3. QUADRATIC EXPRESSIONS-I

Class: IX, MATHEMATICS

FOUNDATION+ SOLUTIONS

2025-26

TEACHING TASK

JEE MAINS LEVEL

01. $2x^2+3x+5=0$

$\Rightarrow \alpha+\beta=-\frac{3}{2}$; $\alpha\beta=\frac{5}{2}$

$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$

$=\left(-\frac{3}{2}\right)^3-3\left(\frac{5}{2}\right)\left(\frac{-3}{2}\right)$

$=\frac{-27}{8}+\frac{45}{4}$

$=\frac{63}{8}$

Ans:—

02. $f(x)=x^2+2x+3=0$

$f(x-2)=(x-2)^2+2(x-2)+3$

$=x^2-2x+3$

$\Rightarrow x=1\pm\sqrt{2}i$

Ans:—

03. $(x-3)(x-4)=0$

$\Rightarrow x^2-7x+12=0$

$\Rightarrow a=1,\ b=-7,\ c=12$

$ax^2+(b-7a)x+(c+2a)=0$

$\Rightarrow x^2-14x+0=0 \Rightarrow x(x-14)=0$

$\Rightarrow x=0$ or 14

Ans:

04 $ax^2 - 4x + 1 = 0$

$\Delta = (-4)^2 - 4 \cdot a \cdot 1 = 0$

$\Rightarrow a = 4$

Ans: A

05 $x^2 + 4x + c = 0$.

Let the roots be $\alpha, \alpha - 2$

$\therefore \alpha + (\alpha - 2) = -4$

$\Rightarrow \alpha = -1$

Now, $\alpha(\alpha - 2) = c$

$\Rightarrow (-1)(-1-2) = c \Rightarrow c = 3$

Ans: A

06 $\frac{1}{\alpha^3} + \frac{1}{\beta^3} = \frac{\alpha^3 + \beta^3}{(\alpha\beta)^3} = \frac{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)}{(\alpha\beta)^3}$

$= \frac{\left(\frac{-b}{a}\right)^3 - 3\left(\frac{c}{a}\right)\left(\frac{-b}{a}\right)}{\left(\frac{c}{a}\right)^3} = \frac{3abc - b^3}{c^3}$

Ans: C

07 $x^2 - (5m-2)x + (4m^2 + 10m + 25) = 0$

perfect square $\Rightarrow \Delta = 0$

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow [-(5m-2)]^2 - 4 \cdot 1 \cdot (4m^2 + 10m + 25) = 0$

