ws-7 vertically Projected body 7th advanced

Task

①

Given, velocity of body = u

It is at the same point at t=6s as well as at t=4s

so let $t_1 = 4s$ and $t_2 = 6s$, body is at a height h.

so $h = ut_1 - \frac{1}{2} g t_1^2 \rightarrow ①$ and

$h = ut_2 - \frac{1}{2} g t_2^2 \rightarrow ②$

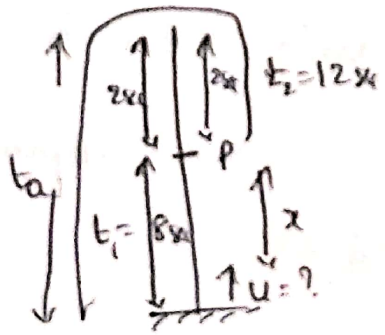
By solving $ut_1 - \frac{1}{2} g t_1^2 = ut_2 - \frac{1}{2} g t_2^2$

$\Rightarrow u(t_1 - t_2) = \frac{1}{2} g (t_1^2 - t_2^2)$

$\Rightarrow u(t_1 - t_2) = \frac{1}{2} g (t_1 + t_2)(t_1 - t_2)$

$\Rightarrow u = \frac{1}{2} \times 10 (4+6) = 5 \times 10 = 50 \text{ m/s}$

②



$t_a = \frac{u}{g}$

$\Rightarrow u = t_a g = 10 \times 10 = 100 \text{ m/s}$

Hence From $s = ut + \frac{1}{2} a t^2$

$\Rightarrow x = 100 \times 10 + \frac{1}{2}(-g)(10)^2$

$= 1000 - \frac{1}{2} \times 10 \times 100$

$\Rightarrow 500 \text{ m} \simeq 480 \text{ m}$

For $(g = 9.8 \text{ m/s}^2)$

③

Given $u = 20\ m/s$; $t = 3\ sec$; $a = -g = -10\ m/s^2$

Distance travelled by the body in 3 sec

$$s = ut + \frac{1}{2}at^2$$

$$\Rightarrow s = 20 \times 3 + \frac{1}{2}(-10)(3)^2$$

$$= 60 - 45 = 15\ m$$

④

Given distance covered in 5^{th} sec = 2 Distance covered in 6^{th} sec

$$\Rightarrow \quad S_5 = 2S_6 \quad [\text{Here } a = -g]$$

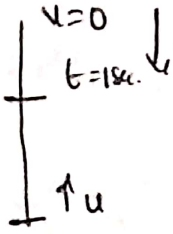
$$\Rightarrow u - \frac{g}{2}(2(5)-1) = 2\left[u - \frac{g}{2}(2(6)-1)\right]$$

$$\Rightarrow u - \frac{9g}{2} = 2u - \frac{22g}{2}$$

$$\Rightarrow 2u - u = \frac{22g}{2} - \frac{9g}{2} \Rightarrow u = \frac{13 \times g}{2} = \frac{13}{2} \times 10 = 65\ m/s$$

⑤ ⑥

let H be max height reached by the body $H = \frac{u^2}{2g}$



From $s = ut + \frac{1}{2}at^2$ $\quad a = -g$

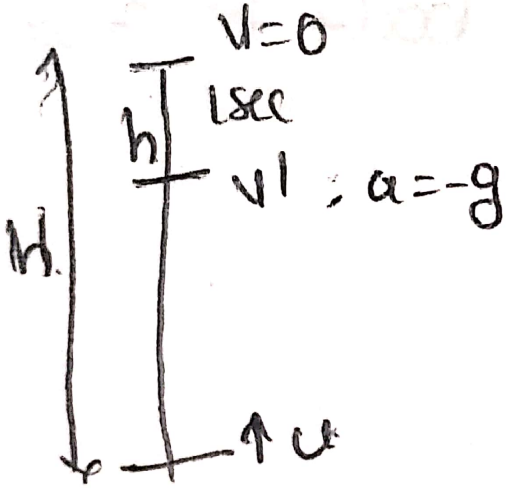
$$\Rightarrow (-h) = 0(1) + \frac{1}{2}(-g)(1)^2$$

$$\Rightarrow -h = \frac{-g}{2} \Rightarrow h = \frac{g}{2} = \frac{9.8}{2} = 4.9\ m$$

The last second of vertically projected body acts as 1^{st} sec of freely falling body

