

4. QUADRATIC EXPRESSIONS

Class: IX. MATHEMATICS - II ①

FOUNDATION + SOLUTIONS

TEACHING TASK

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Q1.

Given $f(x) = \sqrt{x^2 + 4x + 5}$

We have $x^2 + 4x + 5 \geq 0, \forall x \in \mathbb{R}$

Since $\Delta < 0$, Domain of $f = \mathbb{R}$.

$$\begin{aligned} \text{Now, } f(x) &= \sqrt{x^2 + 4x + 5} \\ &= \sqrt{(x+2)^2 + 1} \end{aligned}$$

clearly, $\forall x \in \mathbb{R}, (x+2)^2 + 1 \geq 1$

$$\Rightarrow \sqrt{(x+2)^2 + 1} \geq 1$$

$\therefore$ Range of $f(x) = [1, \infty)$

Ans: B

Q2. $f(x) = (x-2)^2 + 1 \geq 0, \forall x \in \mathbb{R}$

$\therefore$ Range of $f(x) = [1, \infty)$

Ans: A

Q3. $f(x) = \frac{1}{x^2 - 5x + 6}$

f is defined only when, $x^2 - 5x + 6 \neq 0$.

$$\Rightarrow (x-2)(x-3) \neq 0 \Rightarrow x \neq 2 \text{ and } x \neq 3$$

$\therefore$ Domain = $\mathbb{R} - \{2, 3\}$

Ans: A



$$\begin{aligned}
 & x^2 - 2x + 1 \leq 0 \\
 & \Rightarrow (x-1)^2 \leq 0 \\
 & \Rightarrow x = 1
 \end{aligned}$$

②

Ans: B

05. $f(x) = x^2 + Kx + 4$.

Minimum value = $\frac{4ac - b^2}{4a} = 0$

$$\begin{aligned}
 & \Rightarrow 4ac - b^2 = 0 \\
 & \Rightarrow 16 - K^2 = 0 \\
 & \Rightarrow K = \pm 4
 \end{aligned}$$

Ans: D

06. $y = x^2 + 2x + 3$

$y = ax^2 + bx + c$

Max. Value occurs at $x = \frac{-b}{2a} = \frac{-2}{2 \cdot 1} = -1$

Ans: B

07. a, b are the roots of $x^2 + ax + b$

$$\begin{aligned}
 \therefore a + b &= -a & a \cdot b &= b \\
 \Rightarrow 1 + b &= -1 & \Rightarrow a &= 1 \\
 \Rightarrow b &= -2
 \end{aligned}$$

min. value of $x^2 + ax + b = \frac{4 \cdot 1 \cdot b - a^2}{4 \cdot 1}$

$$= \frac{4(-2) - 1^2}{4} = \frac{-9}{4}$$

Ans: B

08. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$$\begin{aligned}
 &= (a-2)^2 + 2(a+1) \\
 &= a^2 - 2a + 6 \\
 &= (a-1)^2 + 5
 \end{aligned}$$

We know, $(a-1)^2 \geq 0$

$$\Rightarrow (a-1)^2 + 5 \geq 5$$

$\therefore$ Least value = 5

Ans: C

09.

$$\frac{14x}{x+1} = \frac{9x-30}{x-1} < 0$$

(2)

$$\Rightarrow \frac{(x-1)(x-6)}{(x+1)(x-1)} < 0$$



$$\therefore x \in (-\infty, -1) \cup (1, 6)$$

Ans. D

10. $\min \{f(x)\} > \max \{g(x)\}$

$$\frac{4 \cdot 1 \cdot 2c^2 - 4b^2}{4 \cdot 1} > \frac{4(-1)b^2 - 4c^2}{4(-1)}$$

$$\Rightarrow 2c^2 - b^2 > b^2 + c^2$$

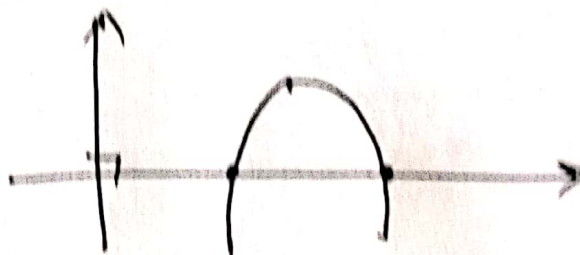
$$\Rightarrow c^2 > 2b^2$$

$$\Rightarrow |c| > |b|\sqrt{2}$$

Ans. B

THE ADVANCED LEVEL

01. Conceptual



A) $a < 0$ (true)

