$\Rightarrow c = \frac{\pi}{2}$$

$\therefore \Delta ABC$ is an right angled triangle also, since $C = \frac{\pi}{2}$

$$\Rightarrow A + B = \frac{\pi}{2}$$

$$\Rightarrow B = \frac{\pi}{2} - A$$

From (i), we have

$$\sin A + \sin B = \frac{a+b}{c}$$

$$\Rightarrow \sin A + \sin\left(\frac{\pi}{2} - A\right) = \frac{a+b}{c}$$

SinA+CosA = $= \dfrac{a+b}{c}$

Ans : B, D

12. Given, in ΔABC

$$\cos\left(\frac{A}{2}\right) = \sqrt{\frac{b+c}{2c}}$$

$$\Rightarrow \cos^2\left(\frac{A}{2}\right) = \frac{b+c}{2c}$$

$$\Rightarrow 2\cos^2\left(\frac{A}{2}\right) = \frac{b+c}{c}$$

$$\Rightarrow 1 + \cos A = \frac{b+c}{c}$$

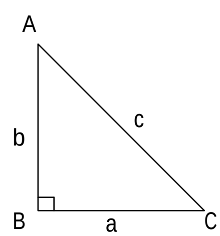
$$\Rightarrow \cos A = \frac{b+c}{c} - 1$$

$$\Rightarrow \cos A = \frac{b}{c}$$

$$\Rightarrow \frac{b^2 + c^2 - a^2}{2bc} = \frac{b}{c}$$

$$\Rightarrow b^2 + c^2 - a^2 = 2b^2$$

$$\Rightarrow a^2 + b^2 = c^2$$



ΔABC is a right angled triangle

$\therefore$ Area of $\Delta ABC = \frac{1}{2}$ x *base* x *height*

$$= \frac{1}{2} \text{ x } a \text{ x } b$$

Also, we know that circum radius $= \frac{1}{2}$ (hypotenuse)

$$= \frac{1}{2}.c$$

Ans : A,B

13. **Statement I:**

Given a,b,c are the sides of a ΔABC

Given $\frac{2a}{b+c-a}+\frac{2b}{c+a-b}+\frac{2c}{a+b-c}$

$$=\frac{2a}{b+c-a}+1+\frac{2b}{c+a-b}+1+\frac{2c}{a+b-c}+1-3$$

$$=\frac{a+b+c}{b+c-a}+\frac{a+b+c}{c+a-b}+\frac{a+b+c}{a+b-c}-3 \qquad \text{.......... (i)}$$

We know that, $A.M \geq H.M$

i.e $\frac{x+y+z}{3} \geq \frac{3}{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}$

$$\Rightarrow \left(\frac{1}{b+c-a}+\frac{1}{c+a-b}+\frac{1}{a+b-c}\right) \geq \frac{3}{b+c-a+c+a-b+a+b-c}$$

$$\Rightarrow \frac{1}{b+c-a}+\frac{1}{c+a-b}+\frac{1}{a+b-c} \geq \frac{9}{a+b+c}$$

$$\Rightarrow \frac{a+b+c}{b+c-a}+\frac{a+b+c}{c+a-b}+\frac{a+b+c}{a+b-c} \geq 9$$

$\therefore$ The minimum value of this quantity is 9.

$\therefore$ The minimum value of expression (i)

= 9 - 3

= 6

$\therefore$ statement I is FALSE

Statement II

$A.M \geq G.M \geq H.M$

Statement II is TRUE

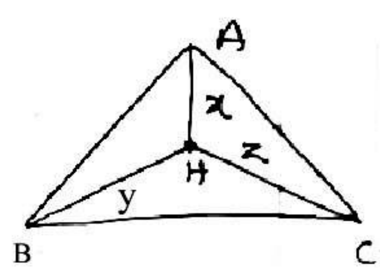
Ans : D

14. **Comprehension type :**

We know, $\Delta = \frac{1}{2}ax$

$\Rightarrow ax = 2\Delta$

Similarly by $= 2\Delta$ and $cz = 2\Delta$



Given $\frac{bx}{c}+\frac{cy}{a}+\frac{az}{b}=\frac{a^2+b^2+c^2}{K}$

$$\Rightarrow \frac{ab^2x+bc^2y+ca^2z}{abc}=\frac{a^2+b^2+c^2}{K}$$

$$\Rightarrow \frac{2\Delta b^2+2\Delta c^2+2\Delta a^2}{abc}=\frac{a^2+b^2+c^2}{K}$$

$$\Rightarrow \frac{2\Delta\left(a^2+b^2+c^2\right)}{abc} = \frac{a^2+b^2+c^2}{K}$$

$$\Rightarrow K = \frac{abc}{2\Delta}$$

$$\Rightarrow K = \frac{abc}{\left(\frac{abc}{2R}\right)} \qquad \text{since } \Delta = \frac{abc}{4R}$$

$$2\Delta = \frac{abc}{2R}$$

$$\Rightarrow K = 2R$$

Ans: C

15. Let ΔABC be an equilateral triangle with side 1 unit

We have A = B = C = 60^0

$$x = y = z = \frac{\sqrt{3}}{2}$$

Given $\cot A + \cot B + \cot C = K\left(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\right)$

$$\Rightarrow \cot 60^0 + \cot 60^0 + \cot 60^0 = K\left(\frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2}+\frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2}+\frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2}\right)$$

$$\Rightarrow \frac{1}{\sqrt{3}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{3}} = K\left(\frac{4}{3}+\frac{4}{3}+\frac{4}{3}\right)$$

$$\Rightarrow \frac{3}{\sqrt{3}} = \frac{12K}{3}$$

$$\Rightarrow K = \frac{\sqrt{3}}{4}$$

We know, Area of an equilateral triangle with side 1 Unit = $\frac{\sqrt{3}}{4}$

$$\therefore K = \Delta$$

Ans : C

16. let ΔABC be an equilateral triangle with side 1 Unit

We have A = B = C = 60^0

a = b = c = 1

x = y = z $= \frac{\sqrt{3}}{2}$

Given $\frac{c\sin B+b\sin C}{x}+\frac{a\sin C+c\sin A}{y}+\frac{b\sin A+a\sin B}{z}$

$$= 3\left(\frac{c\sin B+b\sin C}{x}\right)$$

$$= 3\left(\frac{1.\sin 60^0+1.\sin 60^0}{\left(\frac{\sqrt{3}}{2}\right)}\right)$$

$$= 3\left(\frac{2\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)}\right)$$

