WS-2 T task

① Given For first vector let it be $\vec{A}$, It's components are $A_x = 1$; $A_y = \sqrt{3}$

∴ The angle made by the vector with x-axis is $\tan\theta_A = \frac{A_y}{A_x} \Rightarrow \tan\theta_A = \frac{\sqrt{3}}{1}$

$$\Rightarrow \theta_A = 60^\circ$$

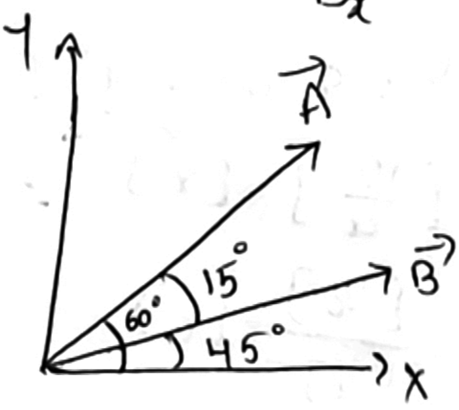
let the second vector is $\vec{B}$, It's components are

$B_x = 2$; $B_y = 2$

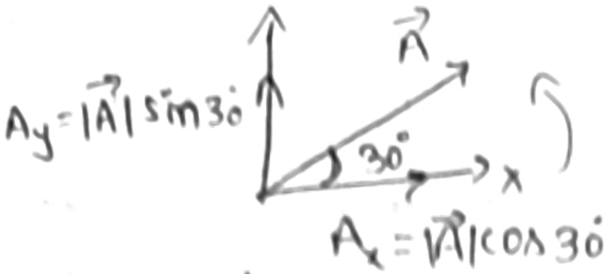
∴ The angle made by the vector with x-axis

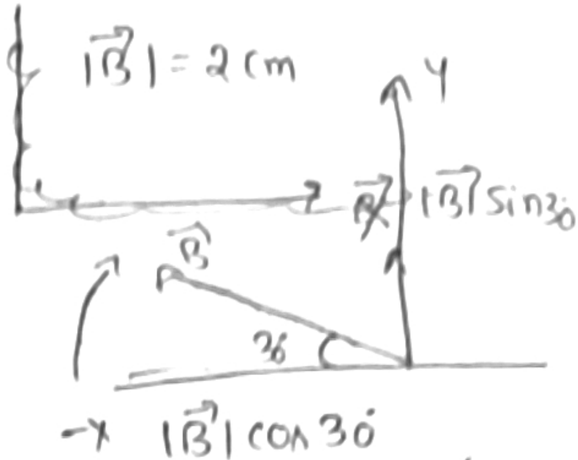
is $\tan\theta_B = \frac{B_y}{B_x} \Rightarrow \tan\theta_B = \frac{2}{2} \Rightarrow \tan\theta_B = 1$

$$\Rightarrow \theta_B = 45^\circ$$



(2) Given length of the vector $\vec{A}$ is 2 cm





$\therefore \vec{A} = A_x \hat{i} + A_y \hat{j}$

$= |\vec{A}| \cos 30° \hat{i} + |\vec{A}| \sin 30° \hat{j}$

$= 2 \times \frac{\sqrt{3}}{2} \hat{i} + 2 \times \frac{1}{2} \hat{j}$

$\vec{A} = \sqrt{3} \hat{i} + \hat{j}$

$\therefore \vec{B} = B_x \hat{i} + B_y \hat{j}$

$= -|\vec{B}| \cos 30 \hat{i} + |\vec{B}| \sin 30 \hat{j}$

$\vec{B} = -2 \times \frac{\sqrt{3}}{2} \hat{i} + 2 \frac{1}{2} \hat{j}$

$\vec{B} = -\sqrt{3} \hat{i} + \hat{j}$

$\therefore \vec{A} + \vec{B} = \sqrt{3} \hat{i} + \hat{j} - \sqrt{3} \hat{i} + \hat{j}$

$= 2\hat{j}$

(3) let A_x be the x-component of a vector = 4 m

A_y be the y-component of a vector = 6 m

x-component of $\vec{A} + \vec{B}$ is $(\vec{A} + \vec{B})_x = 10$ m

y-component of $\vec{A} + \vec{B}$ is $(\vec{A} + \vec{B})_y = 9$ m.

