LIMITS OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS ①

F^+

Class: IX. Mathematics

SOLUTIONS

01. $\underset{x\to 0}{\text{Lt}} \log\left|\dfrac{\log(1+x)}{x}\right|$

$= \underset{x\to 0}{\text{Lt}} \log\left|\dfrac{x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots}{x}\right|$

$= \underset{x\to 0}{\text{Lt}} \log\left|1-\dfrac{x}{2}+\dfrac{x^2}{3}-\dfrac{x^3}{4}+\cdots\right|$

$= \log|1-0+0-0| = \log 1 = 0$ Ans: A

02. $\underset{x\to 0}{\text{Lt}} \dfrac{e^x+\sin x-1}{\log(1+x)}$ $\left(\frac{0}{0}\right)$

$= \underset{x\to 0}{\text{Lt}} \dfrac{e^x+\cos x}{\left(\frac{1}{1+x}\right)} = \dfrac{1+1}{1} = 2$ Ans: D

03. $\underset{x\to 0}{\text{Lt}} \dfrac{10^x-2^x-5^x+1}{x\tan x}$ $\left(\frac{0}{0}\right)$

$= \cancel{\underset{x\to 0}{\text{Lt}} \dfrac{10^x\cdot\log 10-2^x\cdot\log 2-5^x\cdot\log 5}{x\cdot\sec^2 x+\tan x\cdot 1}}$

$= \underset{x\to 0}{\text{Lt}} \dfrac{2^x\cdot 5^x-2^x-5^x+1}{x\tan x}$

$= \underset{x\to 0}{\text{Lt}} \dfrac{(2^x-1)(5^x-1)}{x\tan x}$

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$$= \lim_{x\to 0} \frac{\left(\frac{2^x-1}{x}\right)\cdot\left(\frac{5^x-1}{x}\right)}{\left(\frac{\tan x}{x}\right)} = \log 2 \cdot \log 5 \quad \text{Ans C}$$

04. $$\lim_{x\to\infty} \frac{x - \log x}{x + \log x}$$

$$= \lim_{x\to\infty} \frac{1-\frac{1}{x}}{1+\frac{1}{x}} = \frac{1-0}{1+0} = 1 \quad \text{Ans: A}$$

05 $$\lim_{x\to\infty} \frac{a^x - a^{-x}}{a^x + a^{-x}}$$

$$= \lim_{x\to\infty} \frac{\cancel{a^x}(1-a^{-2x})}{\cancel{a^x}(1+a^{-2x})}$$

$$= \lim_{x\to\infty} \frac{1-\frac{1}{a^{2x}}}{1+\frac{1}{a^{2x}}} = \frac{1-0}{1+0} = 1$$

Ans: A

06 $$\lim_{x\to\infty} \frac{(x+1)^{10} + (x+2)^{10} + \cdots + (x+100)^{10}}{x^{10} + 10^{10}}$$

$$= \lim_{x\to\infty} \frac{\cancel{x^{10}}\left[\left(1+\frac{1}{x}\right)^{10} + \left(1+\frac{2}{x}\right)^{10} + \cdots + \left(1+\frac{100}{x}\right)^{10}\right]}{\cancel{x^{10}}\left(1+\frac{10^{10}}{x^{10}}\right)}$$

$$= (1+0) + (1+0) + (1+0) \cdots + (1+0)$$ (100 times)

$$= 1 \times 100 = 100$$

Ans : B

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07. $$\operatorname*{Lt}_{n\to\infty} \frac{3\cdot 2^{n+1} - 4\cdot 5^{n+1}}{5\cdot 2^{n} + 7\cdot 5^{n}}$$

$$= \operatorname*{Lt}_{n\to\infty} \frac{3\cdot 2^{n}\cdot 2^{1} - 4\cdot 5^{n}\cdot 5^{1}}{5\cdot 2^{n} + 7\cdot 5^{n}}$$

