DISTANCE BETWEEN TWO POINTS

Class: IX, Mathematics

IIT FOUNDATION

TEACHING TASK

01. $A(at_1^2, 2at_1)$, $B(at_2^2, 2at_2)$

$x_1 \quad y_1 \quad x_2 \quad y_2$

$$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$=\sqrt{(at_2^2-at_1^2)^2+(2at_2-2at_1)^2}$$

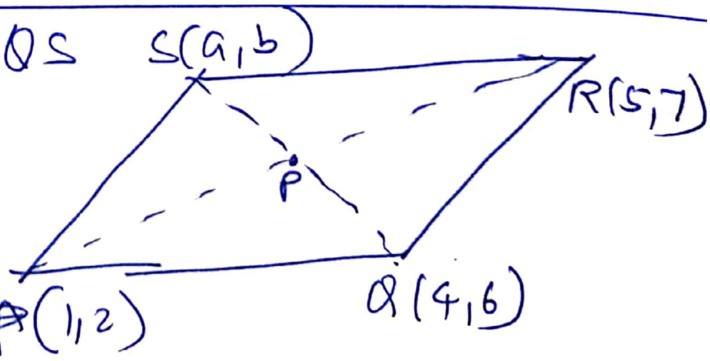
$$=\sqrt{a^2(t_2^2-t_1)^2+4a^2(t_2-t_1)^2}$$

$$=\sqrt{a^2(t_2+t_1)^2(t_2-t_1)^2+4a^2(t_2-t_1)^2}$$

$$=a(t_2-t_1)\sqrt{(t_2+t_1)^2+4}$$

Ans: A

02. mid-point of PR = mid pt of QS



$$\left(\frac{1+5}{2}, \frac{2+7}{2}\right)=\left(\frac{a+4}{2}, \frac{b+6}{2}\right)$$

$\Rightarrow a=2, \quad b=3$

Ans: C

03 $P(at^2, 2at)$, $Q\left(\frac{a}{t^2}, \frac{2a}{t}\right)$, $S(a,0)$ ②

$$SP = \sqrt{(a-at^2)^2 + (0-2at)^2}$$

$$= \sqrt{a^2(1-t^2)^2 + 4a^2t^2}$$

$$= \sqrt{a^2[(1-t^2)^2 + 4t^2]}$$

$$= a\sqrt{(1+t^2)^2} = a(1+t^2)$$

$$SQ = \sqrt{\left(\frac{a}{t^2} - a\right)^2 + \left(\frac{2a}{t} - 0\right)^2}$$

$$= \sqrt{a^2\left(\frac{1}{t^2} - 1\right)^2 + \frac{4a^2}{t^2}}$$

$$= a\sqrt{\left(\frac{1}{t^2} - 1\right)^2 + \frac{4}{t^2}}$$

$$= a\sqrt{\left(\frac{1}{t^2} + 1\right)^2}$$

$$= a\left(\frac{1}{t^2} + 1\right)$$

$$\frac{1}{SP} + \frac{1}{SQ} = \frac{1}{a(1+t^2)} + \frac{1}{a\left(\frac{1}{t^2} + 1\right)}$$

$$= \frac{1}{a(1+t^2)} + \frac{t^2}{a(1+t^2)}$$

$$= \frac{\cancel{1+t^2}}{a\cancel{(1+t^2)}} = \frac{1}{a}$$

Ans: A

04	Let $C(x, y)$, $A(3,4)$, $B(7,7)$
	Option verification method
	Let $C(-5,-2)$.
	$AC = \sqrt{(3+5)^2 + (4+2)^2}$
	$= \sqrt{64+36} = \sqrt{100} = 10$ Ans.: D
05	$A(1,1)$, $B(-2,7)$, $C(3,-3)$
	$AB = \sqrt{(1+2)^2 + (1-7)^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$
	$BC = \sqrt{(3+2)^2 + (-3-7)^2} = \sqrt{25+100} = \sqrt{125} = 5\sqrt{5}$
	$AC = \sqrt{(3-1)^2 + (-3-1)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$
	$\because AB + AC = BC$
	A, B, C are collinear
	Ans: C
06	$A(\underset{x_1}{a\sin\alpha}, \underset{y_1}{-b\cos\alpha})$, $B(\underset{x_2}{-a\cos\alpha}, \underset{y_2}{b\sin\alpha})$
	$AB = \sqrt{(a\sin\alpha + a\cos\alpha)^2 + (b\sin\alpha + b\cos\alpha)^2}$
	$= \sqrt{(a^2+b^2)\sin^2\alpha + (a^2+b^2)\cos^2\alpha + 2(a^2+b^2)\sin\alpha\cos\alpha}$
	$= \sqrt{(a^2+b^2)\left[\sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha\right]}$
	$= \sqrt{a^2+b^2}\,.(\sin\alpha + \cos\alpha)$
	Ans: C