= 6

Ans : D

Comprehension II

17. Let a,b,c be the sides of ΔABC

We know, $S=\frac{a+b+c}{2}$

Also, s, s-a, s-b, s-c will be positive

We have A.M > G.M

$$\frac{s+(s-a)+(s-b)(s-c)}{4} > \left[s(s-a)(s-b)(s-c)\right]^{\frac{1}{4}}$$

$$\Rightarrow \frac{4s-(a+b+c)}{4} > \Delta^{\frac{1}{2}}$$

$$\Rightarrow \frac{2s}{4} > \Delta^{\frac{1}{2}}$$

$$\Rightarrow \frac{s}{2} > \Delta^{\frac{1}{2}}$$

$$\Rightarrow \frac{s^2}{4} > \Delta$$

$$\therefore \Delta < \frac{s^2}{4}$$

Ans : C

18. Given a+b+c = $6\left(\frac{\sin A+\sin B+\sin C}{3}\right)$

$\Rightarrow 2R\sin A+2R\sin B+2R\sin C = 2(\sin A+\sin B+\sin C)$

$\Rightarrow 2R = 2$

$\Rightarrow R = 1$

We have $\frac{a}{\sin A} = 2R$

$\Rightarrow \frac{a}{\sin A} = 2$

$\Rightarrow \frac{1}{\sin A} = 2$ Given a = 1

$\Rightarrow \sin A = \frac{1}{2}$

$\Rightarrow A = \frac{\pi}{6}$

Ans : A

19. Given the angle of ΔABC are in 2:3:7.

We have $A=\frac{2}{12}\text{x}180^0 = 2\text{x}15^0 = 30^0$

similarly B = 45^0

C = 105^0

We have $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

$\Rightarrow a:b:c = \sin A:\sin B:\sin C$

$\Rightarrow a:b:c = \sin 30^0 : \sin 45^0 : \sin 105^0$

$\Rightarrow a:b:c = \frac{1}{2}:\frac{1}{\sqrt{2}}:\frac{\sqrt{3}+1}{2\sqrt{2}}$

$\Rightarrow a:b:c = \sqrt{2}:2:\sqrt{3}+1$

Ans : A

20.

a) Given in ΔABC, a = 5, A = 30^0

We have $\frac{a}{\sin A} = 2R$

$\Rightarrow \frac{5}{\sin 30^0} = 2R$

$$\Rightarrow \frac{5}{\left(\frac{1}{2}\right)} = 2R$$

$$\Rightarrow 2R = 10$$

$$\Rightarrow R = 5$$

$\therefore$ The circum radius = 5

Ans : 5

b) Given in $\Delta ABC,\ a = \sqrt{3}+1,\ B = 30^0, C = 45^0$

We have A+B+C = 180^0

$$\Rightarrow A + 30^0 + 45^0 = 180^0$$

$$\Rightarrow A = 105^0$$

We have, $\frac{a}{\sin A} = 2R$

$$\Rightarrow \frac{\sqrt{3}+1}{\sin 105^0} = 2R$$

$$\Rightarrow \frac{\sqrt{3}+1}{\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)} = 2R$$

$$\Rightarrow 2R = 2\sqrt{2}$$

$$\Rightarrow R = \sqrt{2}$$

Now, $\frac{C}{\sin C} = 2R$

$$\Rightarrow \frac{C}{\sin 45^0} = 2\sqrt{2}$$

$$\Rightarrow \frac{C}{\left(\frac{1}{\sqrt{2}}\right)} = 2\sqrt{2}$$

$$\Rightarrow \sqrt{2}C = 2\sqrt{2}$$

$$\Rightarrow C = 2$$

Ans : 2

21. Let x, y be any two positive real numbers.
Given a is A.M of x, y

$$\therefore a = \frac{x+y}{2} \quad ....\ (i)$$

Given b, c are two G.M's between x and y

$\therefore$ x, b, c, y are in G.P

We have $b^2 = cx$ and $c^2 = by$ .... (ii)

Now, $\frac{\sin^3 B+\sin^3 C}{\sin A.\sin B.\sin C} = \frac{\left(\frac{b}{2R}\right)^3+\left(\frac{c}{2R}\right)^3}{\left(\frac{a}{2R}\right)\left(\frac{b}{2R}\right)\left(\frac{c}{2R}\right)}$

$= \frac{b^3+c^3}{abc}$

$= \frac{b^2.b+c^2.c}{abc}$

$= \frac{cx.b+by.c}{abc} \qquad (from(2))$

$= \frac{bc(x+y)}{bc\left(\frac{x+y}{2}\right)} \qquad (from(1))$

= 2

Ans : C

22. Matrix Matching Type

a) Given (a+b+c)(b+c-a) $= \lambda bc$

$\Rightarrow (a+b+c)(a+b+c-2a) = \lambda bc$

$\Rightarrow (2s)(2s-2a) = \lambda bc$

$\Rightarrow 4s(s-a) = \lambda bc$

$\Rightarrow \frac{s(s-a)}{bc} = \frac{\lambda}{4}$

$\Rightarrow \cos^2\frac{A}{2} = \frac{\lambda}{4}$

We know $0 \le \cos^2\frac{A}{2} \le 1$

$\Rightarrow 0 \le \frac{\lambda}{4} \le 1$

$\Rightarrow 0 \le \lambda \le 4$

$\therefore$ The greatest value of $\lambda = 4$

b) In ΔABC, Given tanA+tanB+Tanc = 9 and $\tan^2A+\tan^2B+\tan^2C$ = K

We know, in a ΔABC, we have TanA+TanB+TanC = TanA.TanB.TanC

$\therefore$ TanA.TanB.TanC = 9

We know $A.M \ge G.M$

$\Rightarrow \frac{Tan^2A+Tan^2B+Tan^2C}{3} \ge \left(Tan^2A.Tan^2B.Tan^2C\right)^{\frac{1}{3}}$

$$\Rightarrow \frac{K}{3} \geq (TanA.TanB.TanC)^{\frac{2}{3}}$$

$$\Rightarrow \frac{K}{3} \geq 9^{\frac{2}{3}}$$

$$\Rightarrow K \geq 3.9^{\frac{2}{3}}$$

$$\Rightarrow K \geq 3.3^{\frac{4}{3}}$$

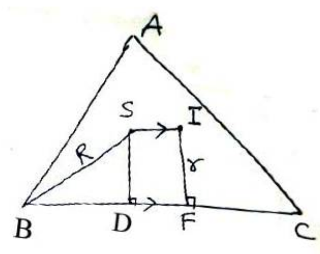
$$\Rightarrow K \geq 3^{\frac{7}{3}}$$

$$\Rightarrow K \geq 9.3^{\frac{1}{3}}$$

Hence, the greates - value of λ *is* $9.3^{\frac{1}{3}}$

c) In ΔABC, Let S be the circumuntre I be the Incentre
Given SI is parallel to BC.
We have SD = RcosA
But, from diagram, SD = IF = r