B) $b^2 - 4ac > 0$ (true)

C) Range = $(-\infty, \text{max. value})$ Ans. A, B, D

02 $(3+2\sqrt{2})^{x^2-4} + (3-2\sqrt{2})^{x^2-4} = 6$ (4)

clearly, $(3+2\sqrt{2}) + (3-2\sqrt{2}) = 6$

and $(3+2\sqrt{2})(3-2\sqrt{2}) = 9-8 = 1$

$\therefore x^2-4 = \pm 1$

$\Rightarrow x^2-4 = 1$ or $x^2-4 = -1$

$\Rightarrow x = \pm\sqrt{5}$ or $x = \pm\sqrt{3}$ Ans: A, B, C, D

03 Statement I: $f(x) = x^2 + 6x + 13$

min. value = $\frac{4ac - b^2}{4a}$

= $\frac{4 \cdot 1 \cdot 13 - 6^2}{4 \cdot 1} = 4$ (True)

Statement II: $f(x) = x^2 + 6x + 13$

min. value occurs at

$x = -\frac{b}{2a}$

= $-\frac{6}{2 \cdot 1} = -3$ (True)

Ans: A

04. Statement I: $(\log_5 x)^2 + (\log_5 x) < 2$

let $\log_5 x = a$

$\therefore a^2 + a - 2 < 0$

$\Rightarrow (a+2)(a-1) < 0$

$\Rightarrow -2 < a < 1$

$\Rightarrow -2 < \log_5 x < 1$

$\Rightarrow 5^{-2} < x < 5^1$

$\Rightarrow \frac{1}{25} < x < 5$

$\therefore x \in (\frac{1}{25}, 5)$

(True)

Statement II: Conceptual (true)

Ans: A (5)

05

Assertion:

$$\text{Given } \sqrt{x^2 - 5x + 7} < 0$$

$$\text{here } x^2 - 5x + 7$$

$$\Delta = -3 < 0$$

i.e. $x^2 - 5x + 7$ does not have real roots

Hence $\sqrt{x^2 - 5x + 7} < 0$ is meaningless (true)

Reason: Conceptual (true)

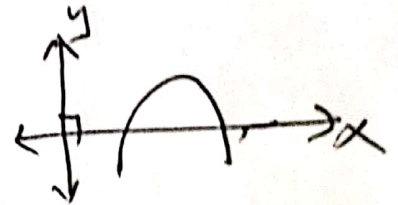
Ans: A

06

Assertion:

$$y = -2x^2 + 3x - 1$$

$$\text{here } a = -2 < 0$$



Reason: Conceptual (true)

Ans: A

07.

$$f(x) = \sqrt{x^2 - 4x + 3}$$

$$\therefore x^2 - 4x + 3 \geq 0$$

$$\Rightarrow (x-1)(x-3) \geq 0$$

$$\text{Domain} = (-\infty, 1] \cup [3, \infty)$$

Ans: B

08

$$f(x) = \sqrt{x^2 - 4x + 3}$$

$$= \sqrt{(x-2)^2 - 3}$$

Minimum value possible = 0

Ans: A

09

$$2 \frac{x^2 - 5x + 8}{x^2 - 5x + 8} + 3 = 13 \quad (6)$$

clearly, $2^2 + 3^2 = 13$

$$\Rightarrow x^2 - 5x + 8 = 2$$

$$\Rightarrow x^2 - 5x + 6 = 0$$

$$\Rightarrow (x-2)(x-3) = 0$$

$$\Rightarrow x = 2, 3$$

Ans: A

10

$$(2\sqrt{2})^{x^2 - 4x + 6} + (3\sqrt{2})^{x^2 - 4x + 6} = 26$$

clearly, $(2\sqrt{2})^2 + (3\sqrt{2})^2 = 26$

$$\therefore x^2 - 4x + 6 = 2$$

$$\Rightarrow x^2 - 4x + 4 = 0$$

$$\Rightarrow x = 2, 2$$

Ans: D

11.

