WS-8

8th foundation. ①

Elastic collision

Thank

①

Given $m_1 = 25kg \rightarrow u_1 = 40 m/s$

$m_2 = 15kg \rightarrow u_2 = 0$

After collision both are moving with same velocity

$v_1 = v_2 = v_{com}$

According to law of conservation of linear momentum

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$\Rightarrow 25 \times 40 + 15(0) = 25 \times v_{com} + 15 v_{com}$

$\Rightarrow 25 \times \cancel{40} = \cancel{40} v_{com}$

$\Rightarrow v_{com} = 25 m/s \rightarrow A$

②

$m_1 = 20gm \rightarrow u_1 = 20 m/s$

$m_2 \qquad \rightarrow u_2 = 5 cm/s$

After collision both are moving with same velocity

$v_1 = v_2 = v_{com} = 10 cm/s.$

According to law of conservation of linear momentum

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$\Rightarrow 20 \times 20 + m_2 \times 5 = 20 \times 10 + m_2 \times 10$

$\Rightarrow 400 + 5m_2 = 200 + 10m_2$

$\Rightarrow 200 = 5m_2$

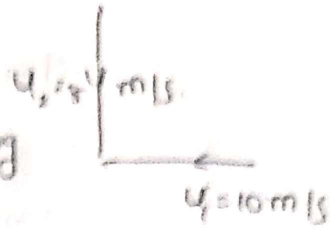
$\Rightarrow m_2 = 40 gm.$

(3)

Given

$m_1 = 3\,kg$; $m_2 = 5\,kg$

$u_1 = 10\,m/s$; $u_2 = 8\,m/s$



After collision both are moving with same velocity $V_1 = V_2 = V_{com}$

According to law of conservation of linear momentum

momentum Before collision = momentum After collision → (1)

Momentum Before collision $= \sqrt{P_1^2 + P_2^2} = \sqrt{(m_1 u_1)^2 + (m_2 u_2)^2}$

$= \sqrt{(3\times10)^2 + (5\times8)^2} = \sqrt{(30)^2 + (40)^2}$

$\Rightarrow 50\ kgm/s$

From (1)

$50 = m_1 V_1 + m_2 V_2$

$\Rightarrow 50 = 3V_{com} + 5V_{com}$

$\Rightarrow 50 = 8V_{com} \Rightarrow V_{com} = \frac{50}{8} = 6.25\,m/s$

(4)

Given $u_1 = 40\,m/s$; $u_2 = 0$

After collision both bodies move with same velocity

$V_1 = V_2 = V_{com} = 30\,m/s$

Acc to law of conservation of linear momentum

$m_1 u_1 + m_2 u_2 = m_1 V_1 + m_2 V_2$

$\Rightarrow m_1 \times 40 + m_2(0) = m_1 \times 30 + m_2 \times 30$

$\Rightarrow 40 m_1 = 30 m_1 + 30 m_2$

$\Rightarrow 10 m_1 = 30 m_2$

$\Rightarrow \boxed{\frac{m_1}{m_2} = \frac{30}{10} = \frac{3}{1}}$

(5)

Given $m_1 = 5\ kg$; $u_1 = 2\hat{i}\ m/s$

$m_2 = 10\ kg$; $u_2 = \sqrt{3}\hat{j}\ m/s$

After collision both are moving with same velocity

$V_1 = V_2 = V_{com}$.

momentum Before collision $= m_1 u_1 + m_2 u_2$

$\Rightarrow 5(2\hat{i}) + 10(\sqrt{3})\hat{j}$

$\Rightarrow 10\hat{i} + 10\sqrt{3}\hat{j}$

$|P_{Before}| = \sqrt{10^2 + (10\sqrt{3})^2} = 10\sqrt{1+3} = 20\ kgm/s$

According to law of conservation of linear momentum

$\Rightarrow m_1 V_1 + m_2 V_2 = m_1 u_1 + m_2 u_2$

$\Rightarrow 5V_{com} + 10V_{com} = 20$

$\Rightarrow 15V_{com} = 20 \Rightarrow V_{com} = \frac{20}{15} = \frac{4}{3}\ m/s \rightarrow C.$

(6)