$\therefore$ r = R cos A



$$\Rightarrow R\cos A = 4R\sin\frac{A}{2}.\sin\frac{B}{2}.\sin\frac{C}{2}$$

$$\Rightarrow \cos A = 4\sin\frac{A}{2}.\sin\frac{B}{2}.\sin\frac{C}{2}$$

$$\Rightarrow \cos A = -1 + \cos A + \cos B + \cos C$$

$$\Rightarrow \cos B + \cos C = 1$$

d) Given a = 5, b = 4 and $\cos(A-B) = \frac{31}{32}$

We know, $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$$\therefore \cos(A-B) = \frac{1-\tan^2\left(\frac{A-B}{2}\right)}{1+\tan^2\left(\frac{A-B}{2}\right)}$$

$$\Rightarrow \frac{31}{32} = \frac{1-x}{1+x}, \quad Let\ x = \tan^2\left(\frac{A-B}{2}\right)$$

$$\Rightarrow 31 + 31x = 32 - 32x$$

$$\Rightarrow 63x = 1$$

$$\Rightarrow \ x = \frac{1}{63}$$

$$\Rightarrow \ Tan^2\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}.\cot\frac{c}{2}$$

$$\Rightarrow \ \frac{1}{\sqrt{63}} = \frac{5-4}{5+4}.\cot\frac{c}{2}$$

$$\Rightarrow \ \cot\frac{c}{2} = \frac{9}{\sqrt{63}}$$

$$\Rightarrow \ Tan\frac{c}{2} = \frac{\sqrt{63}}{9}$$

We know

$$\cos C \ = \ \frac{1-\tan^2\frac{c}{2}}{1+\tan^2\frac{c}{2}}$$

$$= \ \frac{1-\frac{63}{81}}{1+\frac{63}{81}}$$

$$= \ \frac{18}{144}$$

$$= \ \frac{1}{8}$$

We know,

$c^2 = a^2+b^2-2ab\cos C$

$= 25+16 - 2x5x4x\frac{1}{8}$

$= 36$

$\therefore \ c = 6$

Ans : a-p, b-q, c-r, d-s

23.

a) Given A = 60^0

We have $\cos A = \frac{b^2+c^2-a^2}{2bc}$

$$\Rightarrow \cos 60^0 = \frac{b^2+c^2-a^2}{2bc}$$

$$\Rightarrow \frac{1}{2} = \frac{b^2+c^2-a^2}{2bc}$$

$$\Rightarrow bc = b^2+c^2-a^2$$

$$\Rightarrow b^2+c^2 = a^2+bc$$ .... (i)

Now, $\frac{b}{c+a}+\frac{c}{a+b}$

$$= \frac{ab+b^2+c^2+ac}{(c+a)(a+b)}$$

$$= \frac{ab+a^2+bc+ac}{(c+a)(a+b)}$$ from (i)

$$= \frac{a(b+a)+c(b+a)}{(c+a)(a+b)}$$

$$= \frac{(a+b)(a+c)}{(c+a)(a+b)}$$

= 1

b) Given C = 60^0

We have $\cos C = \frac{a^2+b^2-c^2}{2ab}$

$$\Rightarrow \cos 60^0 = \frac{a^2+b^2-c^2}{2ab}$$

$$\Rightarrow \frac{1}{2} = \frac{a^2+b^2-c^2}{2ab}$$

$$\Rightarrow ab = a^2+b^2-c^2$$

$$\Rightarrow a^2+b^2 = ab+c^2$$ .... (i)

Now, $\frac{1}{a+c}+\frac{1}{b+c}-\frac{3}{a+b+c}$

$$= \frac{b+c+a+c}{(a+c)(b+c)}-\frac{3}{a+b+c}$$

$$= \frac{a+b+2c}{ab+ac+bc+c^2}-\frac{3}{(a+b+c)}$$

$$= \frac{(a+b+2c)(a+b+c)-3(ab+bc+ca+c^2)}{(ab+ac+bc+c^2)(a+b+c)}$$

$$= \frac{a^2+ab+ac+ba+b^2+bc+2ac+2bc+2c^2-3ab-3bc-3ca-3c^2}{\left(ab+ac+bc+c^2\right)\left(a+b+c\right)}$$

$$= \frac{a^2+b^2-c^2-ab}{\left(ab+ac+bc+c^2\right)\left(a+b+c\right)}$$

$$= \frac{0}{\left(ab+bc+ca+c^2\right)\left(a+b+c\right)} \qquad \text{[from (i)]}$$

= 0

$$\therefore \frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$$

c) Given B = 60^0

We have $\cos B = \frac{c^2+a^2-b^2}{2ca}$

$$\Rightarrow \cos 60^0 = \frac{c^2+a^2-b^2}{2ca}$$

$$\Rightarrow \frac{1}{2} = \frac{c^2+a^2-b^2}{2ca}$$

$$\Rightarrow c^2+a^2-b^2-ac=0 \qquad \text{............. (i)}$$

Consider $\frac{1}{a+b}+\frac{1}{b+c}-\frac{3}{a+b+c}$

$$= \frac{(b+c)(a+b+c)+(a+b)(a+b+c)-3(a+b)(b+c)}{(a+b)(b+c)(a+b+c)}$$

$$= \frac{ab+b^2+bc+ca+cb+c^2+a^2+ab+ac+ab+b^2+bc-3ab-3ac-3b^2-3bc}{(a+b)(b+c)(a+b+c)}$$

$$= \frac{c^2+a^2-b^2-ac}{(a+b)(b+c)(a+b+c)}$$

$$= \frac{0}{(a+b)(b+c)(a+b+c)} \qquad \text{From (i)}$$

= 0

$$\frac{1}{a+b}+\frac{1}{b+c}=\frac{3}{a+b+c}$$

b) Given c = 60^0

$$\Rightarrow \cos c = \frac{a^2+b^2-c^2}{2ab}$$

$\Rightarrow \cos 60^0 = \dfrac{a^2+b^2-c^2}{2ab}$

$\Rightarrow \dfrac{1}{2} = \dfrac{a^2+b^2-c^2}{2ab}$

$\Rightarrow a^2+b^2-c^2-ab=0$ .... (i)

