INTEGRATED

Class: VIII, MATHEMATICS

3. FACTORISATION OF POLYNOMIALS

Teaching Task (JEE Mains Level)

01. $x^3 - 4x = x(x^2-4) = x(x+2)(x-2)$ Ans: C

02 $x^2 - 4x + 4 = (x-2)^2$ Ans: B

03 $9x^2 - 16 = (3x)^2 - 4^2 = (3x+4)(3x-4)$ Ans: A

04. $2x^2 + 5x + 3$
$= 2x^2 + 2x + 3x + 3$
$= 2x(x+1) + 3(x+1)$
$= (x+1)(2x+3)$ Ans: A

05 $x^3 - 1 = x^3 - 1^3 = (x-1)(x^2+x+1)$ Ans: A

06. $x^6 + y^6 = (x^2)^3 + (y^2)^3 = (x^2+y^2)(x^4 - x^2y^2 + y^4)$ Ans: B

07. $p(y) = y^3 - 7y + 6$
$y-1$ is a factor

$y=1$	1	0	−7	6
	0	1	1	−6
	1	1	−6	0

$\therefore y^2 + y - 6 = (y+3)(y-2)$
$\therefore$ other factors are $(y+3), (y-2)$ Ans: B

08 Given $\alpha + \beta + \gamma = 2 \rightarrow S_1$
$\alpha\beta + \beta\gamma + \gamma\alpha = -7 \rightarrow S_2$
$\alpha\beta\gamma = -14 \rightarrow S_3$
Required eqn is $x^3 - S_1x^2 + S_2x - S_3 = 0$
$x^3 - 2x - 7x + 14 = 0$ Ans: C

09. $P(x) = ax^3 + bx^2 + cx + d$.

product of the zeroes = 0

i.e $\alpha\beta\gamma = 0$

$\Rightarrow -\frac{d}{a} = 0 \Rightarrow d = 0$.

$\therefore P(x) = ax^3 + bx^2 + cx$

Since, there is no constant term

∴ The curve passes through the origin

Also, One of the zero is zero

Ans: B, C

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10. $V(x) = x^3 - 12x^2 + 47x - 60$.

$\therefore \alpha + \beta + \gamma = 12$

Given $\alpha + \beta = 5$

$\therefore \gamma$ = height = 7

α = length

β = breadth

γ = height.

Ans: D

JEE ADVANCED LEVEL

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11. A) $x^2 - 16 = (x-4)(x+4)$ ✓

B) $x^2 + 6x + 9 = (x+3)^2$ ✓

C) $x^2 - 9x + 20 = (x-5)(x-4)$ ✓

D) $x^2 + x + 1 = (x+1)(x+1)$ ✗

Ans: A, B, C

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12. Given $f(x) = 4x^4 + 12x^3 + 2x^2 - 18x - 12$

clearly $f(-1) = 0$, $f(-2) = 0$

−1	4	12	2	−18	−12
	6	−4	−8	6	12
−2	4	8	−6	−12	0
	0	−8	0	12	
	4	0	−6	0	

$\therefore 4x^2 - 6 = 2(2x^2 - 3)$

∴ The factors are, $x+1$, $x+2$, $2(2x^2-3)$ Ans: C, D

13 Statement I: $x^3-27 = x^3-3^3 = (x-3)(x^2+3x+9)$ (True)

Statement II: Mathematically (True) Ans. A

14. Statement I: $f(x) = x^3-x^2-8x+12$

$f(2) = 2^3-2^2-8\cdot 2+12$

$= 8-4-16+12$

$= 0$ (True)

Statement II: Conceptual (True) Ans. A

15 Assertion: $x^2+10x+25 = (x+5)^2$ (True)

Reason: Mathematically (True) Ans. A

16 Assertion: $V(x) = lbh$

$= (x-2)(x+1)(x-4)$

$= (x^2-x-2)(x-4)$

$= x^3-4x^2-x^2+4x-2x+8$

$= x^3-5x^2+2x+8$ (True)