$$y = x^2 - 4x + 5$$

$$\therefore x^2 - 4x + 5 = 0$$

$$\Delta = 16 - 20 = -4 < 0$$

$\therefore$ The graph never touches the x-axis Ans: D

12.

$$x = \frac{\sqrt{7} + \sqrt{3}}{\sqrt{7} - \sqrt{3}} \times \frac{\sqrt{7} + \sqrt{3}}{\sqrt{7} + \sqrt{3}} = \frac{7 + 3 + 2\sqrt{21}}{7 - 3} = \frac{5 + \sqrt{21}}{2}$$

Similarly, $y = \frac{5 - \sqrt{21}}{2}$

$$x^4 + y^4 + (x+y)^4 = \left(\frac{5 + \sqrt{21}}{2}\right)^4 + \left(\frac{5 - \sqrt{21}}{2}\right)^4 + \left(\frac{5 + \sqrt{21}}{2} + \frac{5 - \sqrt{21}}{2}\right)^4$$

$$= 1152$$

Ans: 1152

13

NOTE: Please note, We have to find the "RANGE" of the functions given in column I.

⑦

a) $y = x^2 + 1 \Rightarrow \text{Range} = [1, \infty)$

b) $y = (x-2)^2 - 5 \Rightarrow \text{Range} = [-5, \infty)$

c) Since $(x-2)^2 \geq 0$
 $\Rightarrow (x-2)^2 - 5 \geq -5$

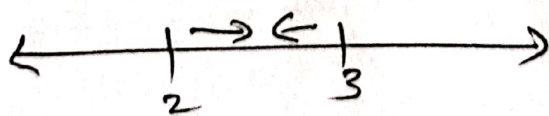
c) $y = -x^2 + 4$
 We have, $x^2 \geq 0$
 $\Rightarrow -x^2 \leq 0$
 $\Rightarrow -x^2 + 4 \leq 4$
 $\therefore \text{Range} = (-\infty, 4]$

d) $y = (x+1)^2$
 We have $(x+1)^2 \geq 0$
 $\therefore \text{Range} = [0, \infty)$

Ans: P, Q, R, S

14.

a) $x^2 - 5x + 6 < 0$
 $\Rightarrow (x-2)(x-3) < 0$



$\therefore 2 < x < 3$

$$b) x^2 + x - 12 > 0$$

$$\Rightarrow (x+4)(x-3) > 0$$

$$\Rightarrow \leftarrow \begin{array}{c} \leftarrow \quad \quad \quad \rightarrow \\ -4 \quad \quad \quad 3 \end{array} \rightarrow$$

$$x < -4 \text{ or } x > 3$$

$$c) x^2 + 9x + 18 < 0$$

$$\Rightarrow (x+6)(x+3) < 0$$

$$\Rightarrow -6 < x < -3$$

$$d) x^2 - x - 6 > 0$$

$$\Rightarrow (x-3)(x+2) > 0$$

$$\leftarrow \begin{array}{c} \leftarrow \quad \quad \quad \rightarrow \\ -2 \quad \quad \quad 3 \end{array} \rightarrow$$

$$x < -2 \text{ or } x > 3$$

Ans: p, r, t, v

LEARNERS TASK (CUB'S)

01. $\alpha\beta = 6, \alpha + \beta = 5$

$$\Rightarrow x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\Rightarrow x^2 - 5x + 6 = 0$$