Now, $\dfrac{b}{c^2-a^2}+\dfrac{a}{c^2-b^2}$

$= \dfrac{b\left(c^2-b^2\right)+a\left(c^2-a^2\right)}{\left(c^2-a^2\right)\left(c^2-b^2\right)}$

$= \dfrac{bc^2-b^3+ac^2-a^3}{\left(c^2-a^2\right)\left(c^2-b^2\right)}$

$= \dfrac{c^2\left(a+b\right)-\left(a^3+b^3\right)}{\left(c^2-a^2\right)\left(c^2-b^2\right)}$

$= \dfrac{c^2\left(a+b\right)-\left(a+b\right)\left(a^2-ab+b^2\right)}{\left(c^2-a^2\right)\left(c^2-b^2\right)}$

$= \dfrac{\left(a+b\right)\left(c^2-a^2+ab-b^2\right)}{\left(c^2-a^2\right)\left(c^2-b^2\right)}$

$= \dfrac{\left(a+b\right)\left(0\right)}{\left(c^2-a^2\right)\left(c^2-b^2\right)}$ [From (i)]

= 0

Ans : A-s, B-r, C-r, d-q

24.

a) Given $\left(a-b\right)^2\cos^2\dfrac{c}{2}+\left(a+b\right)^2\sin^2\dfrac{c}{2}$

$= \left(a-b\right)^2\cos^2\dfrac{c}{2}+\left(a+b\right)^2\left(1-\cos^2\dfrac{c}{2}\right)$

$= \left(a-b\right)^2\cos^2\dfrac{c}{2}+\left(a+b\right)^2-\left(a+b\right)^2.\cos^2\dfrac{c}{2}$

$= \left[\left(a-b\right)^2-\left(a+b\right)^2\right]\cos^2\dfrac{c}{2}+\left(a+b\right)^2$

$= \left(a+b\right)^2-4ab\cos^2\dfrac{c}{2}$

$= a^2 + b^2 + 2ab - 4ab\cos^2 \frac{c}{2}$

$= a^2 + b^2 - 2ab\left(2\cos^2 \frac{c}{2} - 1\right)$

$= a^2 + b^2 - 2ab\cos c$

= c²

b) $\frac{b^2 - c^2}{a^2}.\sin^2 A + \frac{c^2 - a^2}{b^2}.\sin^2 B$

$= \frac{b^2 - c^2}{a^2}\left(\frac{2\Delta}{bc}\right)^2 + \frac{c^2 - a^2}{b^2}\left(\frac{2\Delta}{ac}\right)^2$

$= \frac{b^2 - c^2}{a^2}\left(\frac{4\Delta}{b^2c^2}\right)^2 + \frac{c^2 - a^2}{b^2}\left(\frac{4\Delta^2}{a^2c^2}\right)$

$= \frac{4\Delta^2}{a^2b^2c^2}\left(b^2 - c^2 + c^2 - a^2\right)$

$= \left(b^2 - a^2\right)\frac{4}{a^2b^2c^2} \text{ x } \frac{a^2b^2c^2}{16R^2}$

$= \frac{b^2 - a^2}{4R^2}, \text{ since } \Delta = \frac{abc}{4R}$

c) Let ΔABC be an equilateral triangle with side 1unit.

We have Area of $\Delta ABC = \frac{\sqrt{3}}{4}a^2$

$\therefore \Delta = \frac{\sqrt{3}}{4}$

Now, $\frac{bc\cos A + ca\cos B + ab\cos C}{\cot A + \cot B + \cot C}$

$= \frac{1.1.\cos 60^0 + 1.1.\cos 60^0 + 1.1\cos 60^0}{\cot 60^0 + \cot 60^0 + \cot 60^0}$

$= \frac{\left(\frac{3}{2}\right)}{\left(\frac{3}{\sqrt{3}}\right)} = \frac{\sqrt{3}}{2}$

Now $2\Delta = 2.\frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

d) Let ΔABC, be an equilateral triangle with side 1unit

Now, $\dfrac{\cos C+\cos A}{c+a}+\dfrac{\cos B}{b}$

$= \dfrac{\cos 60^0+\cos 60^0}{1+1}+\dfrac{\cos 60^0}{1}$

$= \dfrac{1}{2} + \dfrac{1}{2}$

Now, $\dfrac{1}{b} = \dfrac{1}{1} = 1$

Ans : a-p, b-s, c-r, d-q

LEARNERS TASK

1. Let a = 3, A = 60^0

We have $\dfrac{a}{\sin A} = 2R$

$\Rightarrow \dfrac{3}{\sin 60^0} = 2R$

$\Rightarrow \dfrac{3}{\left(\dfrac{\sqrt{3}}{2}\right)} = 2R$

$\Rightarrow R = \sqrt{3}$

Ans : B

2. Given a = 10cm, R = 5cm

We have $\dfrac{a}{\sin A} = 2R$

$\Rightarrow \dfrac{10}{\sin A} = 2\text{x}5$

$\Rightarrow \sin A = 1$

$\Rightarrow A = 90^0$

Ans : D

3. Given A=30^0, C = 90^0, $C = 7\sqrt{3}$

We have $\dfrac{c}{\sin C} = 2R$

$\Rightarrow \dfrac{7\sqrt{3}}{\sin 90^0} = 2R$

$\Rightarrow 2R = 7\sqrt{3}$ .... (i)