$$= \operatorname*{Lt}_{n\to\infty} \frac{6\cdot 2^{n} - 20\cdot 5^{n}}{5\cdot 2^{n} + 7\cdot 5^{n}} = \operatorname*{Lt}_{n\to\infty} \frac{\cancel{5^{n}}\left[6\cdot\left(\frac{2}{5}\right)^{n} - 20\right]}{\cancel{5^{n}}\left[5\left(\frac{2}{5}\right)^{n} + 7\right]}$$

$$= \boxed{\cancel{\operatorname*{Lt}_{n\to\infty} 5^{n}\,(6 - 20\,(}} \qquad = \frac{6\cdot 0 - 20}{5\cdot 0 + 7}$$

$$= -\frac{20}{7}$$ Ans.. A

08 $$\operatorname*{Lt}_{x\to\infty} x\left[\log(1+x) - \log x\right]$$

$$= \operatorname*{Lt}_{x\to\infty} x\left[\log\left(\frac{1+x}{x}\right)\right]$$

$$= \operatorname*{Lt}_{x\to\infty} \frac{\log\left(1+\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}$$

Let $\frac{1}{x} = y \Rightarrow x\to\infty,\ y\to 0$

$$\therefore \operatorname*{Lt}_{y\to 0} \frac{\log(1+y)}{y} = 1.$$

Ans: C

09. $$\operatorname*{Lt}_{x\to\infty} x\left(a^{1/x} - b^{1/x}\right)$$

$$= \operatorname*{Lt}_{x\to\infty} \frac{a^{1/x} - b^{1/x}}{\left(\frac{1}{x}\right)}$$

Let $\frac{1}{x} = y$

$$\therefore \operatorname*{Lt}_{y\to 0} \frac{a^{y} - b^{y}}{y} = \log_e\left(\frac{a}{b}\right)$$

Ans: B

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10. $\lim_{x \to 1} \left(\log_3 3x\right)^{\log_x 3} \quad (1^{\infty})$

NOTE: $\lim_{x \to a} (f(x))^{g(x)} = 1^{\infty}$

$= e^{\lim_{x \to a} g(x)[f(x)-1]}$

$$= e^{\lim_{x \to 1} \log_x 3 \left(\log_3 3x - 1\right)}$$

$$= e^{\lim_{x \to 1} \log_x 3 \left(\cancel{\log_3 3} + \log_3 x - \cancel{1}\right)}$$

$$= e^{\lim_{x \to 1} \log_x 3 \left(\log_3 x\right)}$$

$$= e^{\lim_{x \to 1} 1} = e$$

Ans: D

JEE ADVANCED LEVEL

01. $\lim_{x \to 0} \dfrac{\log(1+x)}{3^x - 1}$

$$= \lim_{x \to 0} \frac{\left(\frac{1}{1+x}\right)}{3^x \cdot \log 3} = \frac{1}{\log_e 3} = \log_3 e.$$

Ans: C, D

02. $\lim_{x \to 0} \dfrac{e^{\alpha x} - e^{\beta x}}{\sin \alpha x - \sin \beta x} \quad \left(\frac{0}{0}\right)$

$$= \lim_{x \to 0} \frac{e^{\alpha x} \cdot \alpha - e^{\beta x} \cdot \beta}{\cos \alpha x \cdot \alpha - \sin \beta x \cdot \beta} = \frac{\alpha - \beta}{\alpha - \beta} = 1$$

opt: A $\Rightarrow$ 1 ✓

opt: B $\Rightarrow \lim_{x \to 0} \frac{\sin x}{x} = 1$

opt C $\Rightarrow \lim_{x \to 0} \cos x = 1$

Ans: A, B, C

03. **Statement I:** $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x\cdot\log(1+x)}$ $\left(\frac{0}{0}\right)$

