Thank

①

Given $\vec{A} = 4\hat{i} - 3\hat{j}$ and $\vec{B} = 6\hat{i} - 5\hat{j}$

$\vec{B} - \vec{A} = 6\hat{i} - 5\hat{j} - (4\hat{i} - 3\hat{j}) = 6\hat{i} - 5\hat{j} - 4\hat{i} + 3\hat{j}$

$= 2\hat{i} - 2\hat{j}.$

$|\vec{B} - \vec{A}| = \sqrt{2^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}.$

unit vector Parallel to $\vec{B} - \vec{A} = \dfrac{\vec{B} - \vec{A}}{|\vec{B} - \vec{A}|}$

$= \dfrac{2\hat{i} - 2\hat{j}}{2\sqrt{2}}$

$\Rightarrow \dfrac{2(\hat{i} - \hat{j})}{2\sqrt{2}} = \dfrac{\hat{i} - \hat{j}}{\sqrt{2}}$

②

let the vector Be $\vec{A} = 6\vec{i} - 8\vec{j}$

let $\vec{B}$ is a vector Parallel to $\vec{A}$ and same magnitude as that of $\vec{A}$

$\vec{B} = -\dfrac{\vec{A}}{|\vec{A}|} \times 10$

$\Rightarrow \vec{B} = \dfrac{-[6\hat{i} - 8\hat{j}]}{\sqrt{6^2 + (-8)^2}} \times 10 = -\dfrac{[6\hat{i} - 8\hat{j}]}{\cancel{10}} \times \cancel{10}$

$\Rightarrow \vec{B} = -6\hat{i} + 8\hat{j}.$

(3)

Let $\vec{A} = 3\hat{i} + 4\hat{j}$; $\vec{B} = \hat{j} + \hat{k}$

$\vec{C}$ is a vector which is parallel to $\vec{A}$ and same magnitude as that of $\vec{B}$.

$$\vec{C} = \hat{A} |\vec{B}|$$

$$|\vec{B}| = \sqrt{y^2 + z^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\Rightarrow \vec{C} = \frac{\vec{A}}{|\vec{A}|} |\vec{B}|$$

$$|\vec{A}| = \sqrt{3^2 + 4^2} = \sqrt{9+16} = 5$$

$$\Rightarrow \vec{C} = \frac{3\hat{i} + 4\hat{j}}{5} (\sqrt{2})$$

$$\Rightarrow \vec{C} = \frac{\sqrt{2}(3\hat{i} + 4\hat{j})}{5}$$

(4)

The Given two vectors are

$\vec{A} = \underset{A_x}{2\hat{i}} - \underset{A_y}{y\hat{j}} + \underset{A_z}{3\hat{k}}$ and $\vec{B} = \underset{B_x}{8\hat{i}} + \underset{B_y}{3\hat{j}} + \underset{B_z}{12\hat{k}}$

when $\vec{A}$ and $\vec{B}$ are Parallel then

$$\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}$$

$$\Rightarrow \frac{2}{8} = \frac{-y}{3} = \frac{3}{\cancel{12}\,4} \Rightarrow y = -\frac{3}{4}.$$

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(5)

Given $\vec{A} = 3\hat{i} + 4\hat{j}$: $\vec{B} = 5\hat{i} + 12\hat{j}$.

let $\vec{c}$ is a vector Parallel to $\vec{A}$ and same magnitude as that of $\vec{B}$

$\therefore \vec{c} = \hat{A}\,|\vec{B}|$

Magnitude of $\vec{B} = \sqrt{5^2 + 12^2}$

$= \sqrt{25 + 144}$

$= \sqrt{169}$

$= 13$

$\Rightarrow \vec{c} = \dfrac{\vec{A}}{|\vec{A}|} \times B$

$\Rightarrow \vec{c} = \dfrac{3\hat{i} + 4\hat{j}}{5} \times 13$

Magnitude of $\vec{A} = \sqrt{3^2 + 4^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$= 5$

$\Rightarrow \vec{c} = \dfrac{13\,(3\hat{i} + 4\hat{j})}{5}$

(6)

let $\vec{A} = 6\hat{i} - 8\hat{j}$; $\vec{B}$ is a vector opposite to $\vec{A}$

$\vec{B} = -\hat{A}\,|\vec{B}|$ and Given $|\vec{B}| = 5$

$= -\dfrac{(6\hat{i} - 8\hat{j})}{\sqrt{6^2 + (-8)^2}}\,(5)$

$= \dfrac{-(6\hat{i} - 8\hat{j})}{\cancel{10}\,2} \times \cancel{5}$

$\Rightarrow -\dfrac{\cancel{2}\,(3\hat{i} - 4\hat{j})}{\cancel{2}}$

$\Rightarrow -3\hat{i} + 4\hat{j}$

(7)

let $\vec{A} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ & $\vec{B} = -6\hat{i} + p\hat{j} - 8\hat{k}$