07. $A(2,1)$, $B(1,-2)$, $P(x,y)$ (4)

$$PA = PB$$

$$\Rightarrow PA^2 = PB^2$$

$$\Rightarrow (x-2)^2+(y-1)^2=(x-1)^2+(y+2)^2$$

$$\Rightarrow x^2-4x+\cancel{4}+y^2-2y+\cancel{1}=x^2-2x+\cancel{1}+y^2+4y+\cancel{4}$$

$$\Rightarrow 2x+6y=0$$

$$\Rightarrow x+3y=0$$

Ans: D

08

$$x^2+2ax-b^2=0$$

x_1, x_2

$$x_1+x_2=-2a$$

$$x_1 \cdot x_2=-b^2$$

$$y^2+2py-q^2=0$$

y_1, y_2

$$y_1+y_2=-2p$$

$$y_1 \cdot y_2=-q^2$$

$A(x_1,y_1)$, $B(x_2,y_2)$

$$AB=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

$$=\sqrt{(x_1+x_2)^2-4x_1x_2+(y_1+y_2)^2-4y_1y_2}$$

$$=\sqrt{(-2a)^2-4(-b^2)+(-2p)^2-4(-q^2)}$$

$$=\sqrt{4a^2+4b^2+4p^2+4q^2}$$

$$=2\sqrt{a^2+b^2+p^2+q^2}$$

09 $A(3, 4)$, $B(-2, 3)$ $\rightarrow$ Equilateral triangle ⑤

$x_1 \; y_1 \quad x_2 \; y_2$

$$\text{Third vertex} = \left(\frac{x_1 + x_2 \pm \sqrt{3}(y_2 - y_1)}{2}, \frac{y_1 + y_2 \mp \sqrt{3}(x_2 - x_1)}{2} \right)$$

$$= \left(\frac{3 - 2 \pm \sqrt{3}(3-4)}{2}, \frac{4 + 3 \mp \sqrt{3}(-2-3)}{2} \right)$$

$$= \left(\frac{1 \pm \sqrt{3}(-1)}{2}, \frac{7 \mp \sqrt{3}(-5)}{2} \right)$$

$$= \left(\frac{1 \mp \sqrt{3}}{2}, \frac{7 \pm 5\sqrt{3}}{2} \right)$$

$$= \left(\frac{1 - \sqrt{3}}{2}, \frac{7 + 5\sqrt{3}}{2} \right), \left(\frac{1 + \sqrt{3}}{2}, \frac{7 - 5\sqrt{3}}{2} \right)$$

Ans: D

10. $P(k-1, 2)$, $A(3, k)$, $B(k, 5)$

$$PA = PB$$
$$\Rightarrow PA^2 = PB^2$$
$$\Rightarrow \left[(k-1-3)^2 + (2-k)^2\right] = \left[(k-1-k)^2 + (2-5)^2\right]$$
$$\Rightarrow (k-4)^2 + (2-k)^2 = (-1)^2 + (-3)^2$$
$$\Rightarrow \cancel{k^2} - 8k + 16 + 4 - 4k + \cancel{k^2} = 1 + 9$$
$$\Rightarrow 2k^2 - 12k + 10 = 0$$
$$\Rightarrow k^2 - 6k + 5 = 0$$
$$\Rightarrow (k-1)(k-5) = 0$$
$$\Rightarrow k = 1, 5$$

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11. $A(5, 7)$, $B(2, x)$

$$AB = 5$$
$$\Rightarrow AB^2 = 25$$
$$\Rightarrow (2-5)^2 + (x-7)^2 = 25$$
$$\Rightarrow 9 + x^2 - 14x + 49 = 25$$
$$\Rightarrow x^2 - 14x + 33 = 0$$
$$\Rightarrow (x-11)(x-3) = 0 \Rightarrow x = 3, 11$$

