WS-3

Task

(1) Given vector $\vec{P} = \hat{i} - 2\hat{j} - 2\hat{k}$

$\therefore$ Given vector is in the form of $x\hat{i} + y\hat{j} + z\hat{k}$

whose magnitude is $\sqrt{x^2+y^2+z^2}$

$\therefore |\vec{P}| = \sqrt{(1)^2+(-2)^2+(-2)^2} = \sqrt{1+4+4} = \sqrt{9} = 3$

(2) Given force $\vec{F} = 6\hat{i} - 8\hat{j} + 10\hat{k}$; $a = 1\,m/s^2$

$\therefore |\vec{F}| = \sqrt{6^2+(-8)^2+(10)^2} = \sqrt{36+64+100}$

$= \sqrt{100+100} = \sqrt{200} = 10\sqrt{2}$ N

$\therefore$ Mass $M = \frac{|\vec{F}|}{a} = \frac{10\sqrt{2}}{1} = 10\sqrt{2}$ kg.

(3) Given $\vec{A} = 3\hat{i} + 4\hat{j}$; $\vec{B} = 7\hat{i} + 24\hat{j}$

let $\vec{C}$ be the vector whose magnitude is same as that of B and parallel to $\vec{A}$

$\therefore \vec{C} = |\vec{B}| \times \hat{A}$ $\quad |\vec{A}| = \sqrt{3^2+4^2}$

$\vec{C} = |\vec{B}| \frac{\vec{A}}{|\vec{A}|}$ $\quad = \sqrt{9+16} = \sqrt{25} = 5$

$|\vec{B}| = \sqrt{7^2+24^2}$

$= \sqrt{49+576}$

$\therefore \vec{C} = \frac{\cancel{25}^{5}(3\hat{i}+4\hat{j})}{\cancel{5}}$ $\quad = \sqrt{625} = 25$

$\vec{C} = 15\hat{i} + 20\hat{j}$

(4) Given $0.5\hat{i}+0.8\hat{j}+c\hat{k}$ is a unit vector

$$\Rightarrow \sqrt{(0.5)^2+(0.8)^2+c^2} = 1$$

By doing squaring on both sides

$$\Rightarrow \left(\sqrt{0.25+0.64+c^2}\right)^2 = 1^2$$

$$\Rightarrow 0.89+c^2 = 1$$

$$\Rightarrow c^2 = 1-0.89 \Rightarrow c = \sqrt{0.11}$$

(5) Given $\vec{P} = \hat{i}+3\hat{j}-7\hat{k}$; $\vec{Q} = 5\hat{i}-2\hat{j}+4\hat{k}$

Then length of the vector $\vec{PQ} = \vec{Q}-\vec{P}$

$$= 5\hat{i}-2\hat{j}+4\hat{k}-\hat{i}-3\hat{j}+7\hat{k}$$

$$= 4\hat{i}-5\hat{j}+11\hat{k}$$

$\therefore$ length of $\vec{PQ} = |\vec{PQ}| = \sqrt{4^2+(-5)^2+11^2}$

$$= \sqrt{16+25+121}$$

$$= \sqrt{162}.$$

(6) let the initial velocity $\vec{u} = 2\hat{i}-3\hat{j}+11\hat{k}$ m/s

$a = 10\hat{i}+10\hat{j}+10\hat{k}\ ms^{-2}$; $t = 0.25$ sec.