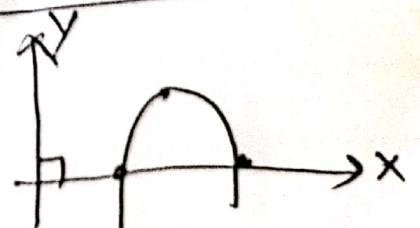
Ans: A

02. Roots Equal $\Rightarrow b^2 - 4ac = 0$
 $\Rightarrow b^2 = 4ac$

Ans: A

03. $y = ax^2 + bx + c$

$$\Rightarrow a < 0 \text{ and } \Delta > 0$$



Ans: C

04. $y = ax^2 + 6x + 9$
 $\Delta < 0$
 $\Rightarrow 6^2 - 4 \cdot a \cdot 9 < 0$
 $\Rightarrow 1 - a < 0$
 $\Rightarrow a > 1$

Ans: B

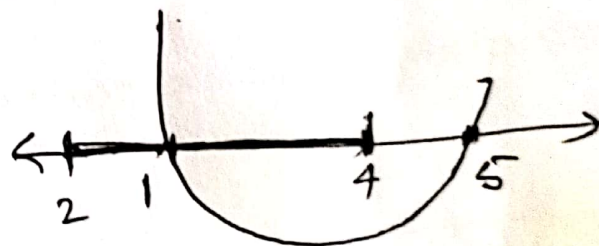
05. $y = x^2 + bx + 4$
 $\Delta = 0$
 $\Rightarrow b^2 - 4 \cdot 1 \cdot 4 = 0$
 $\Rightarrow b = \pm 4$

Ans: C

06. Conceptual

Ans: C

07. $y = x^2 - 6x + 5$
 $= (x-1)(x-5)$



min. value $= \frac{4ac - b^2}{4a}$
 $= -4$

Ans: A

✓ 08. Conceptual

Ans: C

✓ 09. Conceptual

✓ Ans: D

10. Conceptual

Ans: A

JEE MAINS LEVEL

01. $x^2 - (\alpha + \beta)x + \alpha\beta = 0$
 $\Rightarrow x^2 - 6x + 5 = 0$

Ans: A

02.

$$x^2 - 3x - 10 = 0$$

$$\Rightarrow (x-5)(x+2) = 0$$

$$\Rightarrow x = -2 \text{ or } 5$$

NOTE: Change the question

"Solve $x^2 - 3x - 10 = 0$ "

Ans: B

03

$$f(x) = \sqrt{5-x^2}$$

$$\Rightarrow 5-x^2 \geq 0$$

$$\Rightarrow x^2 - 5 \leq 0$$

$$\Rightarrow (x+\sqrt{5})(x-\sqrt{5}) \leq 0$$

$$\Rightarrow -\sqrt{5} \leq x \leq \sqrt{5}$$

$$\therefore \text{Domain} = [-\sqrt{5}, \sqrt{5}]$$

Ans: A

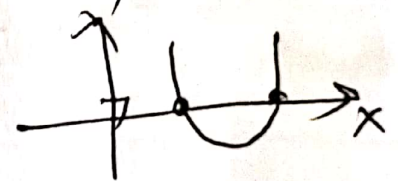
04

$$y = x^2 - 4x + 3$$

$a=1 > 0$, The parabola is open upward

$$\text{Also, } \Delta = 4 > 0$$

The parabola intersects the x-axis at two distinct points



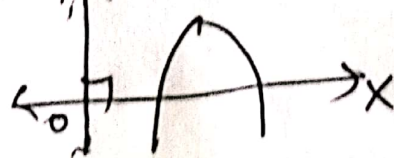
Ans: B

05

$$y = -x^2 + 3x + 4$$

$a = -1 < 0$, The parabola is open downward

$\Delta = 25 > 0$, the parabola intersects the x-axis at two distinct points



Ans: A

06

$$x^2 - 3x + 2 \geq 0$$
$$\Rightarrow (x-1)(x-2) \geq 0$$
$$\Rightarrow x \in (-\infty, 1] \cup [2, \infty)$$

Ans. B

07.

$$f(x) = -2x^2 + 8x + c$$
$$\Rightarrow \frac{4(-2) \cdot c - 8^2}{4 \cdot (-2)} = 10$$
$$\Rightarrow c = 2$$

Ans. B

08

$$-x^2 + x + 6 > 0$$
$$\Rightarrow x^2 - x - 6 < 0$$
$$\Rightarrow (x-3)(x+2) < 0$$
$$\Rightarrow -2 < x < 3$$

