WS-15 motion in a plane 8th foundation + T bable

(1)

Given $\theta = 45^\circ$, $g = 10\ m/s^2$

Horizontal distance $x = 10\ m$

Vertical distance $y = 7.5\ m$.

From $y = (\tan\theta)x - \frac{g}{2u^2\cos^2\theta}x^2$

$\Rightarrow 7.5 = (\tan 45^\circ)(10) - \frac{10}{2u^2\cos^2 45^\circ} \times 10^2$

$\Rightarrow 7.5 = 10 - \frac{1000}{u^2}$ $\Rightarrow u^2 = \frac{1000}{2.5}$

$\Rightarrow u^2 = 400 \Rightarrow u = 20\ m/s$

$\rightarrow C$

(2)

Given $\theta = 45^\circ$; initial velocity $u = 20\sqrt{2}\ m/s$

velocity after '3' sec $v = u\cos\theta\ \hat{i} + (u\sin\theta - gt)\hat{j}$

$\Rightarrow v = 20\sqrt{2}\cos 45^\circ\ \hat{i} + (20\sqrt{2}\sin 45^\circ - 10\times 3)\hat{j}$

$= 20\sqrt{2}\times\frac{1}{\sqrt{2}}\hat{i} + (20\sqrt{2}\times\frac{1}{\sqrt{2}} - 30)\hat{j}$

$\vec{v} = 20\hat{i} - 10\hat{j} \Rightarrow |\vec{v}| = \sqrt{20^2 + (-10)^2}$

$= 20.62 \rightarrow A$

(3)

let u be initial velocity $\vec{u} = u\cos\theta\hat{i} + u\sin\theta\hat{j}$

velocity after 't' sec $\vec{v} = u\cos\theta\ \hat{i} + (u\sin\theta - gt)\hat{j}$.

Given $\vec{u}$ and $\vec{v}$ are perpendicular to each other

$\Rightarrow \vec{u}\cdot\vec{v} = 0$

$\Rightarrow (u\cos\theta\hat{i} + u\sin\theta\hat{j})\cdot(u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}) = 0$

$\Rightarrow u^2\cos^2\theta + u\sin\theta(u\sin\theta - gt) = 0$

$\Rightarrow$

$\Rightarrow u^2\cos^2\theta + u^2\sin^2\theta - u\sin\theta\, g t = 0$

$\Rightarrow u^2(\cos^2\theta + \sin^2\theta) = u\sin\theta\, g t$

$\Rightarrow u^2 = u\sin\theta\, g t$

$\Rightarrow t = \dfrac{u}{g\sin\theta} \rightarrow B$

(4)

Given initial velocity $u = 20\sqrt{2}$ m/s

angle $\theta = 45^\circ$

At maximum height velocity $\vec{V} = u\cos\theta\,\hat{i}$

$= 20\sqrt{2}\cos 45^\circ\,\hat{i}$

$= 20\,\hat{i}$

$\vec{U} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

$= 20\sqrt{2}\cos45^\circ\,\hat{i} + 20\sqrt{2}\sin45^\circ\,\hat{j}$

$\Rightarrow 20\sqrt{2}\times\frac{1}{\sqrt{2}}\,\hat{i} + 20\sqrt{2}\,\frac{1}{\sqrt{2}}\,\hat{j}$

$\Rightarrow 20\,\hat{i} + 20\,\hat{j}$

$<\text{velocity}> = \dfrac{\vec{U}+\vec{V}}{2} = \dfrac{20\hat{i}+20\hat{j}+20\hat{i}}{2}$

$\Rightarrow \cancel{40\hat{j}}\ \dfrac{40\hat{i}+20\hat{j}}{2} = 20\hat{i}+10\hat{j}$

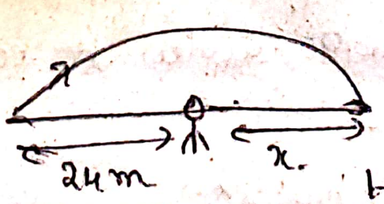
$|\text{velocity}| = \sqrt{20^2+10^2} = \sqrt{500} = 10\sqrt{5}$ m/s $\rightarrow A$