$$= \cancel{\lim_{x\to 0} \frac{\sin x}{\left(\frac{1}{1+x}\right)}}$$

$$= \lim_{x\to 0} \frac{2\sin^2\left(\frac{x}{2}\right)}{x\cdot\log(1+x)}$$

$$= 2\cdot \lim_{x\to 0} \frac{\sin^2\left(\frac{x}{2}\right)}{x^2\cdot \frac{\log(1+x)}{x}}$$

$$= 2\cdot\left(\frac{1}{2}\right)^2 = \frac{1}{2} \quad (\text{True})$$

Statement II: Conceptual (True) Ans: A

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04. **Statement I:** $\lim\limits_{x\to 0} \dfrac{e^{2x}-1}{3x} = \dfrac{1}{3}\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{x}$

$$= \frac{2}{3}$$

Statement II: Conceptual (True) Ans: B

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05 p) $\lim\limits_{x\to 0} \dfrac{e^x-1}{x} = 1$

q) $\lim\limits_{x\to 0} \dfrac{a^x-1}{b^x-1} = \log_b a$

r) $\lim\limits_{x\to 0} (1+x)^{\frac{1}{x}} = e$

s) $\lim\limits_{x\to 0} \dfrac{a^x-1}{x} = \log_e a$

Ans: opt: A

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06. p) $\lim_{x\to 0} \frac{e^{2x}-1}{x} = 2$

q) $\lim_{x\to 0} (1+x)^{\frac{2}{x}} = e^2$

r) $\lim_{x\to 0} \frac{a^{2x}-1}{x} = \log_e a^2$

s) $\lim_{x\to 0} e^x = e^0 = 1$

Ans: D

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07. $\lim_{x\to 0} \frac{x\cdot 2^{3x}-x}{1-\cos(3x)} = \frac{2a}{m^2}\log a$

$= \frac{2.3}{3^2} \cdot \log 2 = \frac{2}{3}\log 2$

Ans: A

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08 $\lim_{x\to 0} \frac{x\cdot 2^{4x}-x}{1-\cos 4x} = \frac{2.4}{4^2}\cdot \log 2 = \frac{1}{2}\log 2$

$= \log\sqrt{2}$

Ans: D

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09. $\lim_{x\to 0} \frac{e^{\sin x}-1}{\sin x}$ here $\lim_{x\to 0} \sin x = 0$

$= \log_e(0) \Rightarrow$ not defined

Ans: D

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10. $\lim_{x\to \frac{\pi}{2}} \frac{e^{\cos x}-1}{\cos x} = \log_e\left(\frac{\pi}{2}\right)$

Ans: B

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LEARNER'S TASK

CUQ'S

01. Conceptual — Ans: A

02. $\lim_{x \to a} \frac{\log[1+(x-a)]}{(x-a)}$

Let $x-a = y \Rightarrow x \to a \Rightarrow y$, $y \to 0$

$\therefore \lim_{y \to 0} \frac{\log(1+y)}{y} = 1$ — Ans: B

03. $\lim_{h \to 0} \frac{\log_{10}(1+h)}{h}$

$= \lim_{h \to 0} \frac{\log_e(1+h)}{h} \times \log_{10} e = 1 \times \log_{10} e$ — Ans: D

04. Conceptual — Ans: A

05. $\lim_{x \to 0} \frac{a^x \cdot b^x - a^x - b^x + 1}{x^2}$

$= \lim_{x \to 0} \frac{(a^x - 1)(b^x - 1)}{x^2}$

$= \lim_{x \to 0} \frac{a^x - 1}{x} \cdot \lim_{x \to 0} \frac{b^x - 1}{x}$

$= \log_e a \cdot \log_e b$ — Ans: C

06 Conceptual Ans: A

07. Conceptual Ans: B

08 Conceptual Ans: D

09. $\underset{x\to\infty}{Lt}\left(1+\frac{2}{x}\right)^{x} = e^{2}$ Ans: D

10. $\underset{x\to 0}{Lt}\frac{\log(1+ax)-\log(1+bx)}{x}$ $\left(\frac{0}{0}\right)$

$$= \underset{x\to 0}{Lt}\frac{\frac{1}{1+ax}\cdot a - \frac{1}{1+bx}\cdot b}{1} = a-b$$