⑥ ⑤ continued

From $H_{max} = \frac{u^2}{2g}$



$h =$

From $v^2 - u^2 = 2as$

$\rightarrow a^2 - v1^2 = 2(-g)h$

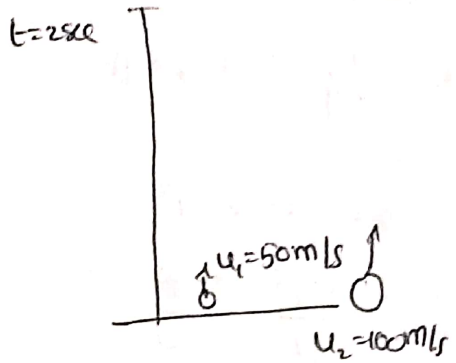
$\rightarrow -v1^2 = -2g\frac{g}{2}$

$\rightarrow v^1 = g.$

⑦

Given $u = 50$ m/s

(7)



$u_1 = u_1 - gt = 50 - 10\times 2$

$= 50 - 20 = 30$ m/s

$u_2 = 100$ m/s

$V_{rel} = V_{21} = 100 - 30 = 70$ m/s

In 2 sec distance travelled by I^{st} body

$H = u_1 t + \frac{1}{2} a t^2$

$= 50(2) - \frac{1}{2}\times 9(2)^2$

$= 100 - 20$

$= 80$ m.

t' be the time of meeting

$t' = \frac{H}{V_{rel}} = \frac{80}{70}$

$= \frac{8}{7}$

Total time taken to meet $= 2 + t' = 2 + \frac{8}{7} = \frac{22}{7}$ sec

(8)

Given $u = 4.9$ m/s

they are asking time taken to reach the ground

$$T = \frac{2u}{g} = \frac{2\times 4.9}{9.8} = 1 \text{ sec}$$

(9)

After 6 sec the stone is at maximum height. In 7^{th} sec the stone act as FFB.

$\therefore$ distance travelled by the vertically projected body in last sec = Distance travelled by it in 7^{th} or Distance travelled by it in 1st sec which act as FFB for return journey $\Rightarrow \frac{g}{2}$. $\therefore \frac{S_1}{S_7} = \frac{\frac{g}{2}}{\frac{g}{2}} = \frac{1}{1} = 1:1$

(10)

let 'u' be the velocity of Projection

$$H_{max} = \frac{u^2}{2g} \Rightarrow u = \sqrt{2gh} = \sqrt{2 \times 10 \times 40} = \sqrt{800}\ m/s$$

$u' = 3u$

we know $H \propto u^2$

$$\Rightarrow \frac{H_2}{H_1} = \left[\frac{u_2}{u_1}\right]^2$$

$$\Rightarrow \frac{H_2}{40} = \left[\frac{3u}{u}\right]^2 \Rightarrow \frac{H_2}{40} = 9$$

$$\Rightarrow H_2 = 360m.$$

(12)

(12)

For vertically Projected body

use $v^2 - u^2 = -2gs$ we know $h = \frac{u^2}{2g}$

$\Rightarrow v^2 = u^2 - 2gs$. $\Rightarrow 2gh = u^2$

(a) For $s = \frac{h}{2}$ $\Rightarrow v_1^2 = u^2 - 2g\frac{h}{2} = u^2 - \frac{u^2}{2} = \frac{u^2}{2}$

$\Rightarrow v_1 = \frac{u}{\sqrt{2}}$

(b) For $s = \frac{3h}{4}$ $\Rightarrow v_2^2 = u^2 - 2g(\frac{3h}{4}) \Rightarrow u^2 - \frac{3u^2}{4} = \frac{u^2}{4}$

$\Rightarrow v_2 = \frac{u}{2}$

(c) For $s = \frac{h}{3}$ $\Rightarrow v_3^2 = u^2 - 2g\frac{h}{3} = u^2 - \frac{u^2}{3} = \frac{2u^2}{3}$

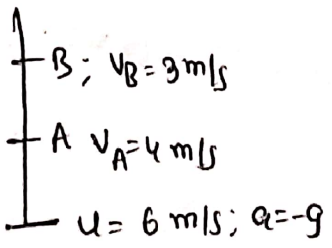
$\Rightarrow v_3 = \sqrt{\frac{2}{3}}\, u$

(d) For $s = \frac{h}{4}$. $\Rightarrow v_4^2 = u^2 - 2g\frac{h}{4} = u^2 - \frac{u^2}{4} = \frac{3}{4}u^2$

$\Rightarrow v_4 = \frac{\sqrt{3}}{2}u$.