$\therefore \vec{A} + \vec{B} = A_x \hat{i} + A_y \hat{j} + B_x \hat{i} + B_y \hat{j}$

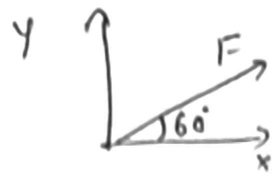
$\Rightarrow (\vec{A} + \vec{B})_x \hat{i} + (\vec{A} + \vec{B})_y \hat{j} = A_x \hat{i} + A_y \hat{j} + B_x \hat{i} + B_y \hat{j}$

$\Rightarrow 10 \hat{i} + 9 \hat{j} = 4 \hat{i} + 6 \hat{j} + B_x \hat{i} + B_y \hat{j}$

$\Rightarrow 6 \hat{i} + 3 \hat{j} = B_x \hat{i} + B_y \hat{j}$

$\therefore B_x = 6$ m ; $B_y = 3$ m.

(16) (4) Given one of the component of a force $F_x = 25\,N$.



$F_x = F\cos\theta = 25\,N$

The angle made by the force with x-axis is $\tan\theta = \frac{F_y}{F_x}$

$\Rightarrow \tan 60° = \frac{F_y}{F_x}$

$\Rightarrow \sqrt{3} = \frac{F_y}{25} \Rightarrow F_y = 25\sqrt{3}\,N$

(5) Given let

x-componet of force $F_x = 5\,N$

y-component of force $F_y = -3\,N$.

z-component of force $F_z = -2\,N$.

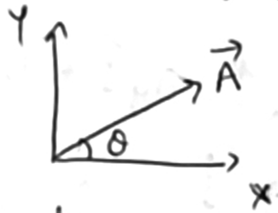
∴ The Force magnitude $= \sqrt{F_x^2 + F_y^2 + F_z^2}$

$= \sqrt{5^2 + (-3)^2 + (-2)^2}$

$= \sqrt{25+9+4} = \sqrt{38}$

(6) If any vector is along horizontal

(i.e)



it's vertical component $A_y = A\sin\theta$

If vector is along x-axis then $\theta = 0°$ $A_y = A\sin 0 = 0$.

that's minimum

(7) If the component of one vector is along the direction of the other vector is zero. then the angle between the vectors $\theta = 90°$.

(10) Given $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = \hat{i} + \hat{j}$

The component of $\vec{A}$ perpendicular to $\vec{B}$

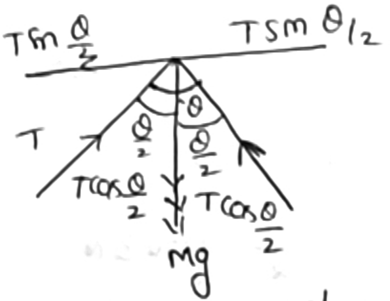
is $= \frac{\vec{A}\cdot\vec{B}}{|\vec{B}|} \cdot \frac{\vec{B}}{|\vec{B}|}$

orce

$$= \frac{(2\hat{i}+3\hat{j})\cdot(\hat{i}+\hat{j})}{\sqrt{1^2+1^2}} \times \frac{(\hat{i}+\hat{j})}{\sqrt{1^2+1^2}}$$

$$= \frac{2+3}{\sqrt{2}} \frac{(\hat{i}+\hat{j})}{\sqrt{2}} = \frac{5}{2}(\hat{i}+\hat{j})$$