Ans: A, C

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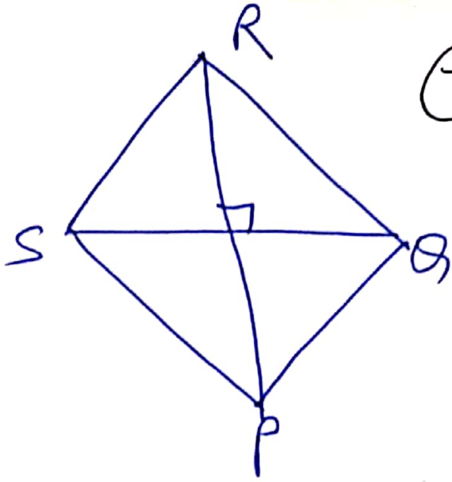
12. $A(-4, 0)$, $B(4, 0)$ $\rightarrow$ Equilateral triangle
$\quad x_1 \ y_1 \qquad x_2 \ y_2$

$$\text{Third vertex} = \left(\frac{x_1 + x_2 \pm \sqrt{3}(y_2 - y_1)}{2}, \frac{y_1 + y_2 \mp \sqrt{3}(x_2 - x_1)}{2}\right)$$
$$= \left(\frac{-4+4 \pm \sqrt{3}(0-0)}{2}, \frac{0+0 \pm \sqrt{3}(4+4)}{2}\right)$$
$$= (0, \pm 4\sqrt{3})$$

Ans: A, B

13 Statement I:



$$PR = \sqrt{(-2-2)^2 + (3+1)^2}$$
$$= \sqrt{16+16}$$
$$= \sqrt{32} = 4\sqrt{2}$$

$$SQ = \sqrt{(3+3)^2 + (4+2)^2}$$
$$= \sqrt{36+36} = 6\sqrt{2}$$

$PR \neq SQ \Rightarrow$ Rhombus (True)

Statement II: Conceptual (True)

Ans: A

14. Statement I: $A(5,2)$, $B(7,9)$, $C(9,16)$

$$AB = \sqrt{(7-5)^2 + (9-2)^2} = \sqrt{53}$$

$$BC = \sqrt{(9-7)^2 + (16-7)^2} = \sqrt{53}$$

$$CA = \sqrt{(9-5)^2 + (16-2)^2} = \sqrt{212} = 2\sqrt{53}$$

$\therefore AB + BC = CA$

Hence, A, B, C are collinear (True)

Statement II: Conceptual (True)

Ans: A

15 $A(2,7)$, $B(8,3)$, $C(14,-1)$

$$AB = \sqrt{(8-2)^2 + (3-7)^2} = \sqrt{52}$$

$$BC = \sqrt{(14-8)^2 + (-1-3)^2} = \sqrt{52}$$

$$CA = \sqrt{(14-2)^2 + (-1-7)^2} = 2\sqrt{52}$$

$\therefore AB + BC = CA$

Ans: A

16 $A(-1,3)$, $B(2,p)$, $C(5,-1)$

$$\text{Area} = \frac{1}{2}\left| x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2) \right| = 0$$

$$\Rightarrow -1(p+1) + 2(-1-3) + 5(3-p) = 0$$

$$\Rightarrow p = 1$$

Ans: A

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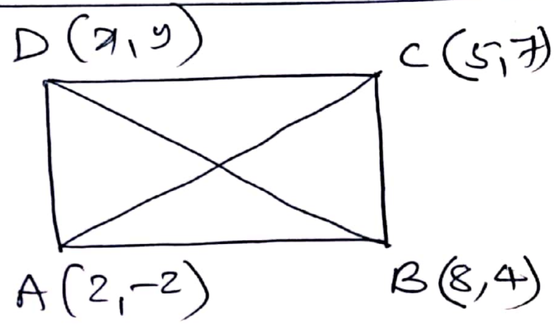
17. mid-point of BD = mid-point of AC



$$\therefore \frac{x+8}{2} = \frac{2+5}{2}$$

$$\Rightarrow x = -1$$

$$\Rightarrow 2x = -2$$

$$\Rightarrow 2x+3 = -2+3 = 1$$

Ans: 1

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18 a) $\sqrt{(-1+6)^2+(-1-7)^2} = \sqrt{25+64} = \sqrt{89}$