Ans: B

JEE MAINS LEVEL

01. $\underset{x\to 0}{Lt}\frac{4^{x}-9^{x}}{x(4^{x}+9^{x})}$

$$= \underset{x\to 0}{Lt}\frac{(4^{x}-1)-(9^{x}-1)}{x} \cdot \underset{x\to 0}{Lt}\frac{1}{4^{x}+9^{x}}$$

$$= \log\left(\frac{4}{9}\right)\times\frac{1}{2} \Rightarrow \log\sqrt{\frac{4}{9}} = \log\left(\frac{2}{3}\right)$$

log B

02 $\underset{x\to\infty}{Lt} 5^{x}\cdot\sin\left(\frac{a}{5^{x}}\right)$

$$= \underset{\cancel{5^{x}}\,x\to\infty}{Lt}\frac{\sin\left(\frac{a}{5^{x}}\right)}{\left(\frac{1}{5^{x}}\right)}$$

let $\frac{1}{5^{x}} = y$

$\underset{y\to 0}{Lt}\frac{\sin(ay)}{y} = a$

Ans: B

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03 $\displaystyle \lim_{n\to\infty} \frac{2^{\frac{1}{n}}-1}{2^{\frac{1}{n}}+1}$

Let $\frac{1}{n} = y \Rightarrow n\to\infty,\ y\to 0$

$\therefore \displaystyle \lim_{y\to 0} \frac{2^{y}-1}{2^{y}+1} = \frac{2^{0}-1}{2^{0}+1} = \frac{1-1}{1+1} = \frac{0}{2} = 0$

Ans.. D

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04. $\displaystyle \lim_{x\to 0} \frac{\log(1+x)-x}{x^{2}} \quad \left(\frac{0}{0}\right)$

$\displaystyle = \lim_{x\to 0} \frac{\frac{1}{1+x}-1}{2x} \quad \left(\frac{0}{0}\right)$

$\displaystyle = \lim_{x\to 0} \frac{\frac{-1}{(1+x)^{2}}}{2} = \frac{-1}{2}$

Ans: B

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05 $\displaystyle \lim_{x\to a} \frac{\log(x-a)}{\log(e^{x}-e^{a})}$

$\displaystyle = \lim_{x\to a} \frac{\left(\frac{1}{x-a}\right)}{\frac{1}{(e^{x}-e^{a})}\cdot(e^{x}-0)}$

$\displaystyle = \lim_{x\to a} \frac{e^{x}-e^{a}}{e^{x}(x-a)}$

$\displaystyle = \lim_{x\to a} \frac{e^{x}}{e^{x}(1)+(x-a)e^{x}} = \frac{e^{a}}{e^{a}+0} = 1$

Ans: A

6. $$\underset{x\to\infty}{Lt} \frac{a^{\sqrt{x}} - a^{\frac{1}{\sqrt{x}}}}{a^{\sqrt{x}} + a^{\frac{1}{\sqrt{x}}}}$$

$$= \underset{x\to\infty}{Lt} \frac{\cancel{a^{\sqrt{x}}}\left(1 - a^{\frac{1}{\sqrt{x}} - \sqrt{x}}\right)}{\cancel{a^{\sqrt{x}}}\left(1 + a^{\frac{1}{\sqrt{x}} - \sqrt{x}}\right)}$$

$$= \underset{x\to\infty}{Lt} \frac{1 - a^{-\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)}}{1 + a^{-\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)}}$$

$\because$ As $x \to \infty$, $\sqrt{x} - \frac{1}{\sqrt{x}} \to \infty$

$$= \frac{1-0}{1+0} = 1$$

Ans : A

07 $$\underset{x\to 0}{Lt} \frac{2^x - 1}{(1+x)^{1/2} - 1} \quad \left(\frac{0}{0}\right)$$

$$= \underset{x\to 0}{Lt} \frac{2^x . \log 2}{\frac{1}{2}(1+x)^{-1/2}} = 2\log 2 = \log 4$$