Reason: Conceptual (True) Ans. A

17. $a^2-b^2 = (a+b)(a-b)$ Ans: B

18. $25y^2-16 = (5y)^2-4^2 = (5y+4)(5y-4)$ Ans: A

19. width $= \dfrac{2x^2+7x+6}{2x+3} = \dfrac{(x+2)(2x+3)}{(2x+3)} = x+2$ Ans. A

20 Area $= 2x^2+7x+6 = 10$

$\Rightarrow 2x^2+7x-4 = 0$

$\Rightarrow 2x^2+8x-x-4$

$\Rightarrow 2x(x+4)-1(x+4) = 0$

$\Rightarrow (x+4)(2x-1) = 0$

$x = -4$ or $x = \frac{1}{2}$

Ans = B

21. $6x^2 - 15x = 3x(2x-5)$ $\therefore c = 5$ (4) Ans-5

22 $f(x) = x^6 - ax^5 + x^4 - ax^3 + 3x - a - 2$

$f(a) = a^5 - a \cdot a^5 + a^4 - a \cdot a^3 + 3 \cdot a - a - 2 = 0$

$\Rightarrow a^6 - a^6 + a^4 - a^4 + 2a - 2 = 0$

$\Rightarrow a = 1$ Ans: 1

23 a) $x^3 - 4x = x(x^2 - 4) = x(x-2)(x+2)$ (P)

b) $2x^3 + 6x^2 = 2x^2(x+3)$ (Q)

c) $3x^4 - 6x^3 = 3x^3(x - 2)$ (R)

d) $x^3 + 27 = x^3 + 3^3 = (x+3)(x^2 - 3x + 9)$ (S)

Ans: P, Q, R, S

24 a) $6x^2 + x - 2 = (3x+2)(2x-1)$ (Q)

b)

$x=3$	1	2	−11	−12
	0	3	15	12
	1	5	4	0

$\therefore x^2 + 5x + 4$ (R)

c)

$x=1$	1	−6	11	−6
	0	1	−5	6
	1	−5	6	0

$\therefore x^2 - 5x + 6$ (S)

d)

$x=-2$	2	9	7	−6
	0	−4	−10	6
	2	5	−3	0

$\therefore 2x^2 + 5x - 3$ (P)

Ans: Q, R, S, P

LEARNERS TASK (CUQ's)

01. $2x^2+5x+3 = 2x^2+2x+3x+3$ | $= 2x(x+1)+3(x+1)$ | $= (x+1)(2x+3)$ Ans: A

02 Conceptual Ans: C

03 Conceptual Ans-B

04. Degree $= 3+2=5$ Ans. B

05 Conceptual Ans: B

06. $2x^2+3xy+2y^2$

since $f(x,y)=f(y,x)$ Ans: A

07. $a^2+b^2+c^2$ Ans-D

08 homogeneous Ans: B

09. $\sum_{i=1}^{n} i = 1+2+3+\cdots+n = \frac{n(n+1)}{2}$ Ans B

10. $f(x)=ax^4+bx^3+cx^2+dx+e.$

divisible by $x^2-1 \rightarrow (x+1)(x-1)$

$f(1) = a+b+c+d+e=0 \rightarrow (1)$

$f(-1) = a-b+c-d+e=0$

$\Rightarrow a+c+e=b+d$

$\Rightarrow a+c+e=0$ & $b+d=0$ (from 1)

Ans: D

JEE MAINS LEVEL

01. $6x^2-15x = 3x(2x-5)$

$\therefore$ The factors are $3, x, 2x-5$ Ans: D

02. $x^3-64 = x^3-4^3 = (x-4)(x^2+4x+16)$ Ans. A

03 $x^2-4x+4 = (x-2)^2$ Ans: B

04. $x^2-25 = x^2-5^2 = (x+5)(x-5)$ Ans. B

05 x^2-2x-8
$= x^2-4x+2x-8$
$= x(x-4)+2(x-4)$
$= (x-4)(x+2)$ Ans: A

06. Given $\alpha+\beta = 2$, $\alpha\beta = \frac{-3}{5}$
We have, $f(y) = y^2-(\alpha+\beta)y+\alpha\beta$
$= y^2-2y-\frac{3}{5}$
$= 5y^2-10y-3$ Ans: A