$\therefore$ The velocity $\vec{v} = \vec{u}+\vec{a}t$

$$\Rightarrow \vec{v} = 2\hat{i}-3\hat{j}+11\hat{k}+(10\hat{i}+10\hat{j}+10\hat{k})0.25$$

$$= 2\hat{i}-3\hat{j}+11\hat{k}+2.5\hat{i}+2.5\hat{j}+2.5\hat{k}$$

$$= 12.5\hat{i}-0.5\hat{j}+13.5\hat{k}$$

$$\therefore |\vec{v}| = \sqrt{(12.5)^2+(-0.5)^2+(13.5)^2} = \frac{1}{2}\sqrt{811}\ ms^{-1}$$

(7) Given $\vec{A} = 6\hat{i}+8\hat{j}+10\hat{k}$

$\therefore$ unit vector parallel to $\vec{A} = \dfrac{\vec{A}}{|\vec{A}|}$

$$\Rightarrow \hat{A} = \frac{6\hat{i}+8\hat{j}+10\hat{k}}{\sqrt{6^2+8^2+10^2}} = \frac{2(3\hat{i}+4\hat{j}+5\hat{k})}{\sqrt{36+64+100}}$$

$$= \frac{2(3\hat{i}+4\hat{j}+5\hat{k})}{\sqrt{200}} = \frac{\cancel{2}(3\hat{i}+4\hat{j}+5\hat{k})}{\cancel{10}_{5}\sqrt{2}}$$

$$= \frac{3\hat{i}+4\hat{j}+5\hat{k}}{5\sqrt{2}}$$

(8) Given forces $\vec{F}_1 = 3\hat{i}-4\hat{j}+2\hat{k}$; $\vec{F}_2 = 2\hat{i}+3\hat{j}-\hat{k}$

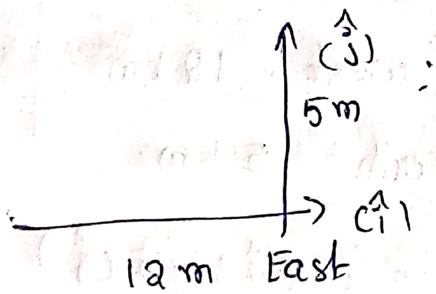
& $\vec{F}_3 = 2\hat{i}+4\hat{j}-5\hat{k}$

$\therefore$ The resultant force $= \vec{F}_1+\vec{F}_2+\vec{F}_3$

$$\Rightarrow \vec{F} = 3\hat{i}\cancel{-4\hat{j}}+2\hat{k}+2\hat{i}+3\hat{j}-\hat{k}+2\hat{i}\cancel{+4\hat{j}}-5\hat{k}$$

$$= 7\hat{i}+3\hat{j}-4\hat{k}$$

(9) Total distance travelled towards east = 12 m



$\therefore$ Displacement $= 12\hat{i}+5\hat{j}$

$$|\text{Displacement}| = \sqrt{12^2+5^2}$$

$$= \sqrt{144+25}$$

$$= \sqrt{169}$$

$$= 13\,m$$

Advanced

(4) If $\vec{a} = 4\hat{i} - 3\hat{j}$ & $\vec{b} = 8\hat{i} - 6\hat{j}$.

$\therefore \vec{a} + \vec{b} = 4\hat{i} - 3\hat{j} + 8\hat{i} - 6\hat{j}$

$= 12\hat{i} - 9\hat{j}$.

$|\vec{a} + \vec{b}| = \sqrt{12^2 + (-9)^2} = \sqrt{144+81} = \sqrt{225}$

$= 15$

(5) $\vec{a} - \vec{b} = 4\hat{i} - 3\hat{j} - 8\hat{i} + 6\hat{j}$

$= -4\hat{i} + 3\hat{j}$

[Given $\vec{a} = 4\hat{i} - 3\hat{j}$, $\vec{b} = 8\hat{i} - 6\hat{j}$]

$\Rightarrow |\vec{a} - \vec{b}| = \sqrt{(-4)^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$

(6) $\vec{b} - \vec{a} = 8\hat{i} - 6\hat{j} - 4\hat{i} + 3\hat{j}$

$= 4\hat{i} - 3\hat{j}$

[Given $\vec{a} = 4\hat{i} - 3\hat{j}$, $\vec{b} = 8\hat{i} - 6\hat{j}$]

$\Rightarrow |\vec{b} - \vec{a}| = \sqrt{4^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$

(8) Given Total distance travelled towards

west = 12 km

Towards north = 5 km.