Ans: B

08 $$\underset{x\to 0}{Lt} \frac{10^x - 2^x - 5^x + 1}{x^2}$$

$$= \underset{x\to 0}{Lt} \frac{2^x . 5^x - 2^x - 5^x + 1}{x^2}$$

$$= \underset{x\to 0}{Lt} \frac{2^x(5^x - 1) - 1(5^x - 1)}{x^2}$$

$$= \underset{x\to 0}{Lt} \frac{(5^x - 1)}{x} \frac{(2^x - 1)}{x}$$

$$= (\log 5).(\log 2)$$

Ans: C

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09. $\lim\limits_{x\to 0} \dfrac{(729)^x - (243)^x - (81)^x + 9^x + 3^x - 1}{x^3}$

$= \lim\limits_{x\to 0} \dfrac{3^{6x} - 3^{5x} - 3^{4x} + 3^{2x} + 3^x - 1}{x^3}$

$= \lim\limits_{x\to 0} \dfrac{3^{5x}(3^x - 1) - 3^{2x}(3^{2x} - 1) + 1(3^x - 1)}{x^3}$

$= \lim\limits_{x\to 0} \dfrac{3^{5x}(3^x - 1) - 3^{2x}(3^x + 1)(3^x - 1) + 1(3^x - 1)}{x^3}$

$= \lim\limits_{x\to 0} \dfrac{(3^x - 1)\left[3^{5x} - 3^{2x}(3^x + 1) + 1\right]}{x^3}$

$= \lim\limits_{x\to 0} \dfrac{(3^x - 1)(3^{5x} - 3^{3x} - 3^{2x} + 1)}{x^3}$

$= \lim\limits_{x\to 0} \dfrac{(3^x - 1)(3^{3x}(3^{2x} - 1) - (3^{2x} - 1))}{x^3}$

$= \lim\limits_{x\to 0} \dfrac{(3^x - 1)}{x} \dfrac{(3^{2x} - 1)}{x} \dfrac{(3^{3x} - 1)}{x}$

$= \log 3 \cdot \log 3 \cdot 2 \cdot \log 3 \cdot 3$

$= 6(\log 3)^3$

Ans: B

10. $\lim_{x\to 0} \dfrac{a^{\tan x} - a^{\sin x}}{\tan x - \sin x}$

$= \lim_{x\to 0} \dfrac{a^{\sin x}\left(a^{\tan x - \sin x} - 1\right)}{(\tan x - \sin x)}$

$= a^{\sin 0} \cdot \log a$

$= \log a$

Ans. A

JEE ADVANCED LEVEL

01. $\lim_{x\to 0} \dfrac{e^{x^2} - \cos x}{x^2}$

$= \cancel{\lim_{x\to 0} e^{x^2}} = \lim_{x\to 0} \dfrac{(e^{x^2} - 1) - (\cos x - 1)}{x^2}$

$= \lim_{x\to 0} \dfrac{e^{x^2} - 1}{x^2} + \lim_{x\to 0} \dfrac{1 - \cos x}{x^2}$

$= 1 + \lim_{x\to 0} \dfrac{2\sin^2\left(\frac{x}{2}\right)}{x^2}$

$= 1 + 2\cdot\left(\frac{1}{2}\right)^2 = 1 + \frac{1}{2} = \frac{3}{2}$

Ans. D

02. $\lim_{x\to\infty} 2^{-x} \cdot \sin(2^x)$

$= \lim_{x\to\infty} \dfrac{\sin(2^x)}{2^x}$

$-1 \le \sin(2^x) \le 1$

$\dfrac{-1}{2^x} \le \dfrac{\sin 2^x}{2^x} \le \dfrac{1}{2^x}$

$\lim_{x\to\infty} \dfrac{\mp 1}{2^x} = \lim_{x\to\infty} \dfrac{1}{2^x} = 0$

$\therefore$ Ans: A, B

03. **Statement I:** Conceptual (true)