Given

$m_1 = 2kg \rightarrow u_1 = i + 2j - 3k\ m/s$

$m_2 = 3kg \rightarrow u_2 = 2i + j + k\ m/s.$

After collision both bodies are moving with same velocity

$V_1 = V_2 = V_{com}.$

According to law of conservation of linear momentum

$m_1 u_1 + m_2 u_2 = m_1 V_1 + m_2 V_2$

$\Rightarrow 2(i + 2j - 3k) + 3(2i + j + k) = 2V_{com} + 3V_{com}$

$\Rightarrow 2i + 4j - 6k + 6\hat{i} + 3j + 3k = 5V_{com}$

$\Rightarrow 8i + 7j - 3k = 5V_{com}$

$\Rightarrow V_{com} = \frac{1}{5}[8i + 7j - 3k] \rightarrow A.$

(7)

let mass of 2nd body is m_2

its velocity is $u_2 = 0$.

Given $m_1 = 6$ kg let its velocity be u_1

and after collision Ist body moves with a velocity

$v_1 = \frac{u_1}{2}$

$\therefore$ we know $v_1 = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] u_1 + \frac{2 m_2 u_2}{m_1 + m_2}$

$\Rightarrow \frac{\cancel{u_1}}{2} = \left[\frac{6 - m_2}{6 + m_2}\right] \cancel{u_1} + \frac{2 m_2 (0)}{6 + m_2}$

$\Rightarrow \frac{1}{2} = \frac{6 - m_2}{6 + m_2}$

$\Rightarrow 6 + m_2 = 12 - 2 m_2$

$\Rightarrow 3 m_2 = 6 \rightarrow m_2 = 2$ kg

(8)

Given $m_1 = 3$ kg ; $u_1 = 4$ m/s

$m_2 = 4$ kg $u_2 = -2$ m/s

After collision both bodies move with same velocity

Initial momentum of the body $= m_1 u_1 + m_2 u_2$

$= 3 \times 4 + 4(-2)$

$\rightarrow 12 - 8$.

$= 4$ kg m/s

9

Given $m_1 = m$; $\rightarrow$ initial velocity $= u_1 = u$

$m_2 = 2m$ $\rightarrow$ initial velocity $u_2 = 0$

velocity of I^th body after collision

$$v_1 = \left[\frac{(m_2 - m_1)u_1 + 2m_2 u_2}{m_1 + m_2}\right]$$

$$\Rightarrow v_1 = \frac{(2m - m)u}{2m + m} = \frac{m}{3m}u = \frac{1}{3}u$$

loss of kinetic energy of colliding particle

$$\Delta K.E = \frac{1}{2}m(v_1^2 - u_1^2)$$

$$= \frac{1}{2}m\left[\left(\frac{1}{3}u\right)^2 - u^2\right)$$

$$= -\frac{1}{2}mu^2\left(\frac{8}{9}\right]$$

$\therefore$ Fractional loss $= \dfrac{\Delta K.E}{K.E_{original}} = \dfrac{-\frac{1}{2}\cancel{m}u^2\left[\frac{8}{9}\right]}{\frac{1}{2}\cancel{m}u^2} = \dfrac{8}{9}$

'-ve' sign shows loss of K.E

10

Given $m_{bullet} = 4 \times 10^{-2}$ kg $\rightarrow$ initial speed $u_1 = 802$ m/s

$m_{block} = 15$ kg $\rightarrow$ initial speed $u_2 = 0$

velocity of bullet + block after collision $v_1 = v_2 = 2$ m/s

$\therefore$ Loss of K.E $= \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 - \left(\frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2\right)$

$$\Rightarrow \frac{1}{2}m_{bullet}v_{com}^2 + \frac{1}{2}m_{block}v_{com}^2 - \frac{1}{2}m_{bullet}u_1^2 -$$

$$\frac{1}{2}m_{block}u_2^2$$

$$\Delta K.E = \frac{1}{2} V_{com}^2 [(15+0.04)] - \frac{1}{2}[4\times10^{-2}\times(802)^2 + 15(0)]$$

$$= \frac{1}{2}(2)^2 [15.04] - \frac{1}{2}\times 4\times10^{-2}\times(802)^2$$

$$\Rightarrow 2\times15.04 - 2\times10^{-2}\times 643204$$

$$\Rightarrow 30.08 - 12864.08$$

$$\Rightarrow 12834 \; J.$$

Advanced level

9, 10

Given $M = 5\,kg \rightarrow$ Initial velocity $= 0$

$m_1 = 3\,kg \rightarrow$ let it's velocity be v_1

$m_2 = 2\,kg \rightarrow$ it's velocity $v_2 = 60\,m/s$

According to law of conservation of linear momentum

$$\Rightarrow Mu = m_1 v_1 + m_2 v_2$$

$$\Rightarrow 5(0) = 3(v_1) + 2(60)$$

$$\Rightarrow 3v_1 = -2\times60 \Rightarrow v_1 = -\frac{2\times60}{3} = -40\,m/s$$