Now, $\frac{a}{\sin A}=2R$

$\Rightarrow \frac{a}{\sin 30^0} = 7\sqrt{3}$

$\Rightarrow a = \frac{7\sqrt{3}}{2}$

Ans : C

4. Given a = 20, b = 21 and $\cos C = \frac{4}{5}$

We have $\cos C = \frac{a^2+b^2-c^2}{2ab}$

$\Rightarrow \frac{4}{5} = \frac{400+441-c^2}{840}$

$\Rightarrow c^2 = 169$

$\Rightarrow c = 13$

Ans : B

5. Given a = 2, b = 3, c = 4

Now, $\cos A = \frac{b^2+c^2-a^2}{2bc}$

$= \frac{9+16-4}{2x3x4}$

$= \frac{7}{8}$

Ans : A

6. Given a = 5, s = 15, $\Delta=30$

Now, $\tan\frac{A}{2} = \frac{\Delta}{S(S-a)}$

$\Rightarrow \tan\frac{A}{2} = \frac{30}{15(15-5)}$

$\Rightarrow \tan\frac{A}{2} = \frac{1}{5}$

Ans : D

7. Given, b = 60, s = 66, $\Delta = 330$

Now, $\tan\frac{B}{2} = \frac{\Delta}{S(S-a)}$

$$= \frac{330}{60(66-60)}$$

$$= \frac{5}{6}$$

$$\therefore \cot\frac{B}{2} = \frac{6}{5}$$

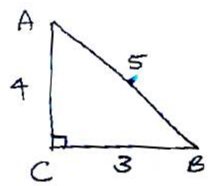
Ans : B

8. Given a = 3, b = 4, c = 5

$$\therefore C = 90^0$$

$$\cos\frac{C}{2} = \cos 45^0$$

$$= \frac{1}{\sqrt{2}}$$



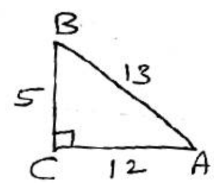
Ans : D

9. Given a = 5, b = 12, c = 13

We have C = 90^0

$$\therefore \sin\frac{c}{2} = \sin 45^0$$

$$= \frac{1}{\sqrt{2}}$$



Ans : A

10. Let ΔABC be an equilateral triangle with side 1 unit

We have $\Delta = \frac{\sqrt{3}}{4}$ and $S = \frac{3}{2}$

Now, abc $\sin\frac{A}{2}.\sin\frac{B}{2}.\sin\frac{C}{2}$

$$= 1.1.1 \sin 30^0.\sin 30^0.\sin 30^0$$

$$= \frac{1}{8}$$

Now, $\frac{\Delta^2}{S} = \frac{3}{16} \text{x} \frac{2}{3} = \frac{1}{8}$

Ans : B

JEE MAINS LEVEL QUESTIONS

1. Let ΔABC be an equilateral triangle with side 1 unit

Now, $\frac{\sin(B-C)}{bc} = \frac{\sin\left(60^0 - 60^0\right)}{1 \text{x} 1}$

$= 0$

$\therefore \sum \frac{\sin\left(B-C\right)}{bc} = 3 \text{x} 0$

$= 0$

Ans : C

2. Let ΔABC be an equilateral triangle with side 1unit

we have $\Delta = \frac{\sqrt{3}}{4}$

Now, $b^2\sin2C+c^2\sin2B$

$= (1)^2\sin2 \text{ x } 60^0 + (1)^2.\sin2 \text{ x } 60^0$

$= \sin120^0+\sin120^0$

$= 2 \sin120^0$

$= 2 \text{x} \frac{\sqrt{3}}{2}$

$= \sqrt{3}$

Now $4\Delta = 4\text{x}\frac{\sqrt{3}}{4} = \sqrt{3}$

Ans : A

3. Given acosA = bcosB

$\Rightarrow$ 2RsinAcosA = 2RsinBcosB

$\Rightarrow$ sin2A = sin2B

$\Rightarrow$ 2A = 2B or $2A = 180^0 - 2B$

$\Rightarrow$ A = B or $A+B = 90^0$

$\Rightarrow C = 90^0$

$\therefore \Delta ABC$ is right angled isoscales triangle

Ans : C

4. Given $\frac{b}{c+a} + \frac{c}{a+b} = 1$

$\Rightarrow \frac{ab+b^2+c^2+ac}{\left(c+a\right)\left(a+b\right)} = 1$

$\Rightarrow ab+b^2+c^2+ac = ac+bc+a^2+ab$

$\Rightarrow ab+b^2+c^2+ac - ac - bc - a^2 - ab = 0$

$\Rightarrow b^2+c^2-a^2 = bc$

$$\Rightarrow \frac{b^2+c^2-a^2}{bc} = 1$$

$$\Rightarrow \frac{b^2+c^2-a^2}{2bc} = \frac{1}{2}$$

$$\Rightarrow \cos A = \frac{1}{2}$$

$$\Rightarrow A = 60^0$$

Ans : A

5. Let $a=\sqrt{2}, b=\sqrt{6}, c=\sqrt{8}$

Now, $\cos A = \frac{b^2+c^2-a^2}{2bc}$

$$= \frac{6+8-2}{2x\sqrt{6}x\sqrt{8}} = \frac{\sqrt{3}}{2}$$

$\therefore A = 30^0$

Similarly B = 60^0, C = 90^0

Ans : C

6. Let ΔABC be an equilateral triangle with side 1unit

$\therefore a=b=c=1$

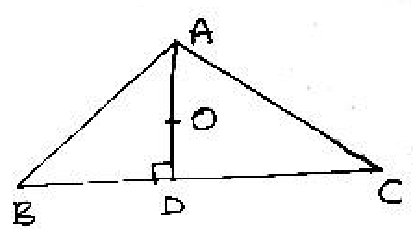
Also $AD = \frac{\sqrt{3}}{2}$

We know, the orthocentre divides the altihide AD in the ratio 2 : 1

$$\therefore AO = \frac{2}{3}xAD$$

$$= \frac{2}{3}x\frac{\sqrt{3}}{2}$$

$$= \frac{1}{\sqrt{3}}$$



$\therefore p=q=r=\frac{1}{\sqrt{3}}$

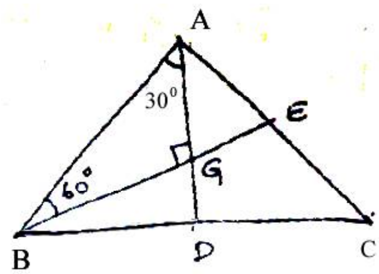
Now aqr = $=1x\frac{1}{\sqrt{3}}x\frac{1}{\sqrt{3}} = \frac{1}{3}$

$\therefore aqr+brp+cpq = 3x\frac{1}{3}=1$

Now abc = 1x1x1 = 1

Ans : B

7. Given AD = 4 since centroid (G) divides AD in the ratio 2 : 1



We have $AG = \frac{2}{3} \text{x} 4 = \frac{8}{3}$

Now $Tan 60^0 = \frac{AG}{BG}$

$\Rightarrow \sqrt{3} = \frac{8}{3.BG}$

$\Rightarrow BG = \frac{8}{3.\sqrt{3}}$

Area of $\Delta ABG = \frac{1}{2} \text{x} BG \text{x} AG$

$= \frac{1}{2} \text{x} \frac{8}{3\sqrt{3}} \text{x} \frac{8}{3}$

$= \frac{32}{9\sqrt{3}}$

We know, Area of $\Delta ABC = 3 \text{x}$ Area of ΔABG

$= 3\text{x} \frac{32}{9\sqrt{3}}$

$= \frac{32}{3\sqrt{3}}$

Ans : C

8. Given (a+b+c) (b+c-a) = 3bc

$\Rightarrow (2s)(2s-2a) = 3bc$

$\Rightarrow 4s(s-a) = 3bc$

$\Rightarrow \frac{s(s-a)}{bc} = \frac{3}{4}$

$\Rightarrow \cos^2 \frac{A}{2} = \frac{3}{4}$

$\Rightarrow \cos \frac{A}{2} = \frac{\sqrt{3}}{2}$

$\Rightarrow \frac{A}{2} = 30^0$

$\Rightarrow A = 60^0$

Ans : A

9. Given $1-\tan\frac{A}{2}.\tan\frac{B}{2}$

$$=1-\sqrt{\frac{(s-b)(s-c)}{s(s-a)}\text{x}\frac{(s-a)(s-c)}{s(s-b)}}$$

$$=1-\frac{s-c}{s}$$

$$=\frac{c}{s}$$

$$=\frac{2c}{2s}=\frac{2c}{a+b+c}$$

Ans : C

10. Given a,b,c are in A.P

$\Rightarrow 2b=a+c$ .... (i)