$\Rightarrow m = -\frac{4}{3}$ or 8

Ans: C

08 $(a^2+b^2)x^2 + 2(bc+ad)x + (c^2+d^2) = 0$

Roots are equal $\Rightarrow \Delta \geq 0$

$\Rightarrow [2(bc+ad)]^2 - 4(a^2+b^2)(c^2+d^2) = 0$

$\Rightarrow ac = bd$

$\Rightarrow (bd)^2 = a^2 \cdot c^2$

$\Rightarrow a^2, bd, c^2$ are in G.P. Ans: B

09 $a^{\sqrt{x}} = (\sqrt{x})^x$

$\Rightarrow x^{\sqrt{x}} = x^{x/2}$

$\Rightarrow \sqrt{x} = \frac{x}{2}$

$\Rightarrow x = \frac{x^2}{4}$

$\Rightarrow x\left(1 - \frac{x}{4}\right) = 0$

$\Rightarrow x = 0$ or $\boxed{x = 4}$

$x = 0$ is not possible

Also, by observation $\boxed{x = 1}$ is a root

$\therefore x \in \{1, 4\}$ Ans. C

10. $(a+b+c)^2 - 2(a+c)x + (a-b+c) = 0$

$\because \Delta = b^2 - 4ac$

$= [-2(a+c)]^2 - 4(a+b+c)(a-b+c)$

$= 4(a^2+c^2+2ac) - 4(a^2 - ab + ac + ba - b^2 + bc + ca - cb + c^2)$

$= 4b^2 > 0$

Hence, the roots are rational Ans: C

JEE ADVANCED LEVEL

01. $x^2 + 3x + 2 = 0 \Rightarrow \alpha + \beta = -3, \quad \alpha\beta = 2$

$\frac{1}{\alpha}, \frac{1}{\beta} \Rightarrow 2x^2 + 3x + 1 = 0$ Ans: A, B, D

02 $\frac{a+b}{2} = 9$ and $\sqrt{ab} = 4$ ④

$\Rightarrow a+b = 18$ and $ab = 16$

$\therefore x^2 - 18x + 16 = 0$ (or)

$2x^2 - 36x + 32 = 0$

Ans: B, C

---

03 Statement I: $f(x) = (x-3)^2$

$f(3) = (3-3)^2 = 0$ (True)

Statement II: Conceptual (True)

Ans: A

---

04 ~~Statement I~~

$(b-c)x^2 + 2(c-a)x + (a-b) = 0$

$\Delta = b^2 - 4ac$

$= [2(c-a)]^2 - 4(b-c)(a-b)$

$= 4[c^2 - 2ac + a^2 - ab + b^2 + ac - bc]$

$= 4[a^2 + b^2 + c^2 - ab - bc - ca]$

$= 2[2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca]$

$= 2[(a-b)^2 + (b-c)^2 + (c-a)^2] \geq 0$

$\therefore$ Roots are real and distinct (True)

Statement II: Conceptual (True)

Ans: B

---

05 Assertion: Conceptual (True)

Reason: Conceptual (True)

Ans: A

---

06. **Assertion:** x^2+2x+5

$a=1>0$, $b^2-4ac=-16<0$.

Hence, $x^2+2x+5>0$, $\forall x \in R$ (True)

Reason: Conceptual (False)

Ans: C

07. $ax^2+bx+c=0$

$\Rightarrow$ Roots are opposite in sign

$\Rightarrow$ product of the roots <0

Ans: B

08 A) $x^2-4=0 \Rightarrow x=\pm 2$

Ans: A

09 $\alpha=2+\sqrt{3}$, $\beta=2-\sqrt{3}$

$\therefore x^2-4x+(4-3)=0$

$\Rightarrow x^2-4x+1=0$

Ans: B

10 $x^2-\left(-\frac{2}{3}+\frac{3}{7}\right)x+\left(-\frac{2}{3}\times\frac{3}{7}\right)=0$

$\Rightarrow 21x^2+5x-6=0$

Ans: D

11. $x^2+px+q=0$

Roots are in the ratio $=2:3$

let the roots be $2\alpha, 3\alpha$

Given $2\alpha \times 3\alpha = 54$

$\Rightarrow \alpha = 3$

Also, sum $= 6+9=-p$

$\therefore p=-15$

Roots are 6, 9

Ans: -15

12. $x^2 - 6x + q = 0$

$|\alpha^2 - \beta^2| = (\alpha+\beta)(\alpha-\beta) = 24$

$\Rightarrow (\alpha+\beta)\sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = 24$

$\Rightarrow 6\sqrt{(6)^2 - 4(q)} = 24$

$\Rightarrow q = 5$

Ans: 5

---

13 Given $ax^2+bx+c=0 \rightarrow$ Roots α, β

a) $cx^2+bx+a=0 \rightarrow$ roots $\frac{1}{\alpha}, \frac{1}{\beta}$

b) $ax^2-bx+c=0 \rightarrow$ roots $-\alpha, -\beta$

c) $a\left(\frac{x}{2}\right)^2+b\left(\frac{x}{2}\right)+c=0 \rightarrow$ roots $2\alpha, 2\beta$

d) $a(2x)^2+b(2x)+c=0 \rightarrow$ roots $\frac{\alpha}{2}, \frac{\beta}{2}$

Ans: t, s, r, q

---

14 a) $f(x) = x^2-4x+3=0 \Rightarrow x = 1$ or 3

b) $f(x-2)=0$

Roots are $1+2, 3+2 \Rightarrow 3, 5$

c) $f(2x)=0$

Roots are $\frac{1}{2}$ or $\frac{3}{2} \Rightarrow 0.5$ or 1.5

d) $f(-x)=0$

roots are -1 and -3.