(14)

Given $u = 6$ m/s



height of A $\Rightarrow v_A^2 - u^2 = -2gh_A$

$\Rightarrow 4^2 - 6^2 = -2 \times 10 \times h_A$

$\Rightarrow (10)(-2) = -20\, h_A$

$h_A = 1$m

height of B $\Rightarrow v_B^2 - u^2 = -2gh_B$

$\Rightarrow 3^2 - 6^2 = -2 \times 10\, h_B$

$\Rightarrow 9 \times (-3) = -20\, h_B$

$h_B = \frac{27}{20} = 1.35$ m.

$h_B - h_A = 1.35 - 1$

$= 0.35$ m

(15)

For a vertically projected body Time of flight $= 2s$

$$\Rightarrow \frac{2u}{g} = 2 \Rightarrow u = g.$$

$$H_{max} = \frac{u^2}{2g} \Rightarrow \frac{g^2}{2g} = \frac{g}{2} = \frac{9.8}{2} = 4.9\,m \simeq 5m$$

(16)

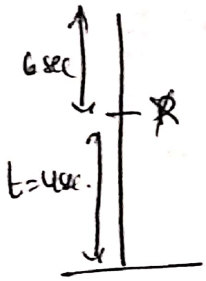
Clearly Time of flight $T = \frac{2u}{g} = 3sec.$

$$u = \frac{3g}{2}$$

$$H_{max} = \frac{u^2}{2g} = \frac{\left(\frac{3g}{2}\right)^2}{2g} = \frac{\frac{3g}{2} \times \frac{3g}{2}}{2g} \Rightarrow \frac{9 \times g}{8} = 4.5m$$

(7)

Given $u = 98\,m/s$



$$T_a = \frac{u}{g} = \frac{98}{g} = \frac{98}{9.8} = 10\ sec.$$

To reach maximum height from ground

Time taken = 10 sec From x to top = 6sec

From top to x → 6sec

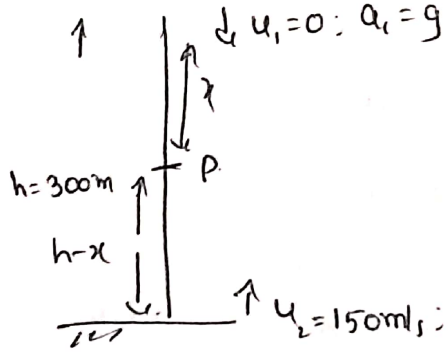
∴ Total time from ground to x in downward journey = 10+6 = 16 sec

LTank

Jee mains

(1)

Given $h = 300$



let P is a meeting point

From $s = ut + \frac{1}{2}at^2$

FFB	UPB
$s = x$; $u = 0$; $a = g$	$s = 300 - x$; $u_2 = 150$; $a = -g$
$x = 0 \times t + \frac{1}{2} g t^2$	$\Rightarrow 300 - x = 150t - \frac{1}{2} g t^2$
$\Rightarrow x = \frac{1}{2} g t^2$ → (1)	→ (2)

From (1) and (2)

$\Rightarrow 300 - \frac{1}{2} g t^2 = 150t - \frac{1}{2} g t^2$

$\Rightarrow 300 = 150t \Rightarrow t = 2$ sec.

From top height $x = \frac{1}{2} \times g t^2$

$= \frac{1}{2} \times 9.8 \times 2^2$

$= 2 \times 9.8 = 19.6$ m.

(2)

Given $H_{max} = g$

From $H = \frac{u^2}{2g} \Rightarrow u^2 = 2gH \Rightarrow u^2 = 2g \times g$

$\Rightarrow u = \sqrt{2}\, g = 14.14 m/s$

(3)

Given $u = 5m$

They asking maximum height reached

$H_{max} = \frac{u^2}{2g} = \frac{5^2}{2 \times 10} = \frac{25}{20} = 1.25m$

(4)

Given $u = 100 m/s$

Time of flight $T = \frac{2u}{g} = \frac{2 \times 100}{10} = 20$ sec.