$A_x\hat{i} + A_y\hat{j} + A_z$ B_x B_y B_z

if $\vec{A}$ and $\vec{B}$ are opposite

$|\vec{A}| = -|\vec{B}|$

$$\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}$$

$$\Rightarrow \frac{3}{-6} = \frac{-2}{p} = \frac{4}{-8} \Rightarrow$$

Take any two $\frac{\cancel{-}3}{\cancel{6}\,2} = \frac{\cancel{-}2}{p}$

$$\Rightarrow \frac{1}{2} = \frac{2}{p} \Rightarrow p = 4$$

(8)

Given $\vec{A} = \sqrt{2}\,\hat{i} + \sqrt{2}\,\hat{j}$; $\vec{B} = \hat{i} + \hat{j}$.

let $\vec{c}$ be the vector Parallel to $\vec{A}$ and same magnitude as that of $\vec{B}$

$$\therefore \vec{c} = \hat{A}\,|\vec{B}|$$

$$= \frac{\vec{A}}{|\vec{A}|}\,|\vec{B}|$$

Magnitude of $\vec{B}$

$$|\vec{B}| = \sqrt{x^2 + y^2}$$

$$= \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$|\vec{A}| = \sqrt{x^2 + y^2}$$

$$= \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2}$$

$$= 2$$

$$\vec{c} = \frac{\sqrt{2}\,\hat{i} + \sqrt{2}\,\hat{j}}{2} \times \sqrt{2}$$

$$\Rightarrow \frac{\cancel{\sqrt{2}}\,(\hat{i} + \hat{j}) \times \cancel{\sqrt{2}}}{\cancel{\sqrt{2}}\ \cancel{2}}$$

$$\vec{c} = \hat{i} + \hat{j}$$

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9

Given vector $0.2\hat{i}+0.3\hat{j}+z\hat{k}$ is a unit vector

i.e. $|0.2\hat{i}+0.3\hat{j}+z\hat{k}|=1$

$\Rightarrow \sqrt{(0.2)^2+(0.3)^2+z^2}=1$

squaring on both sides we get

$\Rightarrow [\sqrt{0.04+0.09+z^2}]^2=1^2$

$\Rightarrow 0.13+z^2=1 \Rightarrow z^2=1-0.13=0.87$

10

If $\vec{A}$ and $\vec{B}$ are two parallel vectors then

$\vec{A}=k\vec{B}$ (or) $\vec{B}=k\vec{A}$ let $\vec{A}=2\hat{i}+3\hat{j}-8\hat{k}$

For option D $\vec{B}=6\hat{i}+9\hat{j}-24\hat{k}$

$=3(2\hat{i}+3\hat{j}-8\hat{k})$

$\Rightarrow \underline{\vec{B}=3\vec{A}}$.

13

$\vec{A}=3\hat{i}+4\hat{j}$; $\vec{B}=12\hat{i}+5\hat{j}$.

$|\vec{A}|=\sqrt{3^2+4^2}$; $|\vec{B}|=\sqrt{12^2+5^2}$

$=\sqrt{9+16}=5$ $\quad = \sqrt{144+25}=13$

unit vector of $\vec{A}=\hat{A}=\frac{\vec{A}}{|\vec{A}|}=\frac{(3\hat{i}+4\hat{j})}{5}$

$\vec{B}$ is not a unit vector Because it's magnitude is not 1

(14), (15), (16)

Given $\vec{A} = 2\hat{i} + 2\hat{j}$; $\vec{B} = 2\hat{i} - 4\hat{j}$ (5)

Magnitude of $\vec{B} = |\vec{B}| = \sqrt{x^2 + y^2} = \sqrt{2^2 + (-4)^2}$

$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

Magnitude of $\vec{A} = |\vec{A}| = \sqrt{x^2 + y^2} = \sqrt{2^2 + 2^2}$

$= 2\sqrt{2}$

unit vector of $\vec{A} = \hat{A} = \dfrac{\vec{A}}{|\vec{A}|} = \dfrac{2\hat{i} + 2\hat{j}}{2\sqrt{2}} = \dfrac{2(\hat{i} + \hat{j})}{2\sqrt{2}} = \dfrac{\hat{i} + \hat{j}}{\sqrt{2}}$

(17)

Given $\vec{A} = 4\hat{i} + 3\hat{j} + 6\hat{k}$ and $\vec{B} = -\hat{i} + 3\hat{j} - 8\hat{k}$

let $\vec{C}$ is the resultant of $\vec{A}$ and $\vec{B}$ then

$\vec{C} = \vec{A} + \vec{B} = 4\hat{i} + 3\hat{j} + 6\hat{k} - \hat{i} + 3\hat{j} - 8\hat{k}$

$= 3\hat{i} + 6\hat{j} - 2\hat{k}$

vector Parallel to resultant of $\vec{A}$ and $\vec{B}$ (ie) to $\vec{C}$ is

$= \dfrac{\vec{C}}{|\vec{C}|} = \dfrac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{3^2 + 6^2 + (-2)^2}}$

$= \dfrac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{9+36+4}} = \dfrac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{49}}$

$= \dfrac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7}$.