Ans. D

09.

$$2^{x^2-3} = 2 \Rightarrow x^2-3=1 \Rightarrow x = \pm 2$$

Ans. A

10

$$(x-2)^2 - 2|x-2| - 15 = 0$$
$$\Rightarrow |x-2|^2 - 2|x-2| - 15 = 0$$

$$\text{let } |x-2| = a$$

$$\Rightarrow a^2 - 2a - 15 = 0$$

$$\Rightarrow (a+3)(a-5) = 0$$

$$\Rightarrow a+3=0 \quad \text{or} \quad a-5=0$$

$$\Rightarrow a = -3 \quad \Rightarrow a = 5$$

$$\Rightarrow |x-2| = -3 \quad \Rightarrow |x-2| = 5$$

$$\text{Not possible} \quad \Rightarrow x-2 = \pm 5$$
$$\Rightarrow x = -3 \text{ or } 7$$

$$\text{Sum} = -3 + 7$$
$$= 4$$

Ans. A

01. $f(x) = 3x^2 - 6x + 7$

$a = 3 > 0$, f has minimum value

Minimum value occurs at $x = \frac{-b}{2a}$
 $= \frac{-(-6)}{2 \cdot 3} = 1$.

Minimum value $= \frac{4ac - b^2}{4a}$
 $= \frac{4 \cdot 3 \cdot 7 - (-6)^2}{4 \cdot 3} = 4$

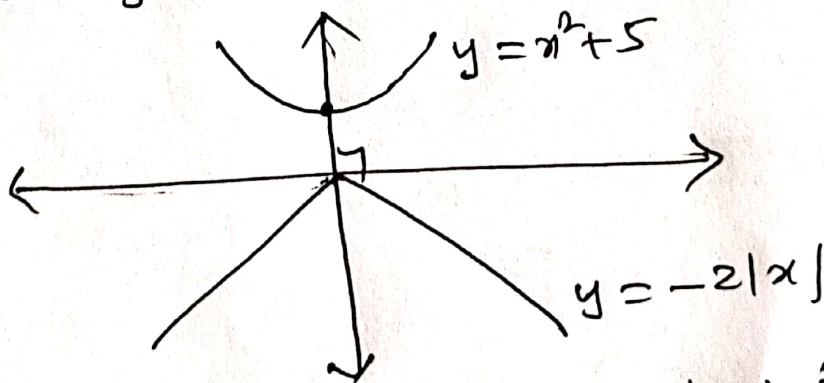
$\therefore \text{Vertex} = (1, 4)$
 $\text{Range} = [4, \infty)$

Ans: A, C, D

02. $x^2 + 2|x| + 5 = 0$

$\Rightarrow x^2 + 5 = -2|x|$

Let $y = x^2 + 5$ and $y = -2|x|$



Both the graphs do not intersect.
 $\therefore$ The given eqn does not have real roots

Ans: A, B

03

Statement I:

$$x^2 - 5x + 6 < 0$$

(13)

$$\Rightarrow (x-2)(x-3) < 0$$

$$\Rightarrow 2 < x < 3$$

$$\Rightarrow x \in (2, 3) \text{ (True)}$$

Statement II: Conceptual (True)

Ans: A

04

Statement I:

$$(\cos p - 1)x^2 + \cos p x + \sin p = 0$$

$$\therefore \Delta \geq 0$$

$$\Rightarrow \cos^2 p - 4(\cos p - 1)(\sin p) \geq 0$$

Clearly, $\cos^2 p \geq 0$, $-1 \leq \cos p \leq 1$
 $\Rightarrow \cos p - 1 \leq 0$

i.e. $\cos^2 p$ is positive, $\cos p - 1$ is negative

$$\Rightarrow \cos^2 p - 4(\cos p - 1)\sin p \geq 0$$

$$\Rightarrow \begin{matrix} \downarrow & \downarrow \\ \text{positive} & \text{negative} \end{matrix}$$

$$\Rightarrow \sin p \geq 0$$

$\Rightarrow p$ lies in either I or II quadrant

$$\Rightarrow p \in [0, \pi]$$

$$\Rightarrow p \in (0, \frac{\pi}{2}) \text{ (True)}$$

Statement II: Conceptual (False)