(5)

let u be the initial velocity $= 20$ m/s

For max Range $\theta = 45^\circ$

'x' is distance covered by the man to catch the ball



Here $x = V_m T$

$T \rightarrow$ Time of flight for ball and Time taken by the man to cover 'x' distance same

Range of ball = ~~24+x~~

$\Rightarrow \frac{u^2}{g}$ ~~$= 24 + v_m T$~~

$R \Rightarrow \frac{20^2}{10}$

$\Rightarrow 24 + x = 40$

$\Rightarrow x = 16$

$\Rightarrow v_m \times T = 16$

$T = \frac{2u\sin\theta}{g}$

$= \frac{2 \times 20 \times \sin 45}{g}$

$\Rightarrow \frac{20\sqrt{2}}{g}$

$\Rightarrow v_m \times \frac{20\sqrt{2}}{g} = 16$

$\Rightarrow v_m = \frac{16}{2\sqrt{2}} = 4\sqrt{2}\ m/s$

$\rightarrow B$

(6)

let Given

u be the initial velocity

$\vec{u} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$.

At heighest point velocity $\vec{V} = u\cos\theta\,\hat{i}$

change in velocity $= |\vec{v} - \vec{u}| = |u\cos\theta\,\hat{i} - u\sin\theta\,\hat{j} - u\cos\theta\,\hat{i}|$

$= |-u\sin\theta\,\hat{j}|$

$= u\sin\theta \rightarrow C.$

(7)

let $u_A = V$; $u_B = \frac{V}{2}$: $\theta_B = 45°$.

$\theta_A = ?$

Given Range of A = Range of B

$\Rightarrow \frac{u_A^2 \sin 2\theta_A}{g} = \frac{u_B^2 \sin 2\theta_B}{g}$

$\Rightarrow V^2 \sin 2\theta_A = (\frac{V}{2})^2 \sin 2(45)$

$\Rightarrow \cancel{V^2} \sin 2\theta_A = \frac{\cancel{V^2}}{4} \times \sin 90°$

$\Rightarrow \sin 2\theta_A = \frac{1}{4}$

$\Rightarrow 2\theta_A = \sin^{-1}(\frac{1}{4}) \Rightarrow \theta_A = \frac{1}{2}\sin^{-1}[\frac{1}{4}]$

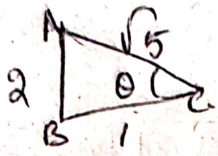
(8)

Given Range $= 2H$

we know $H = \frac{R}{4}\tan\theta$

$\Rightarrow H = \frac{2H}{4_2}\tan\theta$

$\Rightarrow \tan\theta = 2$



$\sin\theta = \frac{2}{\sqrt5}$ $\cos\theta = \frac{1}{\sqrt5}$

$$\text{Range} = \frac{u^2\sin2\theta}{g} = \frac{2u^2\sin\theta\cos\theta}{g}$$

$$= \frac{2\times u^2\times}{g}\frac{2}{\sqrt5}\times\frac{1}{\sqrt5} = \frac{4u^2}{5g}$$

(9)

Here the maximum range is = Area covered by all bullet = Area of Circle formed

$= \pi r^2$.

$r \Rightarrow$ radius of circle formed = Range [Maximum]

$$= \frac{u^2}{g}$$

$\therefore$ Max. Range $= \pi\left(\frac{u^2}{g}\right)^2 \rightarrow A$

(10)

Given Range is maximum $\theta = 45°$

$\therefore$ From $v^2 - u^2 = 2as$ $\quad S = \frac{H}{2} = \frac{u^2\sin^2\theta}{2\,2g}$

$a = -g$ $\quad = \frac{u^2\sin^2 45}{2\times2g} = \frac{u^2}{8g}$

$\Rightarrow v^2 - u^2 = 2(-g)\left(\frac{u^2}{\cancel{8g}_4}\right)$

$\Rightarrow v^2 - u^2 = -\frac{u^2}{4g}$

$\Rightarrow v^2 = \frac{\sqrt3}{2}u$

(2)

At maximum height $v = u\cos\theta$
$= u\cos 45°$
$= \frac{u}{\sqrt{2}}$.