Ans: p, q, r, s

LEARNERS TASK
CUQ'S

01. $x^2 - 6x + 10 = 0 \Rightarrow \Delta < 0$ — Ans: C

02 Conceptual — Ans: D

03 $\alpha, \beta \rightarrow ax^2 + bx + c = 0$

$\frac{1}{\alpha}, \frac{1}{\beta} \rightarrow cx^2 + bx + a = 0$ — Ans: A

04 Conceptual — Ans: A

05 $x^2 + 2x + 5 = 0 \Rightarrow \Delta < 0$ — Ans: C

06 $f(x) = x^2 + 11x + 13 = 0$

$f(x+4) = (x+4)^2 + 11(x+4) + 13 = 0$

$\Rightarrow x^2 + 19x + 73 = 0$ — Ans: B

07 $3 - 2i$ — Ans: A

08 α^2, β^2 — Ans: B

09. $\Delta = a^2 - 4bc$ — Ans: C

10 $ax^2 + bx + c = 0$

$\alpha + \beta = -\frac{b}{a}$ | $\alpha \cdot \beta = \frac{c}{a}$

$\Rightarrow \alpha + \alpha = \frac{-b}{a}$ | $\Rightarrow \alpha \cdot \alpha = \frac{c}{a}$

$\Rightarrow \alpha = \frac{-b}{2a}$ | $\Rightarrow \alpha = \sqrt{\frac{c}{a}}$

Ans: D

JEE MAINS LEVEL

01. $x^2+4x+K=0$

$\Rightarrow$ No real roots

$\Rightarrow b^2-4ac < 0$

$\Rightarrow 16-4K < 0$

$\Rightarrow K > 4$

Ans: A

02 $f(x) \rightarrow \alpha, \beta$

$f(x+2) \rightarrow \alpha-2, \beta-2$

Ans: B

03 Given $x^2+px+1=0$

Let $\alpha, \frac{1}{\alpha}$ be the roots

We have $\alpha+\frac{1}{\alpha} = -p$

We have $\alpha+\frac{1}{\alpha} \geq 2$ or $\alpha+\frac{1}{\alpha} \leq -2$

$\therefore -p \geq 2$ or $-p \leq -2$

$\Rightarrow p \leq -2$ or $p \geq -2$

$\Rightarrow p = -2$

Ans: —

04 $(x-1)^2=0 \Rightarrow x=1$

Ans: B

05 NOTE: Please, change the question.

"If $ax^2+bx+c \geq 0$, [illegible] and $b^2-4ac<0$, then the roots are

A) Not real B) Equal C) Not equal D) None

Ans: A

06 $f(x) = ax^2 + bx + c = 0$

Increasing each root 1, new equation is

$f(x-2) = a(x-1)^2 + b(x-1) + c = 0$

$\Rightarrow ax^2 + (b-2a)x + a - b + c = 0$

$\Rightarrow 2x^2 + 8x + 2 = 0$

$\boxed{a = 2}$ | $b - 2a = 8$ $\Rightarrow \boxed{b = 12}$ | $a - b + c = 2$ $\boxed{c = 12}$

