QUADRATIC EXPRESSIONS - I

Class: IX, Mathematics

F⁺

(1)

SOLUTIONS

TEACHING TASK

01. $\dfrac{x^2-2x-3}{x+1}=0$

$\Rightarrow x^2-2x-3=0$ and $x+1\neq 0 \Rightarrow x\neq -1$

$\Rightarrow (x-3)(x+1)=0$

$\Rightarrow x=3$ or $x=-1$

But $x\neq -1$

$\therefore$ solution set $=\{3\}$ Ans: C

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02. $\sqrt{5x^2-6x+8}-\sqrt{5x^2-6x-7}=1$

Let $5x^2-6x=a$

$\therefore \sqrt{a+8}-\sqrt{a-7}=1$

$\Rightarrow \sqrt{a+8}=1+\sqrt{a-7}$

S.O.B.S

$\Rightarrow \cancel{a}+8=1+\cancel{a}-7+2\sqrt{a-7}$

$\Rightarrow 14=2\sqrt{a-7}$

$\Rightarrow \sqrt{a-7}=7$

$\Rightarrow a-7=49$

$\Rightarrow a=56$

$\Rightarrow 5x^2-6x-56=0$

$\Rightarrow 5x^2-20x+14x-56=0$

$5x(x-4)+14(x-4)=0$

$(x-4)(5x+14)=0$

$x=4$ or $-\dfrac{14}{5}$

Ans: B

03 a, b, c AP $\Rightarrow 2b = a + c$

$ax^2 + bx + c = 0$

2 is a root

$\therefore 4a + 2b + c = 0$

$\Rightarrow 4a + a + c + c = 0$

$\Rightarrow 5a + 2c = 0$

$\Rightarrow \frac{c}{a} = \frac{-5}{2}$

$\Rightarrow 2 \times \beta = \frac{-5}{2} \Rightarrow \beta = \frac{-5}{4}$ Ans : A

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04. $30^\circ + 15^\circ = 45^\circ$

$\Rightarrow \tan(30^\circ + 15^\circ) = \tan 45^\circ$

$\Rightarrow \frac{\tan 30^\circ + \tan 15^\circ}{1 - \tan 30^\circ \cdot \tan 15^\circ} = 1$

$\Rightarrow \cancel{\frac{-p}{1 + q} = 1} \Rightarrow \frac{-p}{1 - q} = 1$

$\Rightarrow \cancel{1 + q + p =}$ $q - p = 1$

$\Rightarrow 2 + q - p = 3$ Ans: D

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05 $f(x) = 2x^2 - 3x - 6 = 0$

$\alpha^2, \beta^2 \Rightarrow f(\sqrt{x}) = 2(\sqrt{x})^2 - 3\sqrt{x} - 6 = 0$

$\Rightarrow 2x - 3\sqrt{x} - 6 = 0$

$\Rightarrow 2x - 6 = 3\sqrt{x}$

$4x^2 - 24x + 36 = 9x$

$\Rightarrow 4x^2 - 33x + 36 = 0$

$\alpha^2+2,\ \beta^2+2 \Rightarrow f(\sqrt{x}-2) = 4(x-2)^2-33(x-2)+36=0$

$\Rightarrow 4x^2-49x+118=0$

Ans: D

06 $\frac{1}{\alpha^3}+\frac{1}{\beta^3} = \frac{\alpha^3+\beta^3}{(\alpha\beta)^3} = \frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{(\alpha\beta)^3}$

$$= \frac{\left(-\frac{b}{a}\right)^3 - 3\frac{c}{a}\left(\frac{-b}{a}\right)}{\left(\frac{c}{a}\right)^3}$$

$$= \frac{3abc-b^3}{c^3}$$

Ans: C

07 $x^2-(5m-2)x+(4m^2+10m+25)=0$

perfect square $\Rightarrow \Delta=0$

$\Rightarrow b^2-4ac=0$

$[-(5m-2)]^2-4(1)(4m^2+10m+25)=0$

$\Rightarrow m=-\frac{4}{3}$ or 8

Ans: C

08 $(a^2+b^2)x^2+2(bc+ad)x+(c^2+d^2)=0$

roots are real $\Rightarrow \Delta \geq 0$

$[2(bc+ad)]^2-4(a^2+b^2)(c^2+d^2)=0$

$\Rightarrow ac=bd$

$\Rightarrow (bd)^2=a^2\cdot c^2$

$\Rightarrow a^2, bd, c^2$ are in G.P.