(3)

change in velocity $= 2u\cos\theta$ &
$= 2u\cos 45 = \frac{2u}{\sqrt{2}}$
$= \sqrt{2}u$

(11)

At highest point direction of velocity is horizontal and acceleration is vertically downwards so they are perpendicular. [90°].
Both A & R are false → D

(12)

Here maximum height $H = \frac{u^2\sin^2\theta}{2g}$
so 'H' is independent on mass.

(13)

For $\theta_1 = \theta$ velocity of projection $= \sqrt{u_x^2 + u_y^2}$
$|\vec{u}_1| = \sqrt{u^2\cos^2\theta + u^2\sin^2\theta} =$
For $\theta_2 = 90 - \theta$ velocity of projection
$|\vec{u}_2| = \sqrt{u^2\cos^2(90-\theta) + u^2\sin^2(90-\theta)}$
$= \sqrt{u^2\sin^2\theta + u^2\cos^2\theta}$
$|\vec{u}_1| = |\vec{u}_2|$.

(a) If $R = H$ we know $H = \frac{R}{4}\tan\theta$

$\Rightarrow H = \frac{H}{4}\tan\theta$

$\Rightarrow \tan\theta = 4 \Rightarrow \theta = \tan^{-1}(4)$

(b) Given $y = Px - Qx^2$ compare with

$y = \tan\theta\, x - \frac{g}{2u^2\cos^2\theta}x^2$

$P = \tan\theta$, $Q = \frac{g}{2u^2\cos^2\theta}$

maximum height $= \frac{u_0^2 \sin^2\theta}{2g} = \frac{1}{4}\frac{P^2}{Q}$

(14)

we know Time of flight increases as the maximum height increases. For A and B Time of flight is same.

ii) The ratio $\frac{u_y}{u_x}$ increases as the range decreases

iii) Time of flight of C is lowest but the range is same as that of B, then in less time C covers the same horizontal displacement as that of B, so it's horizontal velocity is maximum

iv) $T = \frac{2u_y}{g}$; $R = u_x T = \frac{2u_x u_y}{g}$

so the Projectile with Maximum Range has max values of u_x, u_y

(16) Given angles of projection. $\theta_1 = 60°$, $\theta_2 = 30°$

So $R_1 = R_2$ [Because θ_1 and θ_2 ~~can~~ are complimentary angles]

$H = \frac{u^2 \sin^2\theta}{2g} \Rightarrow H \propto \sin^2\theta$ As $\theta_1 > \theta_2$

$\Rightarrow \sin\theta_1 > \sin\theta_2$

$H_1 > H_2$

$T = \frac{u \sin\theta}{g} \Rightarrow T \propto \sin\theta$ Clearly $T_1 > T_2$

16

$\therefore \frac{H_1}{R_1} > \frac{H_2}{R_2}$ and $\frac{H}{T} \propto \frac{\sin^2\theta}{\sin\theta} \Rightarrow \frac{H}{T} \propto \sin\theta$

$$\frac{H_1}{T_1} > \frac{H_2}{T_2}$$

(15)

Maximum height and time of flight depends on the vertical component of initial velocity.

$H_1 = H_2 \Rightarrow u_{y_1} = u_{y_2}$ hence $T_1 = T_2$

Range $= \frac{2u_x u_y}{g}$ $\therefore R_2 > R_1$ because $u_{x_2} > u_{x_1}$ (or) $u_2 > u_1$

Time of flight $= \frac{2u\sin\theta}{g} = \frac{2u_y}{g}$

$H_{max} = \frac{u_y^2}{2g}$

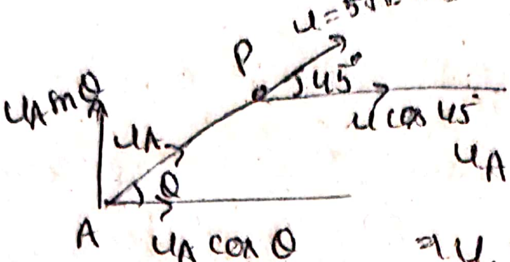
(16), (17), (18)