$\therefore$ The displacement $= 12(-\hat{i}) + 5(\hat{j})$

$= -12\hat{i} + 5\hat{j}$

$\Rightarrow$ $|\text{displacement}| = \sqrt{(-12)^2 + 5^2} = \sqrt{144+25} = 13\text{ m}$

But given For entire journey displacement = 13 km

$\Rightarrow 6.5 z = 13$

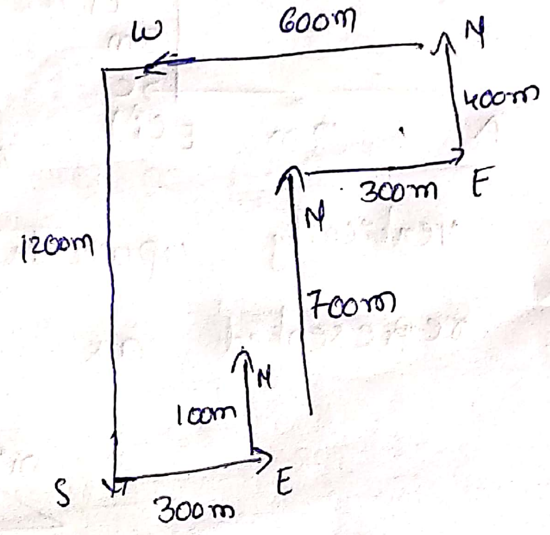
$\Rightarrow z = \frac{13}{6.5} = 2$.

(9) $\hat{i}, -\hat{i}$ are unit vectors along East & west

$\hat{j}, -\hat{j}$ are unit vectors along North & south

Acc to Given data

Displacement =



$= 700\hat{j} + 300\hat{i} + 400\hat{j} - 600\hat{i}$

$- 1200\hat{j} + 300\hat{i} + 100\hat{j}$

$\Rightarrow 1200\hat{j} + 600\hat{i} - 600\hat{i} - 1200\hat{j}$

$= 0$

L Task

SAQs

(11). Given vector $\vec{A} = 5\hat{i} + p\hat{j} + 4\sqrt{2}\,\hat{k}$

It's magnitude i.e) $|\vec{A}| = 11$

$\therefore \sqrt{5^2 + p^2 + (4\sqrt{2})^2} = 11$

By squaring on both sides we get

$\Rightarrow \left[\sqrt{25 + p^2 + 32}\right]^2 = 11^2$

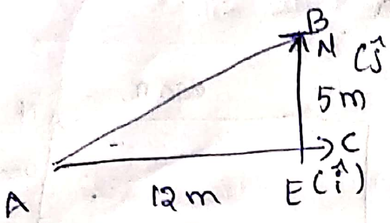
$\Rightarrow p^2 + 57 = 121$

$\Rightarrow p^2 = 121 - 57 = 64$

$\Rightarrow p = \sqrt{64}$

$\Rightarrow p = \pm 8$

(12) Given the Particle directions are like



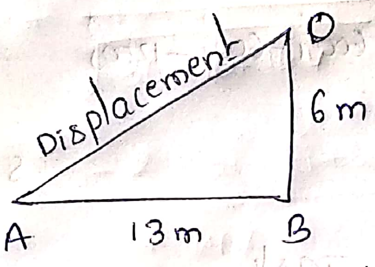
Here the displacement $= 12\hat{i} + 5\hat{j}$

$= \sqrt{12^2 + 5^2}$

$= \sqrt{144+25} = \sqrt{169}$

$= 13\,m$

And then the Particle moves vertically upwards [From AB]. It can be represented as



$\therefore$ The Resultant displacement for entire Journey $= AD = \sqrt{AB^2 + BD^2}$