Now, $\frac{2\sin\frac{A}{2}.\sin\frac{C}{2}}{\sin\frac{B}{2}}$

$$=\frac{2\sqrt{\frac{(s-b)(s-c)}{bc}}.\sqrt{\frac{(s-a)(s-b)}{ab}}}{\sqrt{\frac{(s-a)(s-c)}{ac}}}$$

$$=2.\frac{(s-b)}{b}$$

$$=\frac{2s-2b}{b}$$

$$=\frac{a+b+c-2b}{b}$$

$$=\frac{a+c-b}{b}$$

$$=\frac{2b-b}{b} \quad \text{[from(i)]}$$

$= 1$

Ans : A

11. Let a be the side of the equilateral triangle. Let x be the side of the regular hexagon.

Given perimeters are equal

$\therefore 3a=6x$

$\Rightarrow\ a = 2x$ .... (i)

We know, Area of equilateral triangle

$= \frac{\sqrt{3}}{4}a^2$

Area of regular hexagon $= 6\text{x}\frac{\sqrt{3}}{4}x^2$

Ratio $= \frac{\sqrt{3}}{4}a^2 : 6\text{x}\frac{\sqrt{3}}{4}x^2$

$= a^2 : 6x^2$

$= 4x^2 : 6x^2$ [From (i)]

= 2 : 3

Ans : C

ADVANCED LEVEL QUETIONS

12. Given $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$

$\Rightarrow \frac{\cos A}{2R\sin A} = \frac{\cos B}{2R\sin B} = \frac{\cos C}{2R\sin C}$

$\Rightarrow \cos A = \cos B = \cos C$

$\Rightarrow A = B = C$

$\therefore \Delta ABC$ is an equilateral triangle

Given a = 2 cm

Area $= \frac{\sqrt{3}}{4}\text{x}a^2$

$= \frac{\sqrt{3}}{4}\text{x}(2)^2$

$= \sqrt{3}$

Ans : A,B

13. Let ΔABC, be an equilateral triangle with side 1 unit.

$\therefore\ a = b = c = 1$

We have $S = \frac{a+b+c}{2}$

$\Rightarrow S = \frac{3}{2}$

and $\Delta = \frac{\sqrt{3}}{4}, \quad R = \frac{1}{\sqrt{3}}$

Now, $\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{c}{2}$

$= \cot 30^0+\cot 30^0+\cot 30^0$

$= 3\sqrt{3}$

Now, $\frac{s^2}{\Delta} = \frac{\left(\frac{3}{2}\right)^2}{\left(\frac{\sqrt{3}}{4}\right)} = 3\sqrt{3}$

Also $\frac{(a+b+c)^2}{abc}.R$

$= \frac{(1+1+1)^2}{1.1.1}.\frac{1}{\sqrt{3}}$

$= \frac{9}{\sqrt{3}} = 3\sqrt{3}$

Ans : A,C

Statement Type :

14. **Statement I:**

Given 4s(s-a)(s-b)(s-c) = a^2b^2

$\Rightarrow 2\sqrt{s(s-a)(s-b)(s-c)} = ab$

$\Rightarrow 2\Delta = ab$

$\Rightarrow \Delta = \frac{1}{2}\text{ x } a \text{ x } b$

$\Rightarrow \Delta = \frac{1}{2}$ x base x hypotenuse

$\therefore \Delta ABC$ is a right angled triangle

Hence, statement I is TRUE

Statement II: For $x\in[0,\pi]$

Let f(x) = sinx

We have

$\frac{1}{3}f(A)+\frac{1}{3}f(B)+f(C) \le f\left[\frac{1}{3}(A+B+C)\right]$

$\le f\left(60^0\right)$

$\le \sin 60^0$

$$\leq \frac{\sqrt{3}}{2}$$

The inequality is achieved, when $A = B = C = \frac{\pi}{3}$

Hence, ΔABC is an equilateral triangle.

Statement II is TRUE

Ans : A

5. **Statement I :**

Given the angles of ΔABC are in A.P,

Let A = B = C = 60^0 ($\because$ common difference = 0^0)

Now, cosA+cosB+cosC

= $\cos 60^0 + \cos 60^0 + \cos 60^0$

$= 3\frac{1}{2}$

$= \frac{3}{2}$

Statement I is FALSE

Statement II :

In ΔABC, Given A,B,C are in A.P and cosA+cosB+cosC = 2

Which is not possible for any angles of ΔABC

Hence, Statement II is FALSE

Ans : B

Comprehension Type

Comprehension-III

Given in a ΔABC

$a^2 = b^2+c^2-2bc \cos A$

$b^2 = c^2+a^2-2ac \cos B$

$c^2 = a^2+b^2-2ab \cos C$

16. Given a = 2, b = 4, $\underline{|C} = 60^{00}$

$c^2 = a^2+b^2-2ab \cos C$

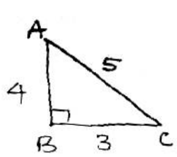
$= 4+16-2.2.4.\cos 60^0$

$= 20 - 16\frac{1}{2}$

= 12

$\therefore\ c = \sqrt{12}$

17. Given a = 3, b = 5, c = 4 clearly $\underline{|B} = 90^0$



Ans : C

18. Let ΔABC be an equilateral triangle with side 1 unit

We have A = B = C = 60^0, a = b = c = 1 and $\Delta = \frac{\sqrt{3}}{4}$

Now, $a^2+b^2+c^2 = 1^2+1^2+1^2 = 3$

Now, $4\Delta(\cot A+\cot B+\cot C)$

$$= 4x\frac{\sqrt{3}}{4}(\cot 60^0+\cot 60^0+\cot 60^0)$$

$$= \sqrt{3}.3.\frac{1}{\sqrt{3}}$$

= 3

Ans : A

Comprehension IV :

Given a = 6, b = 3, cos(A-B) = $\frac{4}{5}$ and AD is the median throngh A,

$\underline{|BAD} = \theta$, CL is perpendicular to AD.