Kinetic energy of 2 kg Piece is

$$K.E_2 = \frac{1}{2} m_2 V_2^2$$

$$= \frac{1}{2}\times 2\times(60)^2$$

$$\Rightarrow (60)^2$$

$$\Rightarrow \underline{3600\,J}$$

LTask
CUQ

5

(5)

we know velocities of two bodies after collision

$$v_1 = \frac{(m_1-m_2)u_1 + 2m_2u_2}{m_1+m_2}$$

$$v_2 = \frac{2m_1u_1 + (m_2-m_1)u_2}{m_1+m_2}$$

Given $m_2 >> m_1$; $u_1 = 0$.

$\therefore$ $v_1 = \frac{2\cancel{m_2}u_2}{\cancel{m_2}} = 2u_2$ m_1 can be neglected

$$v_2 = \frac{+\cancel{m_2}u_2}{\cancel{m_2}} = +u_2$$

so lighter body moves with twice the velocity of havier body and havier body moves with same velocity in same direction

(6)

If $m_1 >> m_2$.

m_1 is havier body. ; $u_1 = 0$.

We know $v_1 = \frac{(m_1-m_2)u_1 + 2m_2u_2}{m_1+m_2}$. m_2 can be neglected

$$v_1 = \frac{(m_1)0 +}{m_1} \simeq 0.$$

$$v_2 = \frac{2mu_1 + (m_2-m_1)u_2}{m_1+m_2} = 0 + \frac{(-\cancel{m_1})u_2}{\cancel{m_1}} = -u_2$$

lighter body retraces its both, havier body does not move.

(7)

Given $m_1 = m_2$ and $u_1 = -u_2$

After collision $v_1 = \dfrac{(m_1 - m_2)u_1 + 2m_2 u_2}{m_1 + m_2}$

$\Rightarrow v_1 = \dfrac{0 u_1 + 2m_1 u_2}{2m_1}$

$v_1 = u_2 = u_1$

$\therefore v_2 = \dfrac{(m_2 - m_1)u_2 + 2m_1 u_1}{m_1 + m_2} = \dfrac{0 \times u_2 + 2m_1 u_1}{2m_1}$

$\boxed{v_2 = u_1} = u_1$

They move away with the same velocity in magnitude

<u>Jee main level</u>

(1)

Given $m_1 = 2\,kg \rightarrow u_1 = 5\,m/s$

$m_2 = 3\,kg \rightarrow u_2 = 0$

After collision both bodies move with same velocity

$v_1 = v_2 = v_{com}$

According to law of conservation of linear momentum

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$\Rightarrow 2 \times 5 + 3(0) = 2 v_{com} + 3 v_{com}$

$\Rightarrow 2 \times 5 = 5 v_{com}$

$\Rightarrow v_{com} = 2\,m/s$

$= u_2 = \ldots$

(2)

Given

$m_1 = 2$ kg ; $m_2 = 8$ kg

$k_1 = 16$ J $k_2 = 24$ J

we know that $k.E = \frac{P^2}{2m}$

$\Rightarrow P = \sqrt{2m\, k.E}$

momentum of first body $P_1 = \sqrt{2m_1\, k.E_1}$

$= \sqrt{2 \times 2 \times 16}$

$\Rightarrow 8$ kgm/s

momentum of 2nd body $P_2 = \sqrt{2m_2\, k.E_2}$

$= \sqrt{2 \times 8 \times 24} = 19.56$ kgm/s

$\Rightarrow \cancel{8\sqrt{6}}$ ~~kg m/s~~

According to law of conservation of linear momentum

Momentum After collision = momentum Before collision

$= P_1 + P_2 = \cancel{8+8\times1.732}$

$= 8 + 19.56 = \cancel{8+19.56} = 27.756$ Ns

(3)