Ans: B

09. $x^{\sqrt{x}} = (\sqrt{x})^{x}$

$\Rightarrow x^{\sqrt{x}} = x^{\frac{x}{2}}$

$\Rightarrow \sqrt{x} = \frac{x}{2}$

$\Rightarrow x = \frac{x^2}{4}$

$\Rightarrow x\left(1 - \frac{x}{4}\right) = 0$

$\Rightarrow x = 0$ or $\boxed{x = 4}$

$x = 0$ can not possible,

Also, By observation 4

$x = 1$ is a root

$\therefore$ solution set $= \{1, 4\}$

Ans: C

~~10.~~ $(a^2+b^2)x^2 + 2(a+b+c)x + (a-b+c) = 0$

~~Now find $\Delta = b^2 - 4ac$~~

~~$\Rightarrow [2(a+c)]^2 - 4(a^2+b^2)(a-b$~~

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10. $(a+b+c)x^2 - 2(a+c)x + (a-b+c) = 0$

find $\Delta = b^2 - 4ac$

$= [-2(a+c)]^2 - 4(a+b+c)(a-b+c)$

$= 4(a^2+c^2+2ac) - 4(a^2 - ab + ac + ba - b^2 + bc + ca - cb + c^2)$

$= 4b^2 > 0$

Hence, the roots are rational

11. $\frac{\alpha+1}{\alpha} + \frac{\beta+1}{\beta}$ (5)

$= 1+\frac{1}{\alpha} + 1+\frac{1}{\beta} \quad = 2+\left(\frac{\alpha+\beta}{\alpha\beta}\right)$

$= 2 + \frac{(-3)}{-2} = 2+\frac{3}{2} = \frac{7}{2}$ (or) $\frac{14}{4}$

Ans: A, C

12. $\frac{a+b}{2} = 9$ & $\sqrt{ab} = 4$

$a+b = 18$ & $ab = 16$

$\therefore x^2 - 18x + 16 = 0$ (or)

$2x^2 - 36x + 32 = 0$

Ans: B, C

13 Statement I: $\left(\frac{\alpha}{\beta} - \frac{\beta}{\alpha}\right)^2$

$= \left(\frac{\alpha^2-\beta^2}{\alpha\beta}\right)^2$

$= \frac{(\alpha+\beta)^2(\alpha-\beta)^2}{(\alpha\beta)^2} = \frac{(\alpha+\beta)^2\left[(\alpha+\beta)^2 - 4\alpha\beta\right]}{(\alpha\beta)^2}$

$= \frac{\left(\frac{-b}{a}\right)^2\left[\left(\frac{-b}{a}\right)^2 - \frac{4c}{a}\right]}{\left(\frac{c}{a}\right)^2}$

$= \frac{b^2(b^2-4ac)}{c^2a^2}$ (True)

Statement II: (conceptual (True))

Ans: A

14. Statement I: $(b-c)x^2+2(c-a)x+(a-b)=0$ 6

$\boxed{\Delta = b^2-4ac} = [2(c-a)]^2 - 4(b-c)(a-b)$

$= 4(c^2-2ac+a^2)-4(ab-b^2-ac+bc)$

$= 4[c^2-2ac+a^2-ab+b^2+ac-bc]$

$= 4[a^2+b^2+c^2-ab-bc-ca]$

$= 2[2a^2+2b^2+2c^2-2ab-2bc-2ca]$

$= 2[(a-b)^2+(b-c)^2+(c-a)^2] \geq 0$

∴ Roots are real and distinct (True)

Statement II: Conceptual (True) Ans: B

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15. $x^2-15x+1=0$

α is a root $\Rightarrow$ $\alpha^2-15\alpha+1=0$

$\Rightarrow 1-15\alpha = -\alpha^2$

Similarly $\Rightarrow 1-15\beta = -\beta^2$

Now $\left(\frac{1}{\alpha}-15\right)^{-2}+\left(\frac{1}{\beta}-15\right)^{-2}$

$= \left(\frac{1-15\alpha}{\alpha}\right)^{-2}+\left(\frac{1-15\beta}{\beta}\right)^{-2}$

$= \left(\frac{-\alpha^2}{\alpha}\right)^{-2}+\left(\frac{-\beta^2}{\beta}\right)^{-2}$