Ans: C

05. Assertion: $y = x^2 + 4x + 7$

$a = 1 > 0$

$\therefore y$ has minimum value (True)

Reason: Conceptual (True)

Ans. A

06. Assertion: $x^2 - 6x + 9 = 0$ (True)

Reason: Conceptual (True)

Ans. A

07. $y = -2x^2 + 8x - 5$

x -Coordinate $= x = \frac{-b}{2a} = \frac{-8}{2(-2)} = 2$

Ans. A

08. Max. value $= \frac{4ac - b^2}{4a}$

$= \frac{4 \cdot (-2) \cdot (-5) - 8^2}{4(-2)} = 3$

Ans. B

09. $-x^2 + 5x + 24 > 0$

$\Rightarrow x^2 - 5x - 24 < 0$

$\Rightarrow (x-8)(x+3) < 0$

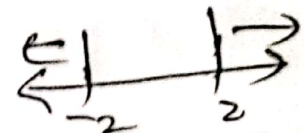
$\Rightarrow -3 < x < 8$

Ans. C

10. $x \leq -2$ or $x \geq 2$

$\Rightarrow x+2 \leq 0$ or $x-2 \geq 0$

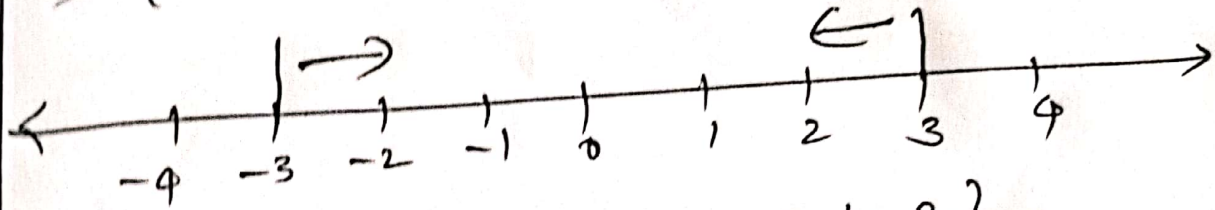
$\Rightarrow x^2 - 4 \geq 0$



Ans. B

11. $x^2 - 9 < 0$

$\Rightarrow (x+3)(x-3) < 0$



Integer value = $\{-2, -1, 0, 1, 2\}$

No. of integer value = 5

Ans: 5

12. $(x+2)(x-3) = 0$

$\Rightarrow x^2 - x - 6 = 0$

min. value = $\frac{4(1)(-6) - (-1)^2}{4 \cdot 1}$

$\therefore x = \frac{-25}{4}$

Ans: 0

$\Rightarrow 4x + 25 = 0$

13 a) $y = -x^2 + 6x - 5$

max. value = $\frac{4(-1)(-5) - 6^2}{4(-1)} = 4 (P)$

b) $y = x^2 - 2x + 3$

min value = $\frac{4 \cdot 1 \cdot 3 - (-2)^2}{4 \cdot 1} = 2 (Q)$

c) Vertex $\left(\frac{-b}{2a}, \frac{4ac - b^2}{4a} \right) = (-4, -4)$

d) Max. value = 2

Ans: P, Q, R, S

$$14 \quad a) -3 < x < 4$$

$$\Rightarrow (x+3)(x-4) < 0$$

$$\Rightarrow x^2 - \underline{x-12} < 0 \quad (\text{?})$$

$$b) x < -3, x > 4$$

$$\Rightarrow (x+3)(x-4) > 0$$

$$\Rightarrow x^2 - x - 12 > 0$$

$$c) -4 < x < -3$$

$$\Rightarrow (x+4)(x+3) < 0$$

$$\Rightarrow x^2 + 7x + 12 < 0$$

$$d) x < -4 \text{ or } x > -3$$

$$\Rightarrow (x+4)(x+3) > 0$$

$$\Rightarrow x^2 + 7x + 12 > 0$$

Ans: P, Q, R, S

$\Rightarrow$ THE END $\Leftarrow$