Given

masses of two bodies are m_1, m_2

let their initial velocities u_1, $u_2=0$

$V_1 = \frac{2}{3}u_1$; v_2 be the final velocity of 2nd body

According to law of conservation of linear momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow m_1 u_1 + m_2 (0) = m_1\left(\frac{2}{3}u_1\right) + m_2 v_2$$

$$\Rightarrow m_1 u_1 - \frac{2m_1 u_1}{3} = m_2 v_2$$

$$\Rightarrow \frac{1}{3} m_1 u_1 = m_2 v_2 \rightarrow (1)$$

According to law of conservation of Energy

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$\Rightarrow m_1 u_1^2 + m_2 (0)^2 = m_1\left(\frac{2}{3}u_1\right)^2 + m_2 v_2^2$$

$$\Rightarrow m_1 u_1^2 = \frac{4}{9} m_1 u_1^2 + m_2 v_2^2$$

$$\Rightarrow \frac{5}{9} m_1 u_1^2 = m_2 v_2^2$$ From (1) $m_2 v_2 = \frac{1}{3} m_1 u_1$

$$\Rightarrow \frac{5}{9} m_1 u_1^2 = (m_2 v_2) v_2$$

$$\Rightarrow \frac{5}{\cancel{9}_3} \cancel{m_1} u_1^{\cancel{2}} = \frac{1}{\cancel{3}} \cancel{m_1} \cancel{u_1} v_2$$

$$\Rightarrow v_2 = \frac{5}{3} u_1$$

Substitute v_2 in (1) we get

$$\Rightarrow \frac{1}{\cancel{3}} m_1 \cancel{u_1} = m_2 \frac{5}{\cancel{3}} \cancel{u_1}$$

$$\Rightarrow \frac{m_1}{m_2} = \frac{5}{1} \rightarrow B$$

(4)

Given mass of bullet $m_1 = 500\ gm = 0.5\ kg$

mass of bag $m_2 = 4.5\ kg$

initial velocity of bullet $u_1 = 200\ m/s$

initial velocity of bag $u_2 = 0$.

After collision both sand and bag move with same velocity $V_1 = V_2 = V$.

According to law of conservation of linear momentum

$\Rightarrow m_1 u_1 + m_2 u_2 = m_1 V_1 + m_2 V_2$

$\Rightarrow 0.5 \times 200 + 4.5(0) = 0.5V + 4.5V$

$\Rightarrow 100 + 0 = 5V$

$\Rightarrow V = \frac{100}{5} = 20\ m/s \rightarrow C$

(5)

Let $m_1 = 6\ kg \rightarrow u_2 = 0$ let $u_1 \rightarrow$ initial velocity of first body

let m_2 be the mass of 2nd body.

Given After collision both bodies travel with a velocity $= \frac{1}{3} u_1 = V_1 = V_2$

$\therefore$ According to law of conservation of linear momentum

$m_1 u_1 + m_2 u_2 = m_1 V_1 + m_2 V_2$

$\Rightarrow 6 \times u_1 + m_2(0) = 6 \frac{u_1}{3} + m_2 \frac{u_1}{3}$

5th continuation

$$\Rightarrow 6 m_2 u_1 = \frac{m u_1}{3} + \frac{m_2 u_1}{3}$$

$$\Rightarrow 6 m u_1 - \frac{6 m u_1}{3} = \frac{m_2 u_1}{3}$$

$$\Rightarrow \frac{18u_1 - 6u_1}{3} = \frac{m_2 u_1}{3}$$

$$\Rightarrow \frac{12 \cancel{u_1}}{\cancel{3}} = \frac{m_2 \cancel{u_1}}{\cancel{3}}$$

$$\Rightarrow m_2 = 12\,kg \rightarrow c$$

(6)

let $m_1 = 2kg$ be the mass of I^{st} body

$u_1 = 40\,m/s$

m_2 be the mass of 2^{nd} body which is at rest ; $u_2 = 0$

After collision two bodies move with a common velocity

$v_1 = v_2 = 25\,m/s$

According to law of conservation of momentum

$$\Rightarrow m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow 2 \times 40 + m_2(0) = 2 \times 25 + m_2(25)$$

$$\Rightarrow 80 = 50 + 25 m_2 \Rightarrow 25 m_2 = 30$$

$$\Rightarrow m_2 = \frac{30}{25} = \frac{6}{5} = 1.2\,kg \rightarrow c$$

(7)

let $m_1 = 5\,kg$ → initial velocity $u_1 = 10\,m/s$

$m_2 = 20\,kg$ → $u_2 = 0$. Given After collision 5 kg

comes to rest (ie) $v_1 = 0$

and velocity of 20 kg mass is $v_2 = v$.

