ws-12- work 9th foundation + Thank ①

①

Given Force $F = 7N$. displacement $s = 8$

F and s are in same direction so $\theta = 0°$

$\omega = F s \cos\theta$

$= 7 \times 8 \times \cos 0°$

$\Rightarrow 56 \times 1 = 56 J \rightarrow D$

②

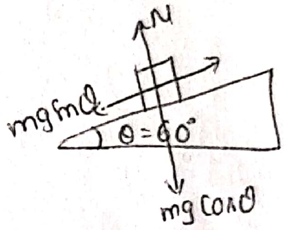
Given mass $= 2 kg$

displacement $= 1m$

work done $= F S$

$= mg\sin\theta \; s$

$\Rightarrow 2 \times 10 \times \sin 60° \Rightarrow 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3} J \rightarrow B$




③

mass $= 200 kg$ ; $h = 5m$

work done in raising a bucket full of water

$W = mgh = 200 \times 9.8 \times 5$

$= 9800 J \rightarrow B$

④

Given $F = 3t\hat{i} + 5\hat{j}$ and $\vec{s} = 4t\hat{i}$

$d\vec{s} = 4\hat{i}dt$

$w = \int F \cdot ds = \int (3t\hat{i} + 5\hat{j}) \cdot (4\hat{i}\, dt)$

$[\hat{i} \cdot \hat{i} = 1 ; \hat{i} \cdot \hat{j} = 0]$

$= \int (3t)(4) dt$

$\frac{d}{dx} \int x^n = \frac{x^{n+1}}{n+1}$

$= \int 12 t\, dt$

$\Rightarrow \cancel{12}^{6} \left[\frac{t^2}{\cancel{2}}\right] \Rightarrow 6t^2$ at $t = 2$ sec

$\Rightarrow \omega = 6(2)^2 = 24 J \rightarrow A$

Scanned with OKEN Scanner

(5)

Given $\vec{F} = 3x^2\hat{i} + 2y\hat{j}$ N; $\vec{r_1} = 2\hat{i} + 3\hat{j}$ m, $\vec{r_2} = 4\hat{i} + 6\hat{j}$ m.

Force is a variable force. $\vec{r} = x\hat{i} + y\hat{j}$

$$W = \int F \cdot d\vec{r} = \int (3x^2\hat{i} + 2y\hat{j})(dx\,\hat{i} + dy\,\hat{j})$$

$$= \int_2^4 (3x^2)\,dx + \int_3^6 2y\,dy \qquad \int x^n dx = \frac{x^{n+1}}{n+1}$$

$$\Rightarrow 3\left[\frac{x^{2+1}}{2+1}\right]_2^4 + 2\left[\frac{y^{1+1}}{1+1}\right]_3^6$$

$$\Rightarrow \cancel{3}\left(\frac{x^3}{\cancel{3}}\right)_2^4 + \cancel{2}\left(\frac{y^2}{\cancel{2}}\right)_3^6 \Rightarrow (4^3 - 2^3) + (6^2 - 3^2)$$

$$\Rightarrow 64 - 8 + 36 - 9 = 83\,J.$$

(6)

Given length $l = 2m$, mass $= 0.5$ kg, $\theta = 60°$

work done in raising other end of the string

$$= mg\frac{l}{2}(1 - \cos\theta)$$

$$\Rightarrow 0.5 \times 10 \times \frac{\cancel{2}}{\cancel{2}}[1 - \cos 60]$$

$$\Rightarrow 5\left(1 - \frac{1}{2}\right) = \frac{5}{2} = 2.5\,J \rightarrow B$$

(7)

Given Force $F = -2x$ N.

work done by variable force $= \int F \cdot dx = \int (-2x)\,dx$

$$= -2\int x\,dx = -\cancel{2}\frac{x^2}{\cancel{2}}$$

$$= -x^2$$

displacement of the block is from $2 \rightarrow -4$ m

$$W = -[(-4)^2 - 2^2] = -[16 - 4] = -12\,J \rightarrow C$$

(8)

Region OA has positive work because slope of graph is increasing [ velocity is increasing ]

Region AB has work zero because slope [ $\frac{v}{t}$ ] = constant
$a = 0$.