(8) From given question



$$Mg = 2T\cos\frac{\theta}{2}$$

$$\Rightarrow T = \frac{Mg}{2\cos\frac{\theta}{2}}$$

~~If θ = 180°~~ the $T = \dfrac{Mg}{2\cos\frac{120}{2}}$

For given options if $\theta = 120^\circ \Rightarrow T = \dfrac{Mg}{2}$ = maximum

(14) (a) let the given vector be $\vec{A} = \hat{i}+\hat{j}$; ie $A_x = 1$, $A_y = 1$

$\therefore$ Angle made by the vector with x-axis is

$$\tan\theta = \frac{A_y}{A_x} \Rightarrow \tan\theta = \frac{1}{1}$$

$$\Rightarrow \theta = 45^\circ$$

(b) In the given vector $3\hat{i}+4\hat{j}$

$\hat{j}$ coefficient is called vertical component = 4.

(c) If $\theta = 60^\circ$

x-component $A_x = A\cos 60^\circ = \dfrac{A}{2} = 0.5A$; y-component $A_y = A\sin 60 = \dfrac{\sqrt{3}}{2}A = 0.866A$

clearly y-component > x-component.

(15) Given $\vec{A} = 5\hat{i}-12\hat{j}+13\hat{k}$ & $\vec{B} = -2\hat{i}+7\hat{j}-\hat{k}$

(i) The unit vector ||el to $\vec{A} = \dfrac{\vec{A}}{|\vec{A}|} = \dfrac{5\hat{i}-12\hat{j}+13\hat{k}}{\sqrt{5^2+(-12)^2+(13)^2}}$

$$= \frac{5\hat{i}-12\hat{j}+13\hat{k}}{\sqrt{169+169}} = \frac{5\hat{i}-12\hat{j}+13\hat{k}}{\sqrt{338}}$$

$$= \frac{5\hat{i}-12\hat{j}+13\hat{k}}{2\sqrt{13}}$$

(ii) The unit vector Parallel to $\vec{B}$ = $\frac{\vec{B}}{|\vec{B}|}$

$$\vec{B} = \frac{-2\hat{i}+7\hat{j}-\hat{k}}{\sqrt{(-2)^2+7^2+(-1)^2}}$$

$$\vec{B} = \frac{-2\hat{i}+7\hat{j}-\hat{k}}{\sqrt{54}}$$

(16) let the vector be $|\vec{A}| = 15N$.

Angle $\theta = 30^\circ$ then it's components are

$$A_x = |\vec{A}|\cos\theta = 15\cos 30^\circ$$

$$\Rightarrow A_x = 15\frac{\sqrt{3}}{2} = 7.5\sqrt{3}\,N$$

$$A_y = |\vec{A}|\,\cancel{\cos 90^\circ}\,\sin 30 = 15 \times \frac{1}{2} = 7.5N$$

(17) let the vector be $\vec{A} = \hat{i} + \sqrt{3}\,\hat{j}$

Angle made by the vector with x-axis $\tan\theta = \frac{A_y}{A_x}$

$$\tan\theta = \frac{\sqrt{3}}{1}$$

$$\tan\theta = \sqrt{3} \Rightarrow \theta = 60^\circ$$

Angle made by the vector y-axis is $90 - \theta = 90 - 60 = 30^\circ$

(18) Given vector is $3\hat{i} + 4\hat{j}$

x-component = 3 ; y-component = 4

Angle made by the vector with $\tan\theta = \frac{y\,comp}{x\,comp}$

$$\tan\theta = \frac{4}{3} \Rightarrow \theta = 53^\circ,$$

with y-axis angle $= 90 - \theta = 90 - 53 = 37^\circ$

6 Task

SAQ's

(1) Here the two components are rectangular i.e. Angle between them is $\theta = 90$