$= \frac{1}{\alpha^2}+\frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{(\alpha\beta)^2} = \frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$

$$= \frac{(15)^2 - 2\cdot 1}{(1)^2} = 225 - 2 = 223 \quad (7)$$

Ans: A

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16 Given $\frac{1}{\alpha} + \frac{1}{\beta} = 4$

$$\Rightarrow \frac{\alpha+\beta}{\alpha\beta} = 4$$

$$\Rightarrow \frac{\left(-\frac{q}{p}\right)}{\left(\frac{\gamma}{p}\right)} = 4$$

$$\Rightarrow \frac{-q}{\gamma} = 4$$

$$\Rightarrow -q = 4\gamma$$

$$\Rightarrow -2q = 8\gamma$$

$$\Rightarrow -(p+\gamma) = 8\gamma$$

$$\Rightarrow -p = 9\gamma$$

$$\Rightarrow \boxed{\frac{\gamma}{p} = \frac{-1}{9}}$$

Also $-q = 4\gamma$

$$\Rightarrow -q = 4\left(-\frac{p}{9}\right)$$

$$\Rightarrow \boxed{\frac{q}{p} = \frac{4}{9}}$$

Now $|\alpha - \beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$$= \sqrt{\left(\frac{-q}{p}\right)^2 - 4\left(\frac{\gamma}{p}\right)}$$

$$= \sqrt{\left(\frac{4}{9}\right)^2 - 4\left(\frac{-1}{9}\right)}$$

$$= \sqrt{\frac{16+36}{81}} = \frac{2\sqrt{13}}{9}$$

Ans: D

17. $\alpha = 2+\sqrt{3}$

$\Rightarrow \beta = 2-\sqrt{3}$

$\therefore x^2 - 4x + (4-3) = 0$

$\Rightarrow x^2 - 4x + 1 = 0$

Ans. B

18. $x^2 - \left(-\frac{2}{3} + \frac{3}{7}\right)x + \left(-\frac{2}{\cancel{3}} \cdot \frac{\cancel{3}}{7}\right) = 0$

$\Rightarrow x^2 - \left(\frac{-14+9}{21}\right)x \cancel{+} - \frac{2}{7} = 0$

$\Rightarrow 21x^2 + 5x - 6 = 0$

Ans: D

19. $22° + 23° = 45°$

$\Rightarrow \tan(22° + 23°) = \tan 45°$

$\Rightarrow \dfrac{\tan 22° + \tan 23°}{1 - \tan 22° \cdot \tan 23°} = 1$

$\Rightarrow \dfrac{-a}{1-b} = 1 \quad \Rightarrow -a = 1-b$

$\Rightarrow a - b = -1$

Ans: -1

20. $|\alpha^2 - \beta^2| = (\alpha+\beta)(\alpha-\beta) = 24$

$= (\alpha+\beta)\sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = 24$

$= (6)\sqrt{(6)^2 - 4q} = 24$

$\Rightarrow 36 - 4q = 16$

$\Rightarrow q = 5$

Ans: 5

21 a) $cx^2+bx+a=0 \rightarrow$ roots $\frac{1}{\alpha}, \frac{1}{\beta}$ (9)

b) $ax^2-bx+c=0 \rightarrow$ roots $-\alpha, -\beta$

c) $a\left(\frac{x}{2}\right)^2+b\left(\frac{x}{2}\right)+c=0 \rightarrow$ roots, $2\alpha, 2\beta$