Region BC has negative work because slope constantly decrease. [ acceleration = −ve ]

(9)

Given $F = 10 + 0.5x$ N

work done $W = \int F\,dx = \int_0^2 (10 + 0.5x)\,dx$

$= \int_0^2 10\,dx + \int_0^2 (0.5)x\,dx$

$\Rightarrow 10[x]_0^2 + (0.5)\left[\frac{x^2}{2}\right]_0^2$ $\left[ \int x^n dx = \frac{x^{n+1}}{n+1} \right]$

$\Rightarrow 10[2-0] + (0.5)\left(\frac{2^{\cancel{2}}}{\cancel{2}}\right)$

$\Rightarrow 20 + 0.5 \times 2 = 21J \rightarrow A$

(10)

Given velocity $v = ax^{3/2}$ : $a = 5\,m/s^2$, $m = 0.5\,kg$

velocity at $x=0 \Rightarrow v_1 = 5(0)^{3/2} = 0$ also $W = 25k$

velocity at $x = 2m \rightarrow v_2 = 5(2)^{3/2} =$

According to work Energy theorem $W = \Delta K.E$

$= \frac{1}{2}m(v_2^2 - v_1^2)$

$= \frac{1}{2} \times 0.5 [(5(2)^{3/2})^2 - 0^2]$

$= \frac{1}{2} \times 0.5 (25 \times 2^3)$

$W = 50J$

$\Rightarrow \cancel{25}k = \overset{2}{\cancel{50}}J$

$\Rightarrow k = 2J \rightarrow C$

(14)

From graph $x=0 \rightarrow x=3$

work done = Area of F-S graph

$= \frac{1}{2} b_1 h_1 + b_2 h_2$

$= \frac{1}{2} \times 2 \times 2 + 1 \times 2 = 4J$

(15)

From graph for $4 \rightarrow 6\,m$

work done = Area under $F \rightarrow S$ graph

$= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 2 \times (-1) = -1J$

(16)

From graph for $x = 3 \rightarrow 4$ displacement $\neq 0$

Force $= 0$

Since $W = FS\cos\theta = 0 \times S\cos\theta = 0$

(17)

Given $F = 2\hat{i} + 3\hat{j}\ N$; line eqn $3y + kx = 5$

$\Rightarrow y + \frac{k}{3}x = \frac{5}{3}$ compare with

$y = mx + c$

$\therefore$ slope $m = \frac{k}{3}$ ; $c = \frac{5}{3}$

$\Rightarrow \frac{dy}{dx} = \frac{k}{3}$ $\Rightarrow$ displacement $= x\hat{i} + y\hat{j} = 3\hat{i} + k\hat{j}$

Given $W = 0$

$\Rightarrow (F \cdot S) = 0$

$\Rightarrow (2\hat{i} + 3\hat{j}) \cdot (3\hat{i} + k\hat{j}) = 0$

$\Rightarrow 2 \times 3 + 3k = 0 \Rightarrow 3k = -6 \Rightarrow k = -2$

(18)

Given $m = 1kg$ : length $l = 1.6 m$

work done in lifting the chain $= -\frac{mgl}{2}$

$= 1 \times 10 \times \frac{1.6}{2} = 8 J$

(19)

Given $m = 5 kg$ , $V = 3t$ , time $t = 2 sec$

acceleration $a = \frac{dv}{dt} = \frac{d}{dt}(3t) = 3 m/s^2$

From Newton's 2nd law $F = ma = 5(3) = 15 N$

velocity at 2 sec

$V = 3 \cdot 2 = 6 m/s$

displacement after 2 sec From $v^2 - u^2 = 2as$

where $u = 0$ (initial velocity) because body starts from rest

$v^2 = 2as \Rightarrow S = \frac{v^2}{2a} = \frac{6^2}{2 \times 3} = 6 m$

work done by the force $W = F \cdot S$

$= 15 \times 6 = 90 J$

LTask

CUQs

(4) There is a electrostatic force of repulsion between the two charges, it means force acts in opposite direction of the movement.

The agent is applying a force to move the charge towards the fixed +ve charge. This force and displacement are in same direction so $\theta = 0°$ & $\cos\theta = \cos 0 = 1$

$W = FS\cos\theta = FS\cos 0° = FS =$ positive.

Scanned with OKEN Scanner

6 , 7

we know that $W = FS\cos\theta$

when $\theta$ is in between $90 \rightarrow 180°$ : $\cos\theta = -ve$

so work done $= -ve$

when force is acting in a direction opposite to motion of the body. then $\theta = 180°$

8 , 9

we know that $W = FS\cos\theta$

when $\theta$ is in between $0 \rightarrow 90°$ $\cos\theta = +ve$.

so work done $= +ve$.

If $\theta = 0°$ $W = FS\cos 0 = FS$ $\rightarrow$ Maximum

10

when a body is in circular motion then displacement and centripetal force are perpendicular to each other

$\theta = 90°$

$W = FS\cos\theta = FS\cos 90°$

$W = FS(0) = 0$

Jee Mains level

1

Given $F = 50\,N$ : $\theta = 30°$ : distance $= 1m$

$\therefore$ work done $W = FS\cos\theta = 50 \times 1 \times \cos 30$

$= 50 \times \frac{\sqrt{3}}{2}$

$\Rightarrow 25\sqrt{3}\,J$

(2)

Given mass $m = 5$ kg ; distance = 4m ; acceleration $a = 3\,m/s^2$

Force $F = ma = 5 \times 3 = 15\,N$

work done = $F \times$ displacement $\Rightarrow 15 \times 4 = 60\,J \rightarrow D$

(3)

Given

work = 64,000 J ; Force = 8,000 N.