19. We know, $Tan^2\left(\frac{A-B}{2}\right) = \frac{1-\cos(A-B)}{1+\cos(A-B)}$

$$= \frac{1-\left(\frac{4}{5}\right)}{1+\left(\frac{4}{5}\right)}$$

$$= \frac{1}{9}$$

$$\therefore\ Tan\left(\frac{A-B}{2}\right) = \frac{1}{3}$$

$$\therefore\ Tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}\cot\left(\frac{C}{2}\right)$$

$$\Rightarrow \frac{1}{3} = \frac{6-3}{6+3}\cot\frac{C}{2}$$

$$\Rightarrow \cot\frac{C}{2} = 1$$

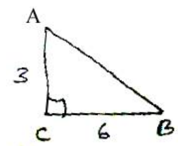
$$\Rightarrow \frac{C}{2} = \frac{\pi}{4}$$

$$\Rightarrow C = \frac{\pi}{2}$$

$\therefore \Delta ABC$ is a right angled triangle

Ans : B

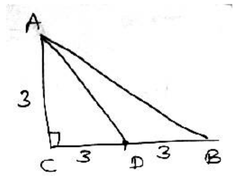
20. Area of $\Delta ABC = \frac{1}{2}.6.3$



$= 9$

Ans : A

21. $AD^2 = 3^2+3^2$



$= 18$

$\therefore\ AD = 3\sqrt{2}$

Ans : C

Integer Answer Type

22. Given $\cos A+\sin A-\frac{2}{\cos B+\sin B}=0$

$\Rightarrow \cos A+\sin A=\frac{2}{\cos B+\sin B}$

$\Rightarrow (\cos A+\sin A)(\cos B+\sin B)=2$

$\Rightarrow \cos A\cos B+\cos A\sin B+\sin A\cos B+\sin A\sin B=2$

$\Rightarrow (\cos A\cos B+\sin A\sin B)+(\sin A\cos B+\cos A\sin B)=2$

$\Rightarrow \cos(A-B)+\sin(A+B)=2$

one of the possibility is

A-B = 0^0 .... (i)

A+B = 90^0 .... (ii)

solving (i) and (ii), we get

A = 45^0, B = 45^0

Also, we have C = 90^0

Now, $\left(\frac{a+b}{c}\right)^4 = \left(\frac{2R\sin A+2R\sin B}{2R\sin C}\right)^4$

$= \left(\frac{\sin A+\sin B}{\sin C}\right)$

$= \left(\frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{\sin C}\right)^4$

$= \left(\frac{2\sin 45^0 \cos 0^0}{\sin 90^0}\right)^4$

$$= \left(\frac{2}{\sqrt{2}} \right)^4$$

$$= \left(\sqrt{2} \right)^4$$

= 4

Ans : 4

23. Given a+b-c = 2 and 2ab-c^2 = 4

Now, 2ab-c^2 = 4

$$\Rightarrow 2ab - c^2 = 2^2$$

$$\Rightarrow 2ab - c^2 = (a + b - c)^2$$

$$\Rightarrow 2ab - c^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$$

$$\Rightarrow a^2 + c^2 - 2ca + b^2 + c^2 - 2bc = 0$$

$$\Rightarrow (a - c)^2 + (b - c)^2 = 0$$

$$\Rightarrow a - c = 0 \quad and \quad b - c = 0$$

$$\Rightarrow a = c = b$$

Hence, ΔABC is an equilateral triangle.

Now, a+b-c = 2

$\Rightarrow$ a+a-a = 2

$\Rightarrow$ a = 2

$$(Area\ of\ \Delta ABC)^2 = \left(\frac{\sqrt{3}}{4} a^2 \right)^2$$

$$= \left(\frac{\sqrt{3}}{4} 2^2 \right)^2$$

= 3

Ans : B

24. **Matrix Matching Type**

a) Let ΔABC be an equilateral triangle with side 1 unit

We have A = B = C = 60^0

a = b = c = 1

and $\Delta = \frac{\sqrt{3}}{4}$

Now, a^2sin2C+c^2sin2A

= 1^2.sin2.60^0 + 1^2.sin2.60^0

= sin 120^0 + sin120^0

$$= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$$

$= \sqrt{3}$

Now, $4\Delta = 4.\frac{\sqrt{3}}{4} = \sqrt{3}$

b) $R^2(\sin 2A+\sin 2B+\sin 2C)$

$= \left(\frac{1}{\sqrt{3}}\right)^2 \left(\sin 120^0 + \sin 120^0 + \sin 120^0\right)$

$= \frac{1}{3} \text{ x } 3 \text{ x} \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{2}$

Now, $2\Delta = 2\text{x}\frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

c) $c\cos^2\frac{A}{2} + a.\cos^2\frac{C}{2}$

$= 1.\cos^2 30^0 + 1.\cos^2 30^0$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$

$= \frac{3}{4} + \frac{3}{4}$

$= \frac{3}{2}$

Now, $S = \frac{a+b+c}{2} = \frac{1+1+1}{2} = \frac{3}{2}$

D) $a\sin^2\frac{C}{2} + c\sin^2\frac{A}{2}$

$= 1.\sin^2 30^0 + 1.\sin^2 30^0$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