d) $a(2x)^2+b(2x)+c=0 \rightarrow$ roots $\frac{\alpha}{2}, \frac{\beta}{2}$

Ans : t, s, r, q

22 a) $x^2-4x+4=0 \Rightarrow (x-2)^2=0 \Rightarrow x=2$

Roots are real and equal

b) $x^2+4x+3=0$

$\Delta = 16-4.1.3 = 16-12 = 4>0$

Roots are real and distinct and rational

c) $x^2+x-3=0$

$\Delta = 1-4.1.(-3) = 1+12 = 13>0$

real and irrational

d) $x^2+x+1=0$

$\Delta = 1-4.1.1 = 1-4 = -3<0$

roots are imaginary

Ans : q, r, s, t

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LEARNERS TASK

CUQ'S

01. Conceptual — Ans: D

02. $(x-2)(8-x)=0$

$8x-x^2-16+2x=0$

$\Rightarrow x^2-10x+16=0$

Linear term $= -10x$ — Ans: B

03. $(x-a)(x-b)=b^2$

$\Rightarrow x^2-bx-ax+ab-b^2=0$

$\Rightarrow x^2-(a+b)x+ab-b^2=0$

$\Delta = [-(a+b)]^2-4(ab-b^2)$

$= (a+b)^2-4ab+4b^2$

$= (a-b)^2+4b^2 > 0.$

Roots real and distinct — Ans: A

~~04~~ 05. $x^2-15-m(2x-8)=0$

$\Rightarrow x^2-2mx+8m-15=0$

$\Delta=0$

$\Rightarrow (-2m)^2-4(1)(8m-15)=0$

$\Rightarrow m^2-8m+15=0$

$\Rightarrow (m-3)(m-5)=0$

$\Rightarrow m=3$ or 5 — Ans: D

04 $x^2+x-1=0$

$\Delta = (1)^2-4(1)(-1)$

$= 1+4$

$= 5>0$, Not perfect square

$\therefore$ roots are Irrational Conjugate

Ans: D

06 $f(x) = x^2+11x+13=0$

$f(x+4) = (x+4)^2+11(x+4)+13=0$

Ans: B

$\Rightarrow x^2+19x+73=0$

Ans: A

07 $3-2i$

Ans: B

08 α^2, β^2

Ans: C

09 $\Delta = a^2-4bc$

10. $ax^2+bx+c=0$

$\alpha+\beta = -\frac{b}{a}$ | $\alpha.\beta = \frac{c}{a}$

$\alpha+\alpha = -\frac{b}{a}$ | $\alpha.\alpha = \frac{c}{a}$

$\Rightarrow \alpha = -\frac{b}{2a}$ | $\alpha = \sqrt{\frac{c}{a}}$

Ans: D

JEE MAINS LEVEL

01. $x^2-S_1x+S_2=0$

Ans. A

02 $\cancel{\Delta = (c+a-b)^2-4(b+c-a)(a+b-c)}$

$\cancel{= c^2+a^2+b^2+2ac}$

12

02. $a+b+c=0$

$(b+c-a)x^2+(c+a-b)x+(a+b-c)=0$

$(-2a)x^2+(-2b)x+(-2c)=0$

$\Rightarrow \quad ax^2+bx+c=0$

$\Delta = b^2-4ac$

$= (-a-c)^2-4ac$

$= (a+c)^2-4ac = (a-c)^2 > 0.$

roots real and distinct

Ans: B

03. $\alpha = 3-2i \Rightarrow \beta = 3+2i$

$x^2-6x+(3-2i)(3+2i)=0$

$\Rightarrow x^2-6x+9+4=0$

$\Rightarrow x^2-6x+13=0$

Ans: C

04. $\sin\theta+\cos\theta = \frac{-b}{a} \rightarrow (1)$

$\sin\theta \cdot \cos\theta = \frac{c}{a} \rightarrow (2)$

$(1)^2 \Rightarrow 1+2\cdot\frac{c}{a} = \frac{b^2}{a^2}$

$\Rightarrow a^2+2ac = b^2$

Ans: A

05. $\frac{-b}{a} = -1$

i.e. $\frac{-(2a+3)}{a+1} = -1$

$\Rightarrow 2a+3 = a+1$

$\boxed{a=-2}$

product $= \frac{3a+4}{a+1}$

$= \frac{3(-2)+4}{-2+1} = \frac{-6+4}{-2+1}$

$= 2$

Ans: B

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06. $a(x-1)^2 + b(x-1) + c = 0$

$\Rightarrow \cancel{ax^2 + bx} + c$

$\Rightarrow a(x^2 - 2x + 1) + b(x-1) + c = 0$

$\Rightarrow ax^2 - 2ax + a + bx - b + c = 0$

$\Rightarrow ax^2 + (b-2a)x + a - b + c = 0$

$2x^2 + 8x + 2 = 0$

$\boxed{a=2}$ | $b - 2a = 8$, $b - 2\cdot 2 = 8$, $\boxed{b = \cancel{4}12}$ | $a - b + c = 2$, $2 - 12 + c = 2$, $\boxed{c = 12}$