From PQ = Range $= \sqrt{20^2 + 15^2} = 25$

$\Rightarrow \frac{u^2 \sin 2\theta}{g} = 25 \Rightarrow u = 5\sqrt{10}$ m/s

$\Rightarrow \frac{u^2 \sin 2(45)}{10} = 25$

Scanned with OKEN Scanner



<u>velocity at A</u>

$u_A \cos\theta = u \cos 45^\circ$

$\Rightarrow u_A \cos\theta = 5\sqrt{10} \times \frac{1}{\sqrt{2}} = 5\sqrt{5}$ m/s

From $v^2 - u^2 = 2as$ $\quad a = -g = -10$; $s = 12.5$

$\Rightarrow (u_A \cos\theta)^2 - (u_A \sin\theta)^2 = 2(-10) \times 12.5$

$\Rightarrow (5\sqrt{5})^2 - u_A^2 \sin^2\theta = -250$

$\Rightarrow u_A^2 \sin^2\theta = 125 + 250 = 375$

$\Rightarrow u_A \sin\theta = \sqrt{375} = 5\sqrt{15}$

$$\frac{\cancel{u_A} \sin\theta}{\cancel{u_A} \cos\theta} = \frac{\cancel{5}\sqrt{15}}{\cancel{5}\sqrt{5}} = \sqrt{3}$$

$\tan\theta = \sqrt{3} \Rightarrow \theta = 60^\circ$

$\therefore$ From $u_A \sin\theta = 5\sqrt{15}$

$\Rightarrow u_A \sin 60 = 5\sqrt{15}$

$\Rightarrow u_A \frac{\sqrt{3}}{2} = 5\sqrt{15}$

$\Rightarrow u_A = 10\sqrt{5}$ m/s

<u>Range</u> $= (AB) = \dfrac{u_A^2 \sin 2\theta}{g}$

$$= \frac{(10\sqrt{5})^2 \sin(2 \times 60)}{10}$$

$$= \frac{500 \times \frac{\sqrt{3}}{2}}{10}$$

$= 25\sqrt{3}$ m

LTask

CUQ's

① As the angle of projection increases from $0 \to 90°$ since $R = \frac{u^2 \sin 2\theta}{g}$

$\therefore$ $\sin 2\theta$ values changes from $0 \to 180°$

since $\sin 2\theta$ value increases from $0 \to 90°$ and then decreases from $90° \to 180°$

② As the angle of projection increases from $0 \to 90° \Rightarrow \sin\theta$ value changes from $0° \to 90°$

we know that $H_{max} = \frac{u^2 \sin^2\theta}{2g}$

$\Rightarrow H_{max} \propto \sin^2\theta$

$\therefore$ H_{max} value increases from $0 \to 90°$

③ The path of projectile as seen by another projectile is a straight line because their relative acceleration $= a_1 - a_2 = g - g = 0$

④ Given angles $(45-\theta)$ and $(45+\theta)$ are complementary angles.

For complementary angles ranges are same

$R_1 = \frac{u^2 \sin 2(45-\theta)}{g} = \frac{u^2 \sin(90-2\theta)}{g} = \frac{u^2 \cos\theta}{g}$

$$R_2 = \frac{u^2 \sin 2(45+\theta)}{g} = \frac{u^2 \sin(90+2\theta)}{g}$$

$$R_2 = \frac{u^2 \cos 2\theta}{g} \Rightarrow R_1 = R_2$$

$$R_1 : R_2 = 1 : 1 \to C$$

(6)

Given parabolic equation

$$y = \sqrt{3}\,x - \frac{g}{2} x^2 \cdot \text{ compare with}$$

$$y = \tan\theta\, x - \frac{g}{2u^2\cos^2\theta} x^2$$

$$\Rightarrow \tan\theta = \sqrt{3} \qquad \frac{\cancel{g}}{2u^2\cos^2\theta} = \frac{\cancel{g}}{2}$$

$$\Rightarrow \theta = 60^\circ \qquad u^2\cos^2\theta = 1$$

$$\Rightarrow u^2 = \frac{1}{\cos^2\theta} \Rightarrow u = \frac{1}{\cos\theta}$$

$$\Rightarrow u = \frac{1}{\cos 60^\circ} = \frac{1}{\frac{1}{2}} = 2 \text{ m/s} \to A$$