(18)

let $\vec{A} = 2\hat{i} - 3\hat{j} - 8\hat{k}$ and $\vec{B} = -\hat{i} + 1.5\hat{j} + c\hat{k}$

$A_x \quad A_y \quad A_z \qquad B_x \quad B_y \quad B_z$

condition for two vectors to be parallel is

$$\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z} \Rightarrow \frac{2}{-1} = \frac{-3}{1.5} = \frac{-8}{c}$$

Take any two ie $\frac{2}{\cancel{-}1} = \frac{\cancel{-}8}{c}^{4} \Rightarrow c = 4.$

L Task

Jee main's level

(1)

let the given vector is $\vec{A} = \frac{1}{2}\hat{i} + \frac{1}{2}\hat{j} + c\hat{k}$

Given $\vec{A}$ is a unit vector $|\vec{A}| = 1$

$$\Rightarrow \left| \frac{1}{2}\hat{i} + \frac{1}{2}\hat{j} + c\hat{k} \right| = 1$$

$$\Rightarrow \sqrt{\left(\tfrac{1}{2}\right)^2 + \left(\tfrac{1}{2}\right)^2 + c^2} = 1$$ on squaring both sides

$$\Rightarrow \left[\sqrt{\frac{1}{4} + \frac{1}{4} + c^2}\,\right]^2 = 1^2$$

$$\Rightarrow \frac{2}{4} + c^2 = 1 \Rightarrow \frac{1}{2} + c^2 = 1 \Rightarrow c^2 = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\Rightarrow c = \frac{1}{\sqrt{2}}$$

(2)

let given $P = (1, 2, -1)$ and $Q = (3, 2, 3)$

$x_1 \; y_1 \; z_1 \qquad x_2 \; y_2 \; z_2$

vector $\vec{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

$= (3-1)\hat{i} + (2-2)\hat{j} + (3-(-1))\hat{k}$

$= 2\hat{i} + 0\hat{j} + 4\hat{k} = 2\hat{i} + 4\hat{k}$

length of $|\vec{PQ}| = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$

(3)

Initial Position $r_1 = 2\hat{i} + 3\hat{j} + 5\hat{k}$ m

Final Position $r_2 = \hat{i} - 2\hat{j} + \hat{k}$ m

Displacement $= \vec{r_2} - \vec{r_1} = \hat{i} - 2\hat{j} + \hat{k} - (2\hat{i} + 3\hat{j} + 5\hat{k})$

$= \hat{i} - 2\hat{j} + \hat{k} - 2\hat{i} - 3\hat{j} - 5\hat{k}$

$\Rightarrow -\hat{i} - 5\hat{j} - 4\hat{k}$

(4)

Given

$0.5\hat{i} + 0.8\hat{j} + c\hat{k}$ is a unit vector

$\Rightarrow |0.5\hat{i} + 0.8\hat{j} + c\hat{k}| = 1$

$\Rightarrow \sqrt{(0.5)^2 + (0.8)^2 + c^2} = 1$ on squaring both side

$\Rightarrow \left[\sqrt{(0.25) + (0.64) + c^2}\right]^2 = 1^2$

$\Rightarrow 0.89 + c^2 = 1 \Rightarrow c^2 = 1 - 0.89 \Rightarrow c^2 = 0.11 \Rightarrow c = \sqrt{0.11}$

(5)

let $\vec{A}$ is a vector having magnitude 5 and opposite to $3\hat{i} + 4\hat{j}$ (let it be $\vec{B}$)

$\therefore \vec{A} = 5 \times [-$ unit vector of given vector$]$ (ie) $\hat{B} = \frac{\vec{B}}{|\vec{B}|}$

$= -5 \times \frac{(3\hat{i} + 4\hat{j})}{|\sqrt{3^2+4^2}|} = -\cancel{5} \times \frac{(3\hat{i} + 4\hat{j})}{\cancel{5}}$

$= -(3\hat{i} + 4\hat{j})$

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(6)

let the Given vector be $\vec{A} = \hat{i} + \hat{j} + \hat{k}$.

unit vector Parallel to $\vec{A} = \hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{1^2+1^2+1^2}}$

$$= \frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$$

(7)

Given $\vec{r_1} = 4\hat{i} + 3\hat{j} + 8\hat{k}$ & $\vec{r_2} = 2\hat{i} + 10\hat{j} + 5\hat{k}$