$= \sqrt{13^2 + 6^2}$

$= \sqrt{169+36}$

$= \sqrt{205} = 14.32\,m$

(13) Given $\vec{A} = 3\hat{i} + 5\hat{j} - 2\hat{k}$ & $\vec{B} = -3\hat{j} + 6\hat{k}$

From the condition $2A + 7B + 4C = 0$

$\Rightarrow 2(3\hat{i} + 5\hat{j} - 2\hat{k}) + 7(-3\hat{j} + 6\hat{k}) + 4C = 0$

$\Rightarrow 6\hat{i} + 10\hat{j} - 4\hat{k} - 21\hat{j} + 42\hat{k} + 4C = 0$

$\Rightarrow 6\hat{i} - 11\hat{j} + 32\hat{k} + 4C = 0$

$\Rightarrow 4C = -6\hat{i} + 11\hat{j} - 38\hat{k}$

$\Rightarrow C = \frac{-6}{4}\hat{i} + \frac{11}{4}\hat{j} - \frac{38}{4}\hat{k}$

$\Rightarrow C = -1.5\hat{i} + 2.75\hat{j} - 9.5\hat{k}$

(14) Given $\vec{A} = 4\hat{i} + 3\hat{j} + 6\hat{k}$

$\vec{B} = -\hat{i} + 3\hat{j} - 8\hat{k}$

$\therefore$ The resultant of $\vec{A}$ & $\vec{B}$ is $\vec{A}+\vec{B}$

$$\Rightarrow 4\hat{i} + 3\hat{j} + 6\hat{k} - \hat{i} + 3\hat{j} - 8\hat{k}$$

$$= 3\hat{i} + 6\hat{j} - 2\hat{k}$$

$\therefore$ The unit vector Parallel to $\vec{A}+\vec{B}$ is

$$= \frac{\vec{A}+\vec{B}}{|\vec{A}+\vec{B}|} = \frac{3\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{3^2+6^2+(-2)^2}}$$

$$= \frac{3\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{9+36+4}} = \frac{3\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{49}}$$

$$= \pm \frac{1}{7}(3\hat{i}+6\hat{j}-2\hat{k}).$$

(15) The coordinates of initial point are $\overset{x_1\ y_1\ z_1}{(1,1,2)} = P$

The coordinates of terminal Point are $\underset{x_2\ y_2\ z_2}{(2,1,3)} = Q$

$\therefore$ The length of the vector $|\vec{PQ}| = |\overrightarrow{Q-P}|$

$$|\vec{PQ}| = |(x_2-x_1)\hat{i} + (y_2-y_1)\hat{j} + (z_2-z_1)\hat{k}|$$

$$= |(2-1)\hat{i} + (1-1)\hat{j} + (3-2)\hat{k}|$$

$$= |\hat{i} + 0\hat{j} + 1\hat{k}| = \sqrt{1^2+0^2+1^2} = \sqrt{2}$$

(16) Given vector is $3\hat{i} + \hat{j} + 2\hat{k}$

For any vector $\vec{A} = x\hat{i} + y\hat{j} + z\hat{k}$, the length in XY plane is $= \sqrt{x^2+y^2}$

$\therefore$ For given vector length in XY plane $= \sqrt{3^2+1^2} = \sqrt{9+1}$

$$= \sqrt{10}$$

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(17) Position vector for point $(-6,-4,-12)$ is