∴ Here the displacement and force are in same direction

$W = FS$

$\Rightarrow 64{,}000 = 8000 \cdot S$

$\Rightarrow S = \frac{64}{8} = 8\,m \rightarrow C$

(4)

mass $m = 3$ kg ; initial velocity $u = 0$

displacement $x = 6m$ ; acceleration $a = 4\,m/s^2$

workdone = $Fx = max$

$= 3 \times \cancel{4 \times 6}$ Area of graph

$\Rightarrow 3\left[4 \times 4 + \frac{1}{2} \times 4 \times \cancel{2}\right]$

$\Rightarrow 3[16 + 4] = 3 \times 20 = 60\,J \rightarrow D$

(5)

Given $T = 200\,N$ ; distance = 20m, $\theta = 60^\circ$

work done = $FS\cos\theta = (200)(20)\cos 60^\circ$

$= 4000 \times \frac{1}{2} = 2000\,J \rightarrow B$

6

Given mass $m = 2\,kg$ ; displacement $x = \frac{t^3}{3}$.

$v = \frac{dx}{dt} = \frac{d}{dt}\left[\frac{t^3}{3}\right] = \frac{3t^2}{3} = t^2$

At $t=0$ → initial velocity $u = (0)^2 = 0$.

At $t = 2\,sec$ → Final velocity $v = (2)^2 = 4\,m/s$

∴ work done $= \frac{1}{2} m v^2 = \frac{1}{2} \times 2 \times 4^2 = 16\,J$ → B

7

Given mass $= m = 5\,kg$ : $\theta = 30^\circ$ : length $l = 10m$.

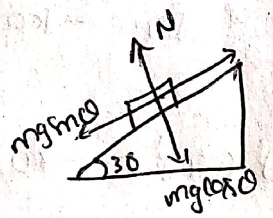
work done by gravity $= mg\,l\,(\cos\theta)$

$= 5 \times 9.8 \times 10 \times \cos 60$

[$\theta = 60^\circ$, it is the angle between Force and displacement]

$= 490 \times \frac{1}{2}$

$= 245\,J$ → A




8

From given data displacement $= (3-1)\hat{i} + (3-3)\hat{j} + (6-2)\hat{k}$

$= 2\hat{i} + 0\hat{j} + 4\hat{k} = 2\hat{i} + 4\hat{k}$

Net force acting $F = F_1 + F_2 = i + 2j + 2k + 3j + 4k$.

$= 5\hat{j} + \hat{i} + 6\hat{k}$.

∴ work done $= \vec{F} \cdot \vec{s} = (\hat{i} + 5\hat{j} + 6\hat{k}) \cdot (2\hat{i} + 4\hat{k})$

$\Rightarrow 2 + 6(4) = 2 + 24$

$= 26\,J$

(9) while moving from $(0,0) \rightarrow (a,0)$

along + x-axis $\Rightarrow \vec{F} = -kx\hat{j}$.

$y = 0$

ie Force is -ve, y-direction, while displacement is in +ve x-direction

Because $F \perp^{er}$ to displacement

Then particle moves from $(a,0)$ to $(a,a)$ along a line parallel to y-axis $(x = +a)$ during this $\vec{F} = -k(y\hat{i} + a\hat{j})$

The first component of force $-ky\hat{i}$ will not contribute any work because this component is along -ve x-direction$(-\hat{i})$ while displacement is in +ve y-direction $(a,0)$ to $(a,a)$.

The second component of force $-ka\hat{j}$ will perform -ve work.

$$\therefore W_2 = (-ka\hat{j})(a\hat{j}) = (-ka)(a) = -ka^2$$

$$\therefore \text{Net work} = W = W_1 + W_2 = 0 + (-ka^2) = \underline{-ka^2}$$

<u>$II^{nd}$</u> <u>method</u>

Given $F = -k(x\hat{j} + y\hat{i})$

we know that $W = \int \vec{F} \cdot d\vec{s} = \int_{(0,0)}^{(a,a)} -k(y\hat{i} + x\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$

$$W = -k\left[\int_{(0,0)}^{(a,a)} y\,dx + \int_{(0,0)}^{(a,a)} x\,dy\right]$$

$$\Rightarrow -k\int_{(0,0)}^{(a,a)} d(xy) \Rightarrow -k[xy]_{(0,0)}^{(a,a)} = -k(a^2)$$

$$= -ka^2$$

(10) (6)