(5)

Time of ascent = 2 sec ; $v = 2$ sec

$\frac{u}{g} = 2 \Rightarrow u = 2g$; $a = -g$

From $v^2 - u^2 = 2as$

$\Rightarrow 2^2 - (2g)^2 = 2(-g)h$

$\Rightarrow 4 - 4g^2 = -2gh$

$\Rightarrow 4 - 4 \times 100 = -20h$

$\Rightarrow -396 = -20h \Rightarrow h = \frac{39.6}{2}$

(6)

Given $T = 4$ sec

$\Rightarrow \frac{2u}{g} = 4$

$\Rightarrow u = 2g = 19.6$ m

(7)

$H_e = 12$ m ; $g_e = g$

$g_m = \frac{1}{6}g$

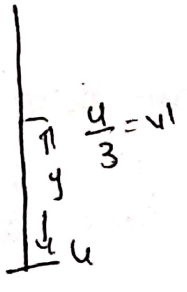
$H_{max} = \frac{u^2}{2g}$; u = same

$\Rightarrow \frac{H_m}{H_e} = \frac{g_e}{g_m}$

$\Rightarrow \frac{H_m}{12} = \frac{g}{\frac{1}{6}g}$

$\Rightarrow H_m = 12 \times 6 = 72$ m

(8)



From $v^2 - u^2 = 2as$

$\Rightarrow (\frac{u}{3})^2 - u^2 = 2(-g)y$

$\Rightarrow -\frac{8u^2}{9} = -2gy$

$\Rightarrow y = \frac{8u^2}{9 \cdot 2g}$

we know $H = \frac{u^2}{2g}$

$\Rightarrow y = \frac{8}{9}H$

$\Rightarrow H = \frac{9y}{8}$.

(9)

Given $H = 9.8\,m$.

we know $H = \frac{u^2}{2g}$

$\Rightarrow u = \sqrt{2gH} = \sqrt{2 \times 9.8 \times 9.8} = 9.8\sqrt{2}\ m/s$

Time of flight $= \frac{2u}{g} = 2 \times \frac{9.8\sqrt{2}}{9.8} = 2\sqrt{2}$ sec.

(10)

let u be velocity of Projection.

at $s = \frac{H}{2}$; $v = 10\ m/s$; $a = -g$

From $v^2 - u^2 = 2as$

$\Rightarrow 10^2 - u^2 = -2g\left(\frac{H}{2}\right)$

$\Rightarrow 100 - u^2 = -g\,\frac{u^2}{2g}$

$\Rightarrow 100 = -\frac{u^2}{2} + u^2 \Rightarrow \frac{u^2}{2} = 100$

$u^2 = 200\ m$

$\therefore H = \frac{u^2}{2g} = \frac{200}{2 \times 10} = 10\ m.$

(4)

(i) Given $u = 100\ m/s$

Time of flight $= T = \frac{2u}{g} = 2 \times \frac{100}{10} = 20$ sec.

(ii)

Given $\frac{u_1}{u_2} = \frac{2}{3}$; we know $H \propto u^2$.

$\frac{H_1}{H_2} = \left[\frac{u_1}{u_2}\right]^2 = \left[\frac{2}{3}\right]^2 = \frac{4}{9}$

(14)

Given $H_{max} = 45m$

$u' = \frac{u}{2}$

$H_{max} = \frac{u^2}{2g} \Rightarrow u = \sqrt{2gH}$

$t = \text{Time} = \frac{u}{g} = \frac{30}{10} = 3 \text{ sec.}$

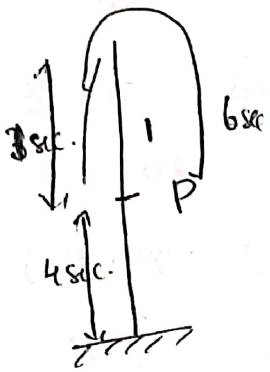
$= \sqrt{2 \times 10 \times 45}$

$= 30 \text{ m/s}$

$\frac{t'}{t} = \frac{u'}{u}$

$\Rightarrow \frac{t'}{3} = \frac{u/2}{u} \Rightarrow t' = \frac{3}{2} = 1.5 \text{ sec}$

(15)



total time of ascent $T = 5$

$\Rightarrow \frac{u}{g} = 5 \Rightarrow u = 5g$

∴ Maximum height reached by it

$H = \frac{u^2}{2g} = \frac{(5g)^2}{2g} = \frac{25g^2}{2g}$

$= \frac{25 \times 10}{2} = 125 m$

(16)

Given $u = 20 m/s$

From $v^2 - u^2 = 2as$

$\Rightarrow v^2 - (20)^2 = 2(-g)\left(\frac{3H}{4}\right)$

$\Rightarrow v^2 - 400 = -2 \times 10 \times 15$

$\Rightarrow v^2 - 400 = -300$

$\Rightarrow v^2 = 100 \Rightarrow v = 10 m/s$

$a = -g : s = H = \frac{u^2}{2g}$

$s = \frac{3}{4} H = \frac{3}{4} \times \frac{u^2}{2g}$

$= \frac{3}{4} \frac{5}{20} \times \frac{20 \times 20}{20}$

$= 15 m$