7th continuation

(8)

According to law of conservation of linear momentum

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$\Rightarrow 5 \times 10 + 20(0) = 5(0) + 20\,v$

$\Rightarrow 50 = 20\,v$

$\Rightarrow v = \frac{50}{20} = \frac{5}{2} = 2.5\ m/s$

(8)

let $m_1 = 6\,kg \rightarrow u_1 = 10\ m/s$

$m_2 = 4\,kg \rightarrow u_2 = -5\ m/s.$

Given After collision 1st body comes to rest i.e. $v_1 = 0$

According to law of conservation of linear momentum

$\Rightarrow m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$\Rightarrow 6 \times 10 + 4 \times (-5) = 6(0) + 4(v_2)$

$\Rightarrow 60 - 20 = 4v_2$

$\Rightarrow 40 = 4v_2 \Rightarrow v_2 = 10\ m/s \rightarrow D$

(9)

let $m_1 = 20\,kg$; $u_1 = 20\ m/s$

$m_2 = 40\,kg$; $u_2 = 10\ m/s$

Given After collision both bodies move with same velocity

i.e. $v_1 = v_2 = v_{com}$

According to law of conservation of linear momentum

$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

$\Rightarrow 20 \times 20 + 40 \times 10 = 20 v_{com} + 40 v_{com}$

$\Rightarrow 400 + 400 = 60\, v_{com}$

$\Rightarrow \frac{60}{3} V_{com} = \frac{\cancel{800}}{40}$

$\Rightarrow V_{com} = \frac{40}{3}$ m/s

(10)

let $m_1 = 2m$; $m_2 = 3m$

$u_1 = u$ $\quad u_2 = 0$.

clearly after collision both sphere's move with same velocity $v_1 = v_2 = V_{com}$.

According to law of conservation of linear momentum

$\Rightarrow m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$\Rightarrow 2mu + 3m(0) = 2m V_{com} + 3m V_{com}$

$\Rightarrow 2\cancel{m}u = 5\cancel{m} V_{com}$

$\Rightarrow V_{com} = \frac{2u}{5} =$

$\therefore$ Loss of k.E $= \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right) - \left[\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right]$.

$\Rightarrow \frac{1}{2}\left[\left(2m V_{com}^2 + 3m V_{com}^2\right] - \left(2mu^2 + 3m(0)\right)\right]$

$= \frac{1}{2}\left[5m V_{com}^2 - 2mu^2\right]$

$= \frac{1}{2}\left[5m \times \left(\frac{2u}{5}\right)^2 - 2mu^2\right]$

$= \frac{1}{2}\left[\cancel{5}m \times \frac{4u^2}{\cancel{25}_5} - 2mu^2\right.$

$\Rightarrow \frac{1}{2}\left[\frac{4mu^2}{5} - 2mu^2\right]$

$\Rightarrow \frac{1}{2} \quad \frac{-6mu^2}{5} \Rightarrow -\frac{6}{5}\left(\frac{1}{2}mu^2\right)$

$= \frac{-\cancel{3}mu^2}{5}$

% Loss of k.E $= \dfrac{-\frac{6}{5}\left(\frac{1}{2}mu^2\right)}{\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2} = \dfrac{-\frac{6}{5} \times \frac{1}{\cancel{2}} mu^2}{\frac{1}{\cancel{2}}(2m)u^2} \times 100\%$

10th continuation

Loss of K.E = $-\frac{6}{8} \times \frac{\cancel{100}^{\cancel{20}}\ 10}{2} = -60\%$

Advanced

9 . 8

let $M_1 = 10$ kg , $u_1 = 10$ m/s

$m_1 = 6$ kg $\rightarrow V_1 = 18$ m/s

$m_2 =$ mass of 2nd piece $= 4$ kg

$V_2 = ?$

According to law of conservation of linear momentum

$Mu = m_1 V_1 + m_2 V_2$

$\Rightarrow 10 \times 10 = 6 \times 18 + 4 V_2$

$\Rightarrow 100 = 108 + 4V_2 \Rightarrow 4V_2 = -8$ m/s

$\Rightarrow V_2 = -2$ m/s

K.E of 2nd Piece after collision

$K.E_2 = \frac{1}{2} m_2 V_2^2$

$= \frac{1}{\cancel{2}} \times \cancel{4}^{2} \times (-2)^2$

$= 2 \times 4 = \underline{8 J}$