b) $A(4,3)$, $B(x,5)$, $C(2,3)$

$$CA = CB$$

$$\Rightarrow CA^2 = CB^2$$

$$\Rightarrow (4-2)^2 + (3-3)^2 = (x-2)^2 + (5-3)^2$$

$$\Rightarrow \cancel{4} = (x-2)^2 + \cancel{4}$$

$$\Rightarrow x = 2$$

c) $PA = PB \Rightarrow PA^2 = PB^2$

$$\Rightarrow (x-2)^2 + (0+5)^2 = (x+2)^2 + (0-9)^2$$

$$\Rightarrow x = -7$$

d) $OA^2 = 3^2 + 4^2 = 25$

Ans: −, p, q, r

LEARNER'S TASK

01. $A(a,0)$, $B(b,0)$

$AB = \sqrt{(b-a)^2+0^2} = \sqrt{(a-b)^2} = |a-b|$ Ans: D

02 $A(0,a)$, $B(0,b)$

$AB = \sqrt{(0-0)^2+(b-a)^2} = \sqrt{(a-b)^2} = |a-b|$ Ans: D

03 $A(\cancel{-3},4)$, $O(0,0)$

$OA = \sqrt{(-3)^2+4^2} = \sqrt{25} = 5$ Ans: B

04. $A(x_1, x_2)$, $B = (y_1, y_2)$

$AB = \sqrt{(x_1-y_1)^2+(x_2-y_2)^2}$ Ans: B

05 $A(a,b)$, $O = (0,0)$

$AO = \sqrt{a^2+b^2}$ Ans: B

06 $A = (a,0)$

Distance from Y-axis $= |x| = |a|$ Ans: C

07 $A = (0, b)$

Distance from Y-axis $= |x| = |0| = 0$

Ans: A

08 $A(\underset{x_1}{a}, \underset{y_1}{0})$, $B(\underset{x_2}{0}, \underset{y_2}{b})$

$AB = \sqrt{(0-a)^2+(b-0)^2} = \sqrt{a^2+b^2}$ Ans: A

09. $P(x_1, y_1)$, $Q(x_2, y_1)$ $= |x_1 - x_2|$

$PQ = \sqrt{(x_2-x_1)^2+(y_1-y_1)^2} = |x_2 - x_1|$ Ans: D

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10 $A = (-3, 4)$

Distance from y-axis $= |x| = |-3| = 3$ Ans: A

JEE MAINS LEVEL

01. $P(x, y)$, $A(a+b, b-a)$, $B(a-b, a+b)$

$\Rightarrow PA = PB$

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (x-(a+b))^2 + (y-(b-a))^2 = (x-(a-b))^2 + (y-(a+b))^2$

$\Rightarrow \cancel{x^2} - 2(a+b)x + \cancel{(a+b)^2} + \cancel{y^2} - 2(b-a)y + \cancel{(b-a)^2}$

$= \cancel{x^2} - 2(a-b)x + \cancel{(a-b)^2} + \cancel{y^2} - 2(a+b)y + \cancel{(a+b)^2}$

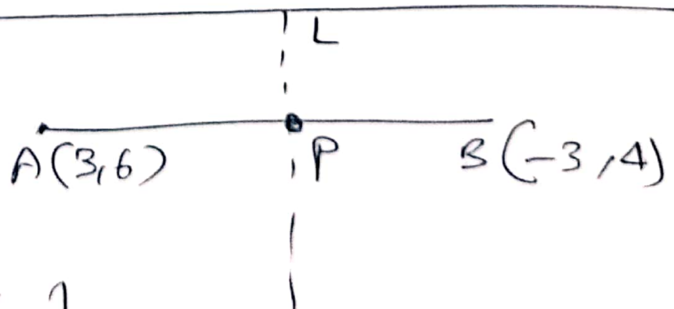
$\Rightarrow (a+b)x + (b-a)y + (a-b)x + (a+b)y = 0$

$\Rightarrow [a+\cancel{b}+a-\cancel{b}]x + [b-\cancel{a}+\cancel{a}+b]y = 0$

$\Rightarrow 2ax + 2by = 0$

$\Rightarrow ax + by = 0$ Ans: D

02 $P = \left(\frac{3-3}{2}, \frac{6+4}{2}\right)$



$P = (0, 5)$

slope of AB $= \frac{4-6}{-3-3} = \frac{1}{3}$

slope of L $= \frac{-1}{m} = -3$

Eqn. $y - y_1 = m(x - x_1)$

$y - 5 = -3(x - 0)$

$3x + y - 5 = 0$

Ans: B

03. Point on Y-axis P(0, y), A(6, 5), B(−4, 3)

$PA = PB \Rightarrow$

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (6-0)^2 + (5-y)^2 = (-4-0)^2 + (3-y)^2$

$\Rightarrow 36 + \cancel{25} - 10y + \cancel{y^2} = \cancel{16} + \cancel{9} - 6y + \cancel{y^2}$