$= \frac{1}{4} + \frac{1}{4}$

$= \frac{1}{2}$

Now, s-b $= \frac{3}{2} - 1 = \frac{1}{2}$

Ans : A-p, B-q, c-r, D-s

ADDITIONAL PRACTICE QUESTIONS FOR STUDENTS

1. Let a = 1, b = 2004

Let the third side be c, which is an Integer

We have a+b > c

$\Rightarrow$ 1 + 2004 > c

$\Rightarrow$ 2005 > c

Also b - a < c

$\Rightarrow$ 2004 -1 < c

$\Rightarrow$ 2003 < c

$\therefore 2003 < c < 2005$

$\therefore \ c = 2004$

Perimeter $\Delta ABC = a+b+c$

= 1 + 2004 + 2004

= 4009

Ans : B

2. Given $a\cos^2\frac{c}{2}+c.\cos^2\frac{A}{2}=\frac{3b}{2}$

$$\Rightarrow a\frac{s(s-c)}{ab}+c.\frac{s(s-a)}{bc}=\frac{3b}{2}$$

$$\Rightarrow s^2-sc+s^2-sa=\frac{3b^2}{2}$$

$$\Rightarrow 2s^2-s(a+c)=\frac{3b^2}{2}$$

$$\Rightarrow 2s^2-s(2s-b)=\frac{3b^2}{2}$$

$$\Rightarrow bs=\frac{3b^2}{2}$$

$$\Rightarrow 2sb=3b^2$$

$$\Rightarrow (a+b+c)b=3b^2$$

$$\Rightarrow ab+b^2+bc=3b^2$$

$$\Rightarrow ab+bc=2b^2$$

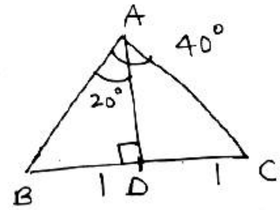
$$\Rightarrow a+c=2b$$

$\Rightarrow a,b,c$ are in A.P

Ans : A

3. Given a regular polygon of 9 sides is inscribed in an equilateral triangle.

We have $\theta = \frac{360^0}{9} = 40^0$



In the diagram AB = radius of the circle

$\sin 20^0 = \frac{BD}{AB}$

$\Rightarrow \sin 20^0 = \frac{1}{AB}$

$\Rightarrow AB = \frac{1}{\sin 20^0}$

$\Rightarrow AB = \operatorname{cosec} 20^0$

$\Rightarrow AB = \operatorname{cosec}\frac{\pi}{9}$

Ans : A

4. Let ΔABC be an equilateral triangle with each side 1 unit

We have a = b = c = 1

Now, $(b-c)\cot\frac{A}{2} + (c-a)\cot\frac{B}{2} + (a-b)\cot\frac{C}{2}$

$= \Sigma(b-c)\cot\frac{A}{2}$

$= \Sigma(1-1)\cot 30^0$

$= \Sigma\ 0.\sqrt{3}$

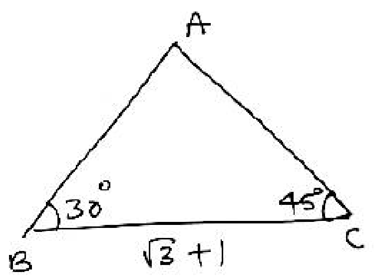
= 0

Ans : D

5. Let B = 30^0, C = 45^0

a = BC = $\sqrt{3}+1$

clearly A = 105^0



We have $\frac{a}{\sin A} = 2R$

$\Rightarrow \frac{\sqrt{3}+1}{\sin 105^0} = 2R$

$\Rightarrow \frac{\sqrt{3}+1}{\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)} = 2R$

$\Rightarrow R = \sqrt{2}$

Now, $\dfrac{b}{\sin B} = 2R$

$\Rightarrow \dfrac{b}{\sin 30^0} = 2\sqrt{2}$

$\Rightarrow \dfrac{b}{\left(\frac{1}{2}\right)} = 2\sqrt{2}$

$\Rightarrow b = \sqrt{2}$

Now, Area of $\Delta ABC = \dfrac{1}{2} ab \sin C$

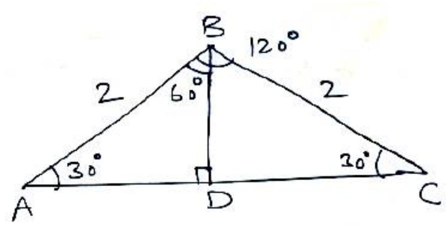
$= \dfrac{1}{2}\left(\sqrt{3}+1\right)\left(\sqrt{2}\right)\sin 45^0$

$= \dfrac{1}{2}\left(\sqrt{3}+1\right)\left(\sqrt{2}\right)\dfrac{1}{\sqrt{2}}$

$= \dfrac{\sqrt{3}+1}{2}$

Ans : A

6.



Now, $\sin 60^0 = \dfrac{AD}{AB}$

$\Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{AD}{2}$

$\Rightarrow AD = \sqrt{3}$

$\therefore b = AC = 2AD = 2\sqrt{3}$

Given a = 2

Area of $\Delta\ ABC = \dfrac{1}{2} ab \sin C$

$= \dfrac{1}{2}.2.2\sqrt{3}.\sin 30^0$

$$= 2\sqrt{3}.\frac{1}{2}$$

$$= \sqrt{3}$$

Ans : B

7. Given $\sin\frac{A}{2}.\cos\frac{B}{2} = \left(\frac{a+c-b}{2c}\right)K$

$$\Rightarrow \sqrt{\frac{(a-b)(s-c)}{bc}\frac{s(s-b)}{ac}} = \frac{(2s-2b)}{2c}K$$

$$\Rightarrow \frac{s-b}{c}\sqrt{\frac{s(s-c)}{ab}} = \frac{s-b}{c}K$$

$$\Rightarrow K = \cos\frac{C}{2}$$

Ans : D

8. Given $\frac{2\cos A}{a}+\frac{\cos B}{b}+\frac{2\cos C}{c} = \frac{a}{bc}+\frac{b}{ca}$

$$\Rightarrow \frac{2\left(b^2+c^2-a^2\right)}{2abc}+\frac{c^2+a^2-b^2}{abc}+\frac{2\left(a^2+b^2-c^2\right)}{2abc} = \frac{a^2+b^2}{abc}$$

$$\Rightarrow b^2+c^2-a^2+c^2+a^2-b^2+a^2+b^2-c^2 = a^2+b^2$$

$$\Rightarrow a^2 = b^2+c^2$$

$\Rightarrow \Delta ABC$ is a right angled triangle with $\underline{|A} = 90^0$

Ans : D

9. Given a,b and A are given for a ΔABC

We have $a^2 = b^2+c^2-2bc\cos A$

$$\Rightarrow c^2-2bc\cos A+\left(b^2-a^2\right) = 0$$

We have $c_1+c_2 = 2b\cos A$

$c_1.c_2 = b^2-a^2$

Sum of the areas of two triangles

$$= \frac{1}{2}bc_1\sin A+\frac{1}{2}bc_2\sin A$$

$$= \frac{1}{2}b\sin A+\left(c_1+c_2\right)$$

$$= \frac{1}{2}b\sin A\left(2b\cos A\right)$$

$$= \frac{1}{2}b^2\sin 2A$$

Ans : B

10. **Statement Type :**

Statement I :

In $\Delta ABC,\ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = K$

Statement I is TRUE

Statement II :

In $\Delta ABC,\ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

$\therefore K = 2R$

$\Rightarrow R = \frac{K}{2}$

Statement II is TRUE

Ans : A