$$\vec{R} = -6\hat{i} - 4\hat{j} - 12\hat{k}$$

$\therefore$ The unit vector parallel to $\vec{R} = \frac{\vec{R}}{|\vec{R}|}$

$$= \frac{-6\hat{i} - 4\hat{j} - 12\hat{k}}{\sqrt{(-6)^2 + (-4)^2 + (-12)^2}}$$

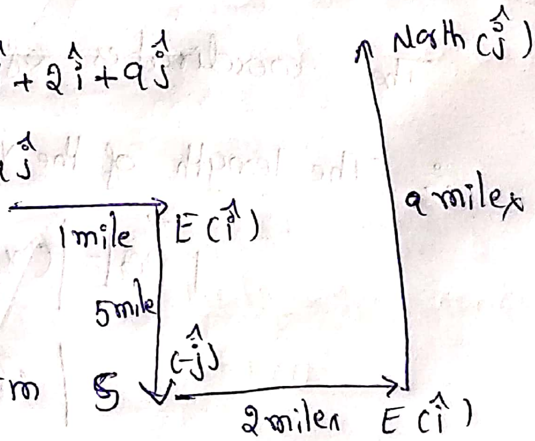
$$= \frac{-2[3\hat{i} + 2\hat{j} + 6\hat{k}]}{\sqrt{36 + 16 + 144}} = \frac{-2(3\hat{i} + 2\hat{j} + 6\hat{k})}{\sqrt{196}}$$

$$= \frac{-\cancel{2}(3\hat{i} + 2\hat{j} + 6\hat{k})}{\pm 14}7 = \pm\frac{1}{7}(3\hat{i} + 2\hat{j} + 6\hat{k})$$

(18) Acc to given question the directions of a particle are represented as

$\therefore$ Displacement $= 1\hat{i} - 5\hat{j} + 2\hat{i} + 9\hat{j}$

$$= 3\hat{i} + 4\hat{j}$$



$|\text{Displacement}| = \sqrt{3^2 + 4^2}$

$$= \sqrt{9+16} = 5m$$

(19) Given $\vec{F} = 2\hat{i} + \hat{j} - \hat{k}$ N ; $u = 0$; $t = 20$ sec

$\vec{V} = 4\hat{i} + 2\hat{j} + 2\hat{k}$ ms^{-1}

$|\vec{F}| = \sqrt{2^2 + 1^2 + (-1)^2}$

$= \sqrt{4+1+1} = \sqrt{6}$ N

From $v = u + at$

$\Rightarrow |\vec{V}| = 0 + a \times t$

$\Rightarrow \sqrt{4^2 + 2^2 + 2^2} = a \times 20$

$= \sqrt{16+4+4} = a \times 20$

$\Rightarrow \sqrt{24} = 20a \Rightarrow a = \frac{\sqrt{24}}{20}$

$\therefore m = \frac{|\vec{F}|}{|\vec{a}|} = \frac{\sqrt{6}}{\frac{\sqrt{24}}{20}}$

$m = \frac{20 \times \sqrt{6}}{\sqrt{4} \times \sqrt{6}} = \frac{20}{2} = 10$ kg

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(20) Given vectors $\vec{A} = 6\hat{i} - 2\hat{k}$

and condition is $\vec{A} - 2\vec{B} = -3(\vec{A} + \vec{B})$

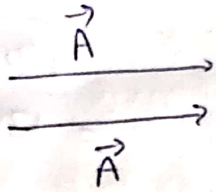
$$\Rightarrow \vec{A} - 2\vec{B} = -3\vec{A} - 3\vec{B}$$

$$\Rightarrow \vec{A} + 3\vec{A} = 2\vec{B} - 3\vec{B}$$

$$\Rightarrow -\vec{B} = 4\vec{A} \Rightarrow \vec{B} = -4\vec{A} = -4(6\hat{i} - 2\hat{k})$$