Given $M_{bucket} = 20\ kg$ depth $(d) = 20\ m$

or $x = 20\ m$

weight of the bucket at the top $m = 10\ kg$

The rate of weight loss per meter $= \frac{\text{Total weight loss}}{\text{depth}}$

$= \frac{10}{20} = 0.5\ kg/m.$

mass of the bucket at depth $x$ can be expressed as

$m(x) = 20 - 0.5x.$

$\therefore$ Force $F(x) = (20 - 0.5x)g = (20 - 0.5x)10$

$= 200 - 5x\ N.$

work done $w = \int_0^{20} F\, dx = \int_0^{20} (200 - 5x)\, dx$

$\Rightarrow \int_0^{20} 200\, dx - \int_0^{20} 5x\, dx$

$\Rightarrow 200[x]_0^{20} - 5\left[\frac{x^2}{2}\right]_0^{20} \Rightarrow 200(20-0) - \frac{5}{2}(20^2 - 0)$

$\Rightarrow 4000 - \frac{5}{2} \times 4^2 \times 100$

$\Rightarrow 4000 - 10^3 = 3000\ J$

(11)

we know $w = FS\cos\theta$

if $F \perp^{er}$ S $\Rightarrow \theta = 90^\circ \Rightarrow w = FS\cos 90^\circ$

$= FS(0) = 0$

If $S = 0 \Rightarrow w = F(0)\cos\theta = 0.$

(14), (15), (16)

Given $M_{block} = 2.5$ kg ; displacement $S_x = 2.2$ m

Force = 16N $\theta = 45°$

work done by applied force $w = FS\cos\theta$

$= 16 \times 2.2 \times \cos 45°$

$= 16 \times 2.2 \times \frac{1}{\sqrt{2}}$

$\Rightarrow 16 \times 1.1 \times \sqrt{2}$

$\Rightarrow 24.886 \Rightarrow 24.9$ J.

work done by gravity $W_g = F_g S \cos\theta$

Here gravity and displacement are Perpendecular

$\theta = 90° \Rightarrow \cos 90° = 0 \Rightarrow W_g = F_g S(0) = 0$ J

Here Normal force and displacement are Perpendicular

so $W = 0$

(17)

Given

$S = t^2 + 2t \Rightarrow$ velocity $v = \frac{ds}{dt} = \frac{d}{dt}[t^2 + 2t]$

$m = 2$ kg

$v = 2t + 2$

Acceleration $a = \frac{dv}{dt} = \frac{d}{dt}[2t+2] = 2\frac{dt}{dt} + \frac{d}{dt}(2) = 2 + 0 = 2\, m/s^2$

$\therefore$ work done by the force $w = \int F\,ds$ $\left( \because v = \frac{ds}{dt},\ ds = v\,dt \right)$

$= \int ma\, v\,dt$

$= \int_2^4 2 \times 2 \times (2t+2)\,dt$

$\Rightarrow \int_2^4 [8t + 8]\,dt$

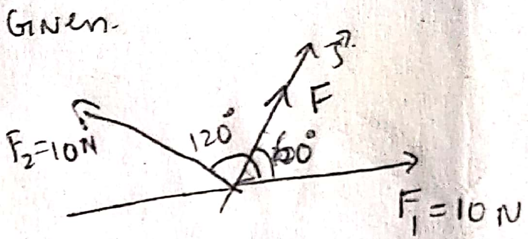
$\Rightarrow \int_2^4 8t\,dt + \int_2^4 8\,dt$

$\Rightarrow 8\left[\frac{t^2}{2}\right]_2^4 + 8[t]_2^4$

$\Rightarrow 8\left[\frac{4^2 - 2^2}{2}\right] + 8[4-2] = 64$ J

18

Given




Displacement $\vec{s} = 10m$

$\therefore \; W = F S \cos\theta$

$= 10 \times 10 \times \cos 60°$

$= 100 \times 1$

$= 100 J$

The resultant force

$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos\theta}$

$F = \sqrt{10^2 + 10^2 + 2(10)(10) \cos 120°}$

$F = \sqrt{10^2 + 10^2 + 2(10)^2(-\frac{1}{2})}$

$= \sqrt{10^2 + 10^2 - 10^2} = \sqrt{10^2} = 10N$

work done by each force

$= F_1 \, s \cos\theta$

$= 10 \times 10 \times \cos 60$

$= 10 \times 10 \times \frac{1}{2} = 50 J$

19

Given

$F = 100 N$, mass $m = 10 kg$ ; distance $S = 5m$

time ~~2~~ $t = 2 sec$

velocity $= \frac{\text{desplacement}}{\text{time}} = \frac{5}{2} = 2.5 \, m/s$

work done $= F \cdot S = 100 \times 5 = 500 J$

weight $\quad w = mg = 10 \times 10 = 100 N$