Statement II: Conceptual (true)

04 **Statement I:** $\lim_{x\to 1} \dfrac{x^3 - x^2\log x + \log x - 1}{x^2-1}$

$= \lim_{x\to 1} \dfrac{(x^3-1) - \log x(x^2-1)}{(x+1)(x-1)}$

$= \lim_{x\to 1} \dfrac{(x-1)(x^2+x+1) - \log x(x+1)(x-1)}{(x+1)(x-1)}$

$= \lim_{x\to 1} \dfrac{(x^2+x+1) - \log x(x+1)}{x+1}$

$= \dfrac{3}{2}$ (True)

Statement II: $\lim_{x\to e} \log_e x = \log_e e = 1$ (False)

Ans: C

5 p) $\lim_{x\to 0} (1+x)^{\frac{1}{x}} = e$

q) $\lim_{x\to 0} \left(\dfrac{x+4}{2-x}\right)^{\frac{x^2+2x-3}{x-1}} = \left(\dfrac{4}{2}\right)^3 = 2^3 = 8$

r) $\lim_{x\to \infty} \dfrac{\log_e x}{x}$

$= \lim_{x\to\infty} \dfrac{\left(\frac{1}{x}\right)}{1} = 0$

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s) $$\lim_{x\to 0} \frac{\sin x + \log(1-x)}{x^2}$$

$$= \lim_{x\to 0} \frac{\cos x + \frac{1}{1-x}(-1)}{2x}$$

$$= \lim_{x\to 0} \frac{-\sin x - \frac{1}{(1-x)^2}}{2}$$

$$= -\frac{1}{2}$$

Ans: A, F, C, D

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p) $$\lim_{x\to 0} \frac{e^x - 1 - x}{x^2}$$

$$= \lim_{x\to 0} \frac{e^x - 1}{2x} = \frac{1}{2}$$

q) $$\lim_{x\to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0} \frac{e^x + e^{-x} - 2}{1 - \cos x}$$

$$= \lim_{x\to 0} \frac{e^x - e^{-x}}{\sin x}$$

$$= \lim_{x\to 0} \frac{e^x + e^{-x}}{\cos x} = \frac{1+1}{1} = 2$$

r) $$\lim_{x\to \infty} 2^{x-1} \cdot \tan\left(\frac{a}{2^x}\right)$$

$$= \lim_{x\to \infty} \frac{2^x}{2} \cdot \tan\left(\frac{a}{2^x}\right)$$

$$= \frac{1}{2} \lim_{x\to \infty} \frac{\tan\left(\frac{a}{2^x}\right)}{\left(\frac{1}{2^x}\right)}$$

put $\frac{1}{2^x} = y$

$\therefore x \to \infty, \quad y \to 0$

$= \frac{1}{2} \lim_{y \to 0} \frac{\tan ay}{y}$

$= \frac{a}{2}$

5) $\lim_{x \to 0} e^x = e^0 = 1$ Ans : F, E, D, C

07 $\lim_{x \to 0} \frac{e^{3x}-1}{x} = 3$ Ans : C

08 $\lim_{x \to 0} \frac{e^{\frac{x}{2}}-1}{x} \quad \left(\frac{0}{0}\right)$

$= \lim_{x \to 0} \frac{e^{\frac{x}{2}} \cdot \frac{1}{2}}{1} = \frac{1}{2}$ Ans. A

09. $\lim_{x \to 0} \frac{4^x - 2^x}{x} = \log\left(\frac{4}{2}\right) = \log_e 2$ Ans. B

10. $\lim_{x \to 0} \frac{2^x - 3^x}{x} \to \log\left(\frac{2}{3}\right)$ Ans. A

$\Rightarrow$ THE END