$\therefore b = c$

Ans: C

07 $x^2 + px + 12 = 0$

$\Rightarrow 4^2 + p(4) + 12 = 0$

$\Rightarrow p = -7$

$x^2 - 7x + q = 0$

Equal Roots $= \Delta = 0$

$\Rightarrow 49 - 4q = 0$

$\Rightarrow q = \frac{49}{4}$

Ans: D

08 $x^{\frac{2}{3}} + x^{1/3} - 2 = 0$

Let $x^{1/3} = a$

$\Rightarrow a^2 + a - 2 = 0$

$\Rightarrow (a+2)(a-1) = 0$

$\Rightarrow a = 1$ or -2

$\Rightarrow$

$x^{1/3} = 1$ $\Rightarrow x = 1$ | $x^{1/3} = -2$ $\Rightarrow x = -8$

$\therefore$ Two roots

Ans: B

09 $f(x) = 4x^2 + 7x + 2 = 0$

$f(\sqrt{x}) = 4(\sqrt{x})^2 + 7(\sqrt{x}) + 2 = 0$

$\Rightarrow 4x + 2 = -7\sqrt{x}$

S.O.B.S and simplifying

$\Rightarrow 16x^2 - 33x + 4 = 0$

Ans: D

10. $x^2+4x+3=0 \Rightarrow x=-1,-3$ Ans: B

JEE ADVANCED LEVEL

01. $3x^2+2x+1=0$

$\alpha+\beta=-\frac{2}{3}$

$\alpha\beta=\frac{1}{3}$

$\Delta = 4-12=-8<0$

Graph never intersects the X-axis Ans: A, B, D

02 $x^2+x-3=0$

$\Delta=(1)^2-4(1)(-3)=13>0$, 13 is not a perfect square Ans: C, D

03 Statement I: Conceptual (true)

Statement II: $x^2+3x=0$

$\Rightarrow (0)^2+3(0)=0$ (true) Ans: A

04 Statement I: $f(x)=ax^2+bx+c$

$f(-x)=ax^2-bx+c=0$ (true)

Statement II: $x^2-3x+4=0$

$\alpha+\beta=3$ (true) Ans: C

05 Assertion: Conceptual (true)

Reason: Conceptual (true) Ans: A

06. Assertion: $(x-1)^2=0 \Rightarrow x=1,1$ (False)

Reason: Conceptual (true) Ans: D

67. $f(x) = x^2 - 5x + 6 = 0$

$f(x-2) = (x-2)^2 - 5(x-2) + 6 = 0$

$\Rightarrow x^2 - 9x + 20 = 0$

$\Rightarrow (x-4)(x-5) = 0$

$\Rightarrow x = 4$ or 5

Ans:

08 New equation $\Rightarrow x^2 - 9x + 20 = 0$ Ans: A

09 $x^2 + x + 3 = 0 \Rightarrow \Delta < 0$ Ans: C

10. conceptual for all options $\Delta = 0$ Ans: D

11. $x^2 + 2x + a = 0$

Let the roots be $\alpha, 2\alpha$

$\therefore \alpha + 2\alpha = -2$

$\Rightarrow \alpha = -\frac{2}{3}$

$\Rightarrow 2\alpha = -\frac{4}{3}$

product: $\alpha \cdot 2\alpha = a$

$\Rightarrow 2\left(-\frac{2}{3}\right)^2 = a$

$\Rightarrow a = \frac{8}{3} = 2.6$

$\therefore$ The integral value of $a = 2$ Ans: 2

12 $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (5)^2 - 2.6$

$= 25 - 12$

$= 13$ Ans: 13

13 a) $\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$= \left(-\frac{b}{a}\right)^3 - 3\left(\frac{c}{a}\right)\left(-\frac{b}{a}\right)$

$= \frac{3abc - b^3}{a^3}$

b) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \frac{b^2-2ac}{a^2}$

c) $|\alpha-\beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \frac{\sqrt{b^2-4ac}}{|a|}$

Ans: t, p, s, q

d) $\alpha+\beta = -\frac{b}{a}$

14 Given α, β are roots of $f(x)=0$

a) $k\alpha, k\beta \Rightarrow f\left(\frac{x}{k}\right)=0$

b) $\frac{\alpha}{k}, \frac{\beta}{k} \Rightarrow f(kx)=0$

c) $\alpha^2, \beta^2 \Rightarrow f(\sqrt{x})=0$

d) $\alpha+k, \beta+k \Rightarrow f(x-k)=0$

Ans: s, p, q, r

$\Rightarrow$ THE END $\Leftarrow$