$\therefore$ The resultant is vector is $\sqrt{A_y^2 + A_x^2}$

For (b) if $A_x = 8 ; A_y = 15$ Then $A = \sqrt{15^2 + 8^2}$

$= \sqrt{225 + 64} = \sqrt{289}$

$A = 17N$

(2) let us take the two forces are F_1 & F_2

$\therefore \frac{F_1}{F_2} = \frac{7}{3}$ Maximum force $= F_1 + F_2$

Minimum force $= F_1 - F_2$

$\therefore$ Given $\frac{F_1 + F_2}{F_1 - F_2} = \frac{7}{3} \Rightarrow 3F_1 + 3F_2 = 7F_1 - 7F_2$

$\Rightarrow 7F_2 + 3F_2 = 7F_1 - 3F_1$

$\Rightarrow 10F_2 = 4F_1$

$\Rightarrow 5F_2 = 2F_1$

$\therefore \frac{F_1}{F_2} = \frac{5}{2}$

(3) let the component be $P_x = \frac{3P}{5}$

we know that $P = \sqrt{P_x^2 + P_y^2} \Rightarrow P^2 = P_x^2 + P_y^2$

$\Rightarrow P^2 = \left[\frac{3P}{5}\right]^2 + P_y^2$

$\Rightarrow P^2 = \frac{9P^2}{25} + P_y^2$

$\Rightarrow P_y^2 = P^2 - \frac{9P^2}{25}$

$\Rightarrow P_y^2 = \frac{25P^2 - 9P^2}{25}$

$\Rightarrow P_y^2 = \frac{16}{25} P^2$

$\Rightarrow P_y = \frac{4}{5} P$

(4) let the force be $F = 100\,N$.

It makes an angle $30°$ with x-axis is

$$F_x = F\cos\theta \Rightarrow 100\cos 30$$
$$\Rightarrow 100\,\frac{\sqrt{3}}{2} = 50\sqrt{3}\,N.$$

$$F_y = F\sin\theta \Rightarrow 100\sin 30°$$
$$\Rightarrow 100 \times \frac{1}{2} = 50\,N$$

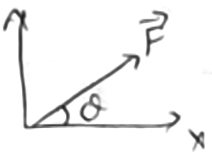
(5) Given that x-component of vector $= \sqrt{3}$ y-component

$$\Rightarrow \cancel{A}\cos\theta = \sqrt{3}\,\cancel{A}\sin\theta$$
$$\Rightarrow \cos\theta = \sqrt{3}\sin\theta$$
$$\Rightarrow \frac{\cos\theta}{\sin\theta} = \sqrt{3}$$
$$\Rightarrow \cot\theta = \sqrt{3} \Rightarrow$$
$$\Rightarrow \theta = 30°$$

(6) let F_x - horizontal components $= x$
F_y = vertical components $= y$

$$\Rightarrow F = \sqrt{F_x^2 + F_y^2} \Rightarrow F^2 = F_x^2 + F_y^2$$
$$\Rightarrow F^2 = x^2 + y^2$$
$\therefore$ y-component $y^2 = F^2 - x^2$
$$\Rightarrow y = \sqrt{F^2 - x^2}$$

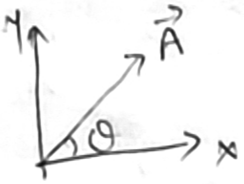
(7) let the vector be $\vec{F}$



Then the vertical component

is $y = F\sin\theta \Rightarrow F = \frac{y}{\sin\theta} \Rightarrow \boxed{F = y\,\mathrm{cosec}\,\theta}$

(8)



let the vector be : $\vec{A}$

Horizontal component $= H$
$$\Rightarrow A\cos\theta = H$$
$$\Rightarrow A = \frac{H}{\cos\theta}$$

Vertical component $A_y = A\sin\theta$

$$A_y = \cancel{A}\,\frac{H}{\cos\theta}\sin\theta = H\tan\theta$$

(9) Given that horizontal and vertical components are equal, let $\vec{A}$ be the vector