(7)

Given velocity of Projection $u = 19.6$ m/s

$\theta = 30^\circ$

$$\text{Time of flight} = \frac{2u\sin\theta}{g} = \frac{2 \times \overset{2}{\cancel{19.6}} \times \sin 30}{\cancel{9.8}}$$

$$T = 2 \times 2 \times \frac{1}{2} = 2 \text{ sec} \to B$$

(8)

Given velocity of Projection $u = 200$ m/s

$\theta = 30^\circ$ with vertical. so with horizontal

Angle of Projection $\theta' = 60^\circ$

in $t = 3$ sec

horizontal distance $= x = u\cos\theta\, t$

$$\Rightarrow x = 200 \times \cos 60 \times 3 = 300 \text{ m} \to A$$

(9)

Given velocity of Projection $u = 20$ m/s

$\theta = 60^\circ$

velocity of vector $\vec{u} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

$\vec{u} = 20\cos 60\,\hat{i} + 20\sin 60\,\hat{j}$

$= 20 \times \frac{1}{2}\,\hat{i} + 20\frac{\sqrt{3}}{2}\,\hat{j}$

$\vec{u} = 10\hat{i} + 10\sqrt{3}\,\hat{j} \rightarrow 3$

(10)

we know $T = \frac{2u\sin\theta}{g}$; $H = \frac{u^2\sin^2\theta}{2g}$

$\therefore H \propto T^2$

As time of flight doubled

$H' \propto (2T)^2 \Rightarrow H' \propto 4T^2$

$\Rightarrow$ ~~4H~~

Maximum height attained becomes four

times $\rightarrow$ D

(11)

Given $y = \frac{x}{\sqrt{3}} - \frac{x^2}{60}$ compare with

$y = \tan\theta\, x - \frac{g}{2u^2\cos^2\theta} x^2$

$\therefore \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^\circ \rightarrow A$

(12)

Given $\theta = 30^\circ$ velocity $u = 50$ m/s

Time of flight $T = \frac{2u\sin\theta}{g} = \frac{2 \times 5\cancel{0} \times \sin 30}{\cancel{10}}$

$= \cancel{2} \times 5 \times \frac{1}{\cancel{2}} = 5$ sec $\rightarrow$ B

(13)

Given initial velocity $u = 60$ m/s & $\theta = 30°$

After $t = 3$ sec velocity

$$\vec{v} = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\hat{j}$$

$$\Rightarrow \vec{v} = 60\cos 30\,\hat{i} + (60\sin 30 - 10\times 3)\hat{j}$$

$$= \cancel{60}^{30}\times\frac{\sqrt{3}}{\cancel{2}}\,\hat{i} + (\cancel{60}^{30}\times\frac{1}{\cancel{2}} - 30)\hat{j}$$

$$= 30\sqrt{3}\,\hat{i} + (30-30)\hat{j}$$

$$\vec{v} = 30\sqrt{3}\,\hat{i} \rightarrow D$$

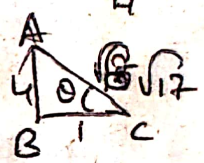
(14)

Given

$$H = R$$

From $H = \frac{R}{4}\tan\theta \Rightarrow R = \frac{R}{4}\tan\theta$

$$\Rightarrow \tan\theta = 4$$



$$\sin\theta = \frac{4}{\sqrt{17}}$$

$$H = \frac{u^2\sin^2\theta}{2g} = \frac{u^2}{2g}\left[\frac{4}{\sqrt{17}}\right]^2$$

$$H = \frac{u^2}{2g}\times\frac{16}{17} = \frac{8u^2}{17g} \rightarrow D$$

(15)

Initial kinetic energy of the ball $= k$

$$k = \frac{1}{2}mu^2$$

Angle of Projection $\theta = 60°$

K.E at the top of the trajectory

$$= \frac{1}{2}mu^2\cos^2\theta$$

$$= \frac{1}{2}m u^2\cos^2 60°$$

$$\Rightarrow k\times\left(\frac{1}{2}\right)^2$$

$$= \frac{k}{4} \rightarrow C$$

(16)

$V_{max} = 20\,m/s^{-1}$

Dangerous distance = Maximum Range

$= \frac{u^2}{g} = \frac{(20)^2}{g} = \frac{400}{10} = 40\,m$

$\rightarrow D$

(17)

$H = 10m.$ Range = maximum means.