$\therefore b = c$

Ans: C

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07. $x^2 + px + 12 = 0$

$4^2 + p(4) + 12 = 0$

$\Rightarrow p = -7$

$x^2 - 7x + q = 0$

Equal roots $\Delta = 0$

$49 - 4q = 0$

$\Rightarrow q = \dfrac{49}{4}$

Ans: D

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08. $x^{\frac{2}{3}} + x^{\frac{1}{3}} - 2 = 0$

let $x^{1/3} = a$

$\therefore a^2 + a - 2 = 0$

$\Rightarrow (a+2)(a-1) = 0$

$\Rightarrow a = 1$ or -2

$x^{1/3} = 1$, $x = 1$ | $x^{1/3} = -2$, $x = -8$

$\therefore$ Two roots

Ans: B

09 $f(x) = 4x^2 + 7x + 2 = 0$

$f(\sqrt{x}) = 4(\sqrt{x})^2 + 7(\sqrt{x}) + 2 = 0$

$4x + 7(\sqrt{x}) + 2 = 0$

$\Rightarrow \quad 4x + 2 = -7\sqrt{x}$

S.O.B.S

$16x^2 + 16x + 4 = 49x$

$\Rightarrow 16x^2 - 33x + 4 = 0$

Ans: D

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10 $x^2 + 4x + 3 = 0$

$(x+1)(x+3) = 0 \quad \Rightarrow x = -1, -3$

Ans: B

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11. $x^2 + 3x - 1 = 0$

$x = \dfrac{-3 \pm \sqrt{9+4}}{2} = \dfrac{-3 \pm \sqrt{13}}{2}$

Ans: A, B

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12. $x^2 + x - 3 = 0$

$\Delta = (1)^2 - 4(1)(-3)$

$= 13 > 0$, Not a perfect square

Ans: C, D

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13 Statement I: $x^2 - 4x + 4 = 0 \Rightarrow (x-2)^2 = 0$

$\Rightarrow x = 2, 2$ (True)

Statement II: Conceptual (True)

Ans: A

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14. Statement I: $f(x) = ax^2 + bx + c = 0$

$f(-x) = a(-x)^2 + b(-x) + c = 0$

$= ax^2 - bx + c = 0$ (True)

Statement II: $x^2 - 3x + 4 = 0 \Rightarrow$ Sum $= 3$ (False)

Ans: C

15. $f(x) = x^2+x+1=0$

$f(x-1) = (x-1)^2+(x-1)+1=0$

$\Rightarrow x^2-x+1=0$

Ans : C

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16. $f(x) = 2x^2-3x+6=0$

α^2, β^2 $f(\sqrt{x}) = 2(\sqrt{x})^2-3(\sqrt{x})-6=0$

$\Rightarrow 4x^2-33x+36=0$

$\alpha^2+2, \beta^2+2 \Rightarrow f(\sqrt{x-2}) = 4(x-2)^2-33(x-2)+36x$

$= 4x^2-49x+118=0$

Ans : B

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17. $x^2+x+3=0$

$\Delta = 1-4\cdot1\cdot3 = 1-12 = -11 < 0$

Ans : C

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18. (Conceptual) for all options $\Delta = 0$

Ans : D

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19. $(a^2-5a+3)x^2+(3a-1)x+2=0$

α, 2α

$\alpha+2\alpha = \dfrac{1-3a}{a^2-5a+3}$

$\Rightarrow \alpha = \dfrac{1-3a}{3(a^2-5a+3)}$

$\alpha\cdot2\alpha = \dfrac{2}{a^2-5a+3}$

$\alpha^2 = \dfrac{1}{a^2-5a+3}$

$\Rightarrow \left[\dfrac{1-3a}{3(a^2-5a+3)}\right]^2 = \dfrac{1}{a^2-5a+3}$

$\Rightarrow \dfrac{(1-3a)^2}{9} = a^2-5a+3$

$\Rightarrow 3a = 2$

Ans : 2

20. $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (5)^2 - 2\cdot 6$

$= 25 - 12$

$= 13$

Ans: 13

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21 a) $\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$= \left(-\frac{b}{a}\right)^3 - 3\left(\frac{c}{a}\right)\left(-\frac{b}{a}\right)$

$= \dfrac{3abc - b^3}{a^3}$

b) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right)$

$= \dfrac{b^2-2ac}{a^2}$

c) $|\alpha-\beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$= \sqrt{\left(-\frac{b}{a}\right)^2 - 4\left(\frac{c}{a}\right)} \quad = \dfrac{\sqrt{b^2-4ac}}{|a|}$

d) $\alpha+\beta = -\frac{b}{a}$

Ans: t, p, s, q

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22 a) $K\alpha, K\beta \Rightarrow f\left(\frac{x}{K}\right)=0$

b) $\frac{\alpha}{K}, \frac{\beta}{K} \Rightarrow f(Kx)=0$

c) $\alpha^2, \beta^2 \Rightarrow f(\sqrt{x})=0$

d) $\alpha+K, \beta+K \Rightarrow f(x-K)=0$

$\Longrightarrow$ THE END

Ans: s, p, q, r.