$\Rightarrow \quad y = 9 \quad \therefore (0, 9)$

Ans: D

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04. Let $P(2x, x)$, $Q(2, -5)$, $R(-3, 6)$

$\Rightarrow PQ = PR$

$\Rightarrow PQ^2 = PR^2$

$\Rightarrow (2x-2)^2 + (x+5)^2 = (2x+3)^2 + (x-6)^2$

$\Rightarrow x = 8$

$\therefore P(16, 8)$

Ans. B

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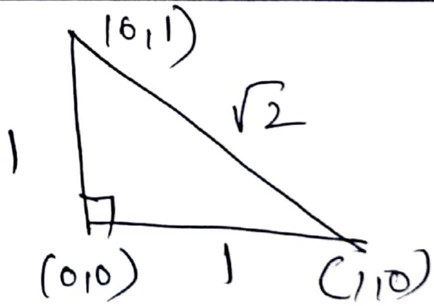
05. perimeter $= 1 + 1 + \sqrt{2}$

$= 2 + \sqrt{2}$



Ans: B

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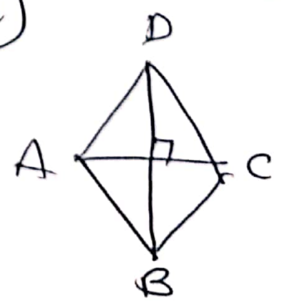
06. $A(2,-1)$, $B(3,4)$, $C(-2,3)$, $D(-3,-2)$

We obsreve $AB = BC = CA = DA$

$AC \neq BD$

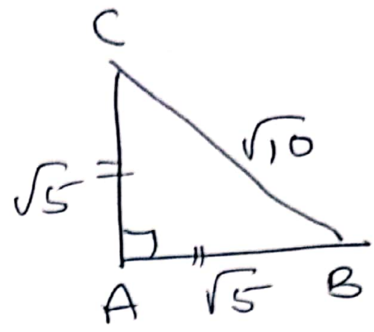
Hence, ABCD is a Rhombus, but not square.



Ans. B

07 $A(0,-1)$, $B(-2,0)$, $C(-1,-3)$

We observe



$AB = \sqrt{4+1} = \sqrt{5}$

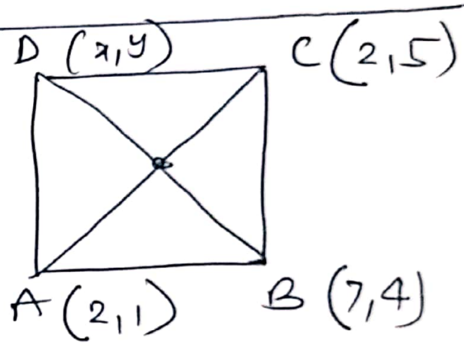
$BC = \sqrt{1+9} = \sqrt{10}$

$CA = \sqrt{1+4} = \sqrt{5}$

i.e $BC^2 = AB^2 + CA^2$

It is right angled isosceles triangle Ans: B

08 mid-point of BD = mid-point of AC



$\Rightarrow \left(\frac{x+7}{2}, \frac{y+4}{2}\right) = \left(\frac{2+2}{2}, \frac{1+5}{2}\right)$

$\Rightarrow (x,y) = (-3, 2)$ Ans: C

09. $A(\underset{x_1}{-4}, \underset{y_1}{0})$, $B(\underset{x_2}{4}, \underset{y_2}{0})$ → Equilateral triangle

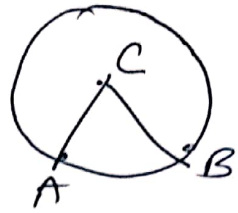
Third vertex $= \left(\frac{x_1+x_2 \pm \sqrt{3}(y_2-y_1)}{2}, \frac{y_1+y_2 \mp \sqrt{3}(x_2-x_1)}{2}\right)$