$\Rightarrow \frac{u^2 \sin^2\theta}{2g} = 10$

$\Rightarrow u^2 (\sin\theta)^2 = 2g \times 10$

$\Rightarrow u \sin\theta = \sqrt{2} \times 10$

$\Rightarrow u \sin 45 = 10\sqrt{2}$

$\Rightarrow u \frac{1}{\sqrt{2}} = 10\sqrt{2}$

$\Rightarrow u = 20\,m/s$

$\theta = 45^\circ$

$R = \frac{u^2}{g} = \frac{(20)^2}{10}$

$=$

(17)

$h_{max} = \frac{u^2}{2g} = 10 \Rightarrow \theta = 90^\circ$ for maximum height

$\Rightarrow$ Range $= \frac{u^2}{g} = 2\,h_{max}$

$= 2 \times 10 = 20m \rightarrow D$

(18)

here grass hopper can jump maximum horizontal distance = Maximum Range = 0.3 m

$\theta = 45^\circ \qquad R = 0.3$

$\frac{u^2}{g} = 0.3 \Rightarrow u^2 = 0.3g$

$\Rightarrow u = \sqrt{3}\,m/s$

Horizontal component of velocity $= u\cos\theta$

$= \sqrt{3}\cos 45 = \frac{\sqrt{3}}{\sqrt{2}}$

$= \sqrt{\frac{3}{2}} \rightarrow B$

(19)

$\theta_1 = \theta$, $\theta_2 = 90 - \theta$

$H_1 = 20m$ $\quad$ $H_2 = 10m$

$\Rightarrow \frac{u^2 \sin^2\theta}{2g} = 20m \rightarrow ①$ $\quad$ $\frac{u^2 \sin^2(90-\theta)}{2g} = 10$

$\Rightarrow \frac{u^2 \cos^2\theta}{2g} = 10m \rightarrow ②$

① + ②

$\therefore \frac{u^2 \sin^2\theta}{2g} + \frac{u^2 \cos^2\theta}{2g} = 20+10$

$\Rightarrow \frac{u^2}{2g}[\sin^2\theta + \cos^2\theta] = 30$

$\Rightarrow \frac{u^2}{2g} = 30 \Rightarrow \frac{u^2}{g} = 2\times30$

$\Rightarrow R_{max} = 60m \rightarrow A$

(8), (9), (10)

Given $u = 60m/s$; $\theta = 30°$

Initial velocity vector $\vec{u} = u\cos\theta\hat{i} + u\sin\theta\hat{j}$

$\vec{u} = 60\cos 30\hat{i} + 60\sin 30\hat{j}$

$= 60\left[\frac{\sqrt3}{2}\hat{i} + \frac{1}{2}\hat{j}\right] \Rightarrow \frac{60}{2}[\sqrt3\hat{i}+\hat{j}]$

$= 30[\sqrt3\hat{i}+\hat{j}] \rightarrow C$

velocity after '3' sec

$\vec{v} = (u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$

$= 60\cos30°\hat{i} + (60\sin30 - 10\times3)\hat{j}$

$= 60\times\frac{\sqrt3}{2}\hat{i} + (60\times\frac{1}{2} - 30)\hat{j}$

$\Rightarrow 30\sqrt3\hat{i} + (30-30)\hat{j} = 30\sqrt3\hat{i}$

displacement after 2 sec

$\vec{r} = u\cos\theta t\,\hat{i} + (u\sin\theta t - \frac{1}{2}gt^2)\hat{j}$

$= 60\cos30\times2\hat{i} + (60\sin30\times2 - \frac{1}{2}\times10\times2^2)\hat{j}$

$\Rightarrow 60\sqrt3\hat{i} + 40\hat{j}$