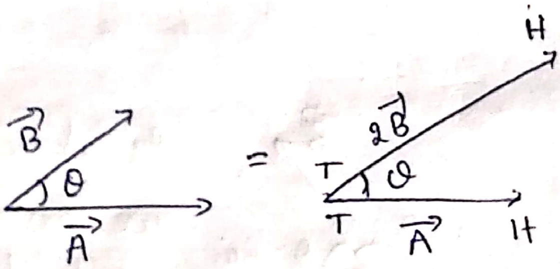
$$= -24\hat{i} + 8\hat{k}$$

(24) let us represent $\vec{A}$ in graphical form



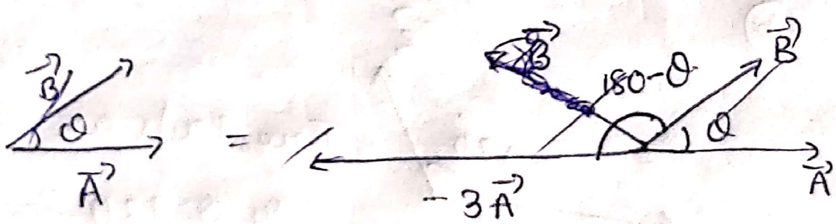
The angle b/w two Parallel vectors is 180°

(25)

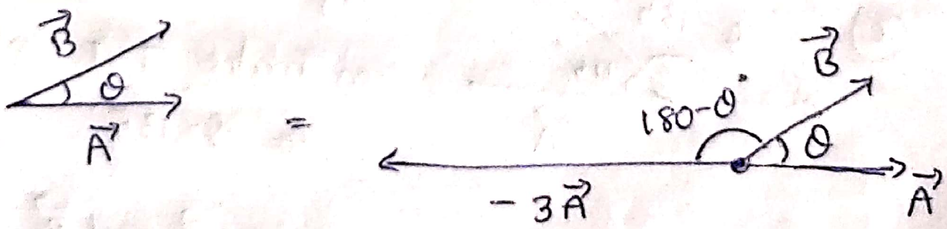


The smallest angle b/w two vectors either H-H (or) T-T can be taken as angle.

(26)

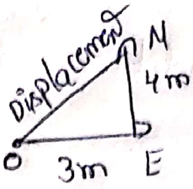


(26)



(27) Acc to given data

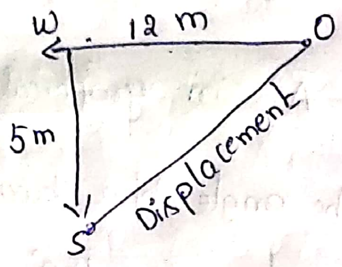
$\therefore$ Displacement $= 3\hat{i} + 4\hat{j}$



$|\text{Displ}| = \sqrt{3^2+4^2}$

$= \sqrt{9+16} = \sqrt{25} = 5\text{m}$

(28) Acc to given data



$\therefore$ Displacement $= -12\hat{i} - 5\hat{j}$

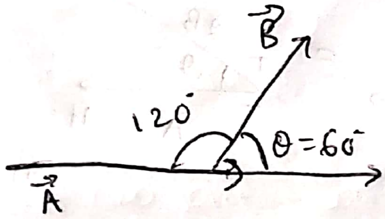
$|\text{Displacement}| = \sqrt{(-12)^2+(-5)^2}$

$= \sqrt{144+25} = \sqrt{169}$

$= 13\text{m}$

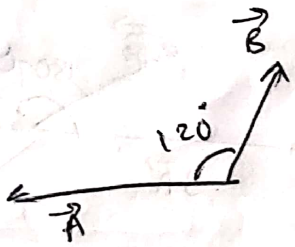
(29)

(a)



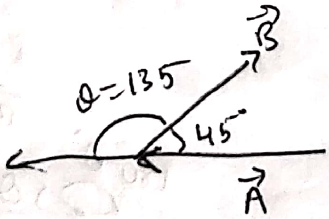
By extending $\vec{A}$ to right we can make T-T contact so $\theta = 60°$

(b)



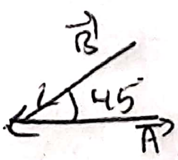
Two Tails are in contact so $\theta = 120°$

(c)



By extending $\vec{A}$ to left we can make T-T contact so $\theta = 135°$

(d)



For both $\vec{A}$ and $\vec{B}$ Heads are in contact so $\theta = 45°$