Position of a vector 1 with respect to 2 is $= -(r_2 - r_1)$

$$= -[2\hat{i} + 10\hat{j} + 5\hat{k} - (4\hat{i} + 3\hat{j} + 8\hat{k})]$$

$$\Rightarrow -[2\hat{i} + 10\hat{j} + 5\hat{k} - 4\hat{i} - 3\hat{j} - 8\hat{k}]$$

$$\Rightarrow -[-2\hat{i} + 7\hat{j} - 3\hat{k}] = 2\hat{i} - 7\hat{j} + 3\hat{k}$$

(8)

Given $r = 3t^2\hat{i} + 4t^2\hat{j} + 7\hat{k}$ m

At $t = 10$ sec $\vec{r} = 300\hat{i} + 400\hat{j} + 7\hat{k}$

Displacement $\Rightarrow |\vec{r}| = \sqrt{(300)^2 + (400)^2 + 7^2}$

$$= \sqrt{90000 + 160000 + 49}$$

$$\Rightarrow \sqrt{250049} \simeq 500 \text{ m}$$

(9)

let $\vec{A} = 6\hat{i} - 4\hat{j} + 8\hat{k}$ and $\vec{B} = -12\hat{i} + P\hat{j} - 16\hat{k}$
$A_x \quad A_y \quad A_z \qquad\qquad B_x \quad B_y \quad B_z$

For $\vec{A}$ and $\vec{B}$ to be opposite condition is

$$\frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z} \Rightarrow \frac{6}{-12} = \frac{-4}{P} = \frac{8}{-16}$$

Take any two $\frac{\cancel{6}}{-\cancel{12}\,4} = \frac{-\cancel{4}\,2}{P} \Rightarrow \frac{-1}{4} = \frac{-2}{P}$

$\Rightarrow P = 8$

(10)

Given let $\vec{A} = 2\hat{i} + 2\hat{j}$; $\vec{B} = \hat{i} + \hat{j} + \hat{k}$

let $\vec{C}$ is a vector parallel to $\vec{A}$

Given $|\vec{C}| = |\vec{B}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$$\therefore \vec{C} = |\vec{C}| \times \hat{A} = \sqrt{3}\frac{\vec{A}}{|\vec{A}|} = \sqrt{3}\frac{(2\hat{i} + 2\hat{j})}{\sqrt{2^2 + 2^2}}$$

$$\vec{C} = \frac{\sqrt{3} \times 2(\hat{i} + \hat{j})}{2\sqrt{2}} = \sqrt{\frac{3}{2}}(\hat{i} + \hat{j})$$

(15), (16)

Given $\vec{F_1} = 8\hat{i} + 6\hat{j}\,N$; $\vec{F_2} = 4\hat{i} - 8\hat{j}\,N.$

then $\vec{F_1} + \vec{F_2} = 8\hat{i} + 6\hat{j} + 4\hat{i} - 8\hat{j} \rightarrow 12\hat{i} - 2\hat{j}.$

and $\vec{F_1} - \vec{F_2} = 8\hat{i} + 6\hat{j} - (4\hat{i} - 8\hat{j}) = 8\hat{i} + 6\hat{j} - 4\hat{i} + 8\hat{j}$

$= 4\hat{i} + 14\hat{j}$

Magnitude of resultant of $\vec{F_1}$ & $\vec{F_2}$ = $|\vec{F_1}+\vec{F_2}|$

$= |\, 12\hat{i} - 2\hat{j}\, |$

$= \sqrt{(12)^2 + (-2)^2}$

$= \sqrt{144+4} = \sqrt{148}$ N

(18)

Given $\vec{A} = 2\hat{i} + 2\hat{j}$ & $\vec{B} = 2\hat{i} - 4\hat{j}$

Resultant of $\vec{A}$ and $\vec{B}$ is $\vec{C} = \vec{A} + \vec{B}$

$\Rightarrow \vec{C} = 2\hat{i} + 2\hat{j} + 2\hat{i} - 4\hat{j} \Rightarrow 4\hat{i} - 2\hat{j}$

unit vector parallel to $\vec{C}$ is $\Rightarrow \dfrac{\vec{C}}{|\vec{C}|} = \dfrac{4\hat{i} - 2\hat{j}}{\sqrt{4^2 + (-2)^2}}$

$= \dfrac{4\hat{i} - 2\hat{j}}{\sqrt{16+4}} = \dfrac{4\hat{i} - 2\hat{j}}{\sqrt{20}} = \dfrac{\cancel{2}(2\hat{i} - \hat{j})}{\cancel{2}\sqrt{5}}$

$= \dfrac{2\hat{i} - \hat{j}}{\sqrt{5}}$.