07. $4(a+b)^2-9(a-b)^2$
$= [2(a+b)]^2-[3(a-b)]^2$
$= (2a+2b)^2-(3a-3b)^2$
$= (2a+2b+3a-3b)(2a+2b-3a+3b)$
$= (5a-b)(5b-a)$ Ans: D

08 $f(x) = 2x^4-7x^3+ax+b$
$f(3) = 2\cdot 3^4-7\cdot 3^3+3a+b = 0$
$= 162-189+3a+b = 0$
$\therefore 3a+b = 27$ Ans. A

09 $x^2-5x+6 = 0 \Rightarrow (x-2)(x-3) = 0 \Rightarrow x = 2, 3$
$\therefore \alpha = 2, \beta = 3 \Rightarrow (\alpha-\beta)^2 = (2-3)^2 = 1$ Ans. A

10. $P(x) = x^2 + x + C$

$\alpha + \beta = -1, \quad \alpha\beta = C$

Now, $(1+\alpha)(1+\beta)$

$= 1 + (\alpha+\beta) + \alpha\beta$

$= 1 + (-1) + C = C$ Ans. D

JEE ADVANCED

11. Conceptual Ans: A, C, D

12. $f(x) = 2x^4 - 3x^3 - 3x^2 + 6x - 2$

Let $\alpha = \sqrt{2}, \quad \beta = -\sqrt{2}$

$\therefore x^2 - (\alpha+\beta)x + \alpha\beta = x^2 - 2$

Dividing $f(x)$ with $x^2 - 2$, we get quotient as $2x^2 - 3x + 1 = (x-1)(2x-1) = 0$

$\therefore x = 1$ or $\frac{1}{2}$ Ans. A, D

13. Statement I: Degree $= 3 + 2 = 5$ (True)

Statement II: Conceptual (true) Ans. A

14. Statement I: Conceptual (True)

Statement II: Conceptual (True) Ans. A

15. Assertion: $x^3 + 27 = x^3 + 3^3 = (x+3)(x^2 - 3x + 9)$ (False)

Reason: Conceptual (True) Ans. D

16. Assertion: $x^4 - 5x^2 + 4 = 0$

$= x^4 - x^2 - 4x^2 + 4 = 0$

$= x^2(x^2 - 1) - 4(x^2 - 1) = 0$

$= (x^2 - 1)(x^2 - 4) = 0$

$\Rightarrow x = \pm 1, \pm 2$ (True)

Reason: Conceptual, Mathematical (True) Ans: A

17. $x^3+27 = x^3+3^3 = (x+3)(x^2-3x+9)$ Ans: A

18. $64a^3-b^3 = (4a)^3-b^3$
$= (4a-b)(16a^2+4ab+b^2)$ Ans: B

19. $27p^3-8q^3$
$= (3p)^3-(2q)^3$
$= (3p+2q)(9p^2+6pq+4q^2)$
Ans: A

20. $1000+y^3 = 10^3+y^3 = (10+y)(100-10y+y^2)$
Ans: A

21. $x^2+Kx-12 = (x+4)(x-3) = x^2+x-12$
$\therefore K = 1$ Ans: 1

22 $(2x^3+5x-1)(4x^4-3x^2+x) = 8x^7+\text{-----}$
Degree = 7 Ans: 7

23
a) $x^2-25 = (x+5)(x-5)$ (r)
b) $x^2+6x+9 = (x+3)^2$ (p)
c) $x^2-16 = (x+4)(x-4)$ (q)
d) $x^2+5x = x(x+5)$ (s)
Ans: r, p, q, s

24
a) $x^3-64 = x^3-4^3 = (x-4)(x^2+4x+16)$ (p)
b) $x^3+27 = x^3+3^3 = (x+3)(x^2-3x+9)$ (s)
c) $8x^3-1 = (2x)^3-1^3 = (2x-1)(4x^2+2x+1)$ (r)
d) $x^3-8 = x^3-2^3 = (x-2)(x^2+2x+4)$ (q)
Ans: A s, r, q

⇒ THE END ∈