Acc to the given condition

$$A_x = A_y$$

$$\Rightarrow A\cos\theta = A\sin\theta$$

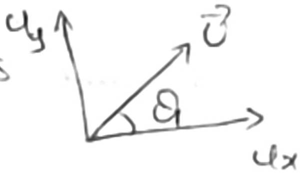
$$\Rightarrow \cos\theta = \sin\theta \Rightarrow \theta = 45^\circ$$

(10). let The vector for I^{st} ball is $\vec{U} = U_x\hat{i} + U_y\hat{j}$

$$\Rightarrow \vec{U} = 1\hat{i} + \sqrt{3}\hat{j}\ ms^{-1}$$

the other vector for the ball is $\vec{V} = V_x\hat{i} + V_y\hat{j}$

$$\vec{V} = 2\hat{i} + 2\hat{j}\ ms^{-1}$$



Angle made by $\vec{U}$ with X-axis

$$\tan\theta_1 = \frac{\vec{U_y}}{\vec{U_x}} \Rightarrow \tan\theta_1 = \frac{\sqrt{3}}{1}$$

$$\Rightarrow \tan\theta_1 = \sqrt{3} \Rightarrow \theta_1 = 60^\circ$$

Angle made by $\vec{V}$ with X-axis

$$\tan\theta_2 = \frac{\vec{V_y}}{\vec{V_x}} \Rightarrow \tan\theta_2 = \frac{2}{2}$$

$$\Rightarrow \tan\theta_2 = 1 \Rightarrow \theta_2 = 45^\circ$$

$\therefore$ the angle between two vectors is $\theta_1 - \theta_2 = 60 - 45$
$= 15^\circ$

(14) (a) If $r_x = 5$; $r_y = -7$ then $\vec{r} = r_x\hat{i} + r_y\hat{j}$

$$\therefore \vec{r} = 5\hat{i} - 5\hat{j}$$

(c) unit vector of $2\hat{i} + \hat{j}$ is $\frac{2\hat{i} + \hat{j}}{\sqrt{2^2 + 1^2}} = \frac{2\hat{i} + \hat{j}}{\sqrt{5}}$

(15) let $\vec{A} = \hat{i} + 3\hat{j} + 2\hat{k}$; $\vec{B} = 2\hat{i} - \hat{j} + \hat{k}$. and
$\vec{C} = -\hat{i} + 2\hat{j} + 3\hat{k}$

(i) The resultant of $\vec{A}$ and $\vec{B}$ is $\Rightarrow \vec{A} + \vec{B}$

$$= \hat{i} + 3\hat{j} + 2\hat{k} + 2\hat{i} - \hat{j} + \hat{k}$$

$$\Rightarrow 3\hat{i} + 2\hat{j} + 3\hat{k}$$

$\therefore$ unit vector Parallel to $\vec{A} + \vec{B}$ $= \dfrac{\vec{A} + \vec{B}}{|\vec{A} + \vec{B}|}$

$$= \frac{3\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{3^2 + 2^2 + 3^2}}$$

$$= \frac{3\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{22}}$$

(ii) unit vector Parallel to $\vec{C}$ is

$$= \frac{\vec{C}}{|\vec{C}|} = \frac{-\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{(-1)^2 + 2^2 + 3^2}}$$

$$= \frac{-\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{14}}$$

(16) Given Horizontal component is $x = 12$ units
vertical component is $y = 5$ units

$\therefore$ The resultant is $= \sqrt{x^2 + y^2} = \sqrt{12^2 + 5^2}$

$$= \sqrt{144 + 25} = \sqrt{169} = 13 \text{ units}$$

(17) let the vector be $\sqrt{2}\,\hat{i} + \sqrt{2}\,\hat{j}$

The angle made by the ~~vetor~~ vector with x-axis is

$$\tan\theta = \frac{y}{x} = \frac{\sqrt{2}}{\sqrt{2}}$$

$$\Rightarrow \tan\theta = 1 \Rightarrow \theta = 45^\circ$$

$\therefore$ Angle made by the vector with y-axis is $90 - \theta$

$$\Rightarrow 90 - 45^\circ = 45^\circ$$