$= \left(\frac{-4+4 \pm \sqrt{3}(0-0)}{2}, \frac{0+0 \mp \sqrt{4+4}}{2}\right)$

$= \left(0, \pm \frac{2\sqrt{2}}{2}\right)$

$= (0, \pm\sqrt{2})$

10 $C(2, -3y)$, $A(-1, y)$, $B(5, 7)$



$CA = CB$

$\Rightarrow CA^2 = CB^2$

$\Rightarrow (2+1)^2 + (-3y-y)^2 = (2-5)^2 + (-3y-7)^2$

$\Rightarrow \cancel{9} + 16y^2 = \cancel{9} + 9y^2 + 42y + 49$

$\Rightarrow 7y^2 - 42y - 49 = 0$

$\Rightarrow y - 6y - 7 = 0$

$\Rightarrow (y-7)(y+1) = 0$

$\Rightarrow y = -1$ or 7

$\therefore$ Let $y = -1$

$C(2, 3)$, $A(-1, -1)$

$r = CA = \sqrt{(2+1)^2 + (3+1)^2} = 5$

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11. $A(x_1, y_1)$, $B(x_2, y_2)$

All the four options are correct. Ans: A, B, C, D

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12 Statement I: $P(2, 2)$, $A(-2, k)$, $B(-2k, -3)$

$PA = PB$

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (-2-2)^2 + (k-2)^2 = (2+2k)^2 + (2+3)^2$

$\Rightarrow k = -1$ or -3 (True)

Statement I: Not always true.

Ans: C

13 $P(x, y)$, $A(3, 6)$, $B(-3, 4)$

$PA = PB$

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$

$\Rightarrow -6x + \cancel{9} - 12y + 36 = 6x + \cancel{9} - 8y + 16$

$\Rightarrow 12x + 4y - 20 = 0$

$\Rightarrow 3x + y - 5 = 0$

Ans: B

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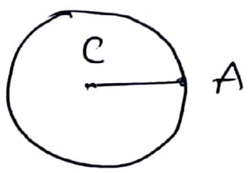
14 $C(2a, a-7)$, $A(11, -9)$



$r = \sqrt{(2a-11)^2 + (a-7+9)^2}$

$= \sqrt{(2a-11)^2 + (a+2)^2} = 5\sqrt{2}$

S.O.B.S

$\Rightarrow (2a-11)^2 + (a+2)^2 = 50$

$\Rightarrow 4a^2 - 44a + 121 + a^2 + 4a + 4 - 50 = 0$

$\Rightarrow 5a^2 - 40a + 75 = 0$

$\Rightarrow a^2 - 8a + 15 = 0$

$\Rightarrow (a-3)(a-5) = 0$

$\Rightarrow a = 3, 5$

Ans: 5

15 a) $X(x,0)$, $A(5,9)$, $B(-4,6)$ 15

$XA = XB$

$\Rightarrow XA^2 = XB^2$

$\Rightarrow (x-5)^2 + (0-9)^2 = (x+4)^2 + (0-6)^2$

$\Rightarrow \cancel{x^2} - 10x + 25 + 81 = \cancel{x^2} + 8x + 16 + 36$

$\Rightarrow 18x - 54 = 0$

$\Rightarrow x = 3 \quad \therefore (3,0)$

b) $Y(0,y)$, $A(2,3)$, $B(-4,1)$

$YA = YB$

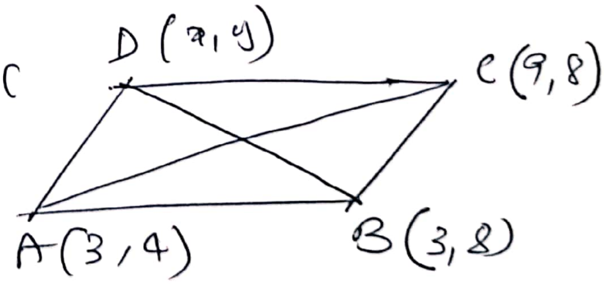
$\Rightarrow YA^2 = YB^2 =$

$\Rightarrow (0-2)^2 + (y-3)^2 = (0+4)^2 + (y-1)^2$

$\Rightarrow 4 + \cancel{y^2} - 6y + 9 = 16 + \cancel{y^2} - 2y + 1$

$\Rightarrow 4y + 4 = 0 \Rightarrow y = -1 \quad \therefore (0,-1)$

c) mid-point BD = mid-point of AC



$\left(\frac{x+3}{2}, \frac{y+8}{2}\right) = \left(\frac{3+9}{2}, \frac{4+8}{2}\right)$

$\Rightarrow (x,y) = (9,4)$

d) $PA^2 = PB^2$

$\Rightarrow (3-0)^2 + (k-2)^2 = (k-0)^2 + (5-2)^2$

$\Rightarrow \cancel{9} + \cancel{k^2} - 4k + 4 = \cancel{k^2} + \cancel{9}$

$\Rightarrow k = 1$

$\Rightarrow$ THE END Ans: S, P, Q, t