

7. LOGARITHMIC

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FORMULAE, CHARACTERISTICS

AND MANTISSA

2025/26

Class: VII, MATHEMATICS

FOUNDATION[†] SOLUTIONS

TEACHING TASK

TEE MAINS LEVEL

01 $0.001 = 10^x \Rightarrow \frac{1}{1000} = 10^x$
 $\Rightarrow 10^{-3} = 10^x \Rightarrow x = -3$ Ans. A

02 $\log_{(a^2-b^2)}(a^2-b^2) = 1$ Ans. A

03 $0.0001 = 10^x$ (or) $10^{-4} = 10^x$ Ans. A

04 $\log_3 [\log_3 (\log_5 b)] = 0$
 $\Rightarrow \log_3 (\log_5 b) = 3^0 = 1$
 $\Rightarrow \log_5 b = 3 \Rightarrow b = 5^3 = 125$ Ans. B

05 $\log_{10}(\log_2(x^2 - 3x)) = 0$

(2)

$$\Rightarrow \log_2(x^2 - 3x) = 10^0 = 1$$

$$\Rightarrow x^2 - 3x = 2$$

$$\Rightarrow x^2 - 3x - 2 = 0$$

$$x = \frac{3 \pm \sqrt{17}}{2}$$

Ans: A

06 $\log_{10} x = -3.5196$

$$= -3 - 0.5196$$

$$= -3 - 1 + 1 - 0.5196$$

$$= -4 + 4.804$$

$$= 4.804$$

Ans: C

07 $2 \log 5\sqrt{5} + 3 \log 2 - \log 2 - \log 50 - \log 3$

$$= \log(5\sqrt{5})^2 + \log 2^3 - \log 2 - \log 50 - \log 3$$

$$= (\log 75 + \log 8) - (\log 2 + \log 50 + \log 3)$$

$$= \log\left(\frac{75 \times 8}{2 \times 50 \times 3}\right) = \log 2$$

Ans: D

08 $\log_3 27 = a \Rightarrow 27 = 3^a \Rightarrow a = 3$

$$\log_2 16 = \log_2 2^4 = 4$$

opt: A: $6 \times \left(\frac{3+a}{3a}\right) = 6 \times \frac{6}{9} = 4$

Ans: A

09 Let $N = 3^{40}$

(3)

$$\Rightarrow \log N = 40 \times \log 3$$

$$= 40 \times 0.477$$

$$= 19.08$$

$\therefore$ Characteristic of N is 19

Hence, N contains 20 digits

Ans. A

10 $a^x = b^y = c^z = d^w$

$$\Rightarrow a^x = b^y$$

$$\Rightarrow x \log a = y \log b$$

$$\Rightarrow \frac{\log a}{\log b} = \frac{y}{x}$$

$$\Rightarrow \frac{\log b}{\log a} = \frac{x}{y}$$

$$\Rightarrow \log_a b = \frac{x}{y}$$

Similarly

$$\log_a c = \frac{x}{z}, \log_a d = \frac{x}{w}$$

$$\therefore \log_a bcd = \log_a b + \log_a c + \log_a d$$

$$= x \left(\frac{1}{y} + \frac{1}{z} + \frac{1}{w} \right)$$

Ans. C

JEE ADVANCED LEVEL

01. $\log_5 x^2 + 9 = 2 \Rightarrow x^2 + 9 = 5^2 \Rightarrow x = \pm 4$

Ans. A, C

02 $\log_{\sqrt{2}} 15 = 2 \log_{2^{1/2}} 15 = \frac{1}{2} \log_2 15$

$$= 2 \log_2 15 = \log_2 (15)^2 = (15)^2 = 225 \text{ (or)}$$

Ans. A, B

(or) $\sqrt[3]{(225)^3}$

03 statement I:

(4)

$$\sqrt[5]{a^x} = y \Rightarrow a^{\frac{x}{5}} = y$$

$$\Rightarrow \frac{x}{5} \log a = \log y$$

$$\Rightarrow x \log a = 5 \log y \text{ (True)}$$

statement II: Conceptual (True)

Ans. A

04 Statement I: Conceptual (False)

Statement II: Conceptual (True)

Ans: D

05 Assertion: $\log_{10} 10000 = \log_{10} (10)^4 = 4$ (True)

Reason: Conceptual (False)

Ans. C

06 Assertion: Conceptual (True)

Reason: Conceptual (True)

Ans. A

07 $\log_{abc} a^2 + \log_{abc} b^2 + \log_{abc} c^2$

$$= \log_{abc} (abc)^2 = 2$$

Ans. A

08 $\log_b a \cdot \log_c b \cdot \log_d c \cdot \log_a d$

$$= \frac{1}{-1} \cdot \log_b a \cdot \log_c b \cdot \log_d c \cdot \log_a d$$

$$= -1 \cdot \log_a a$$

$$= -1$$

Ans. C

09 $\log_{10} 10 = 1.0000$

Ans: A

(or) $\log_{10} 2 + \log_{10} 5 = 0.3010 + 0.6990$
 $= 1.0000$

Ans: A

10 option: B.

Ans: B

11 $\log_x 2^{-1/3} = -\frac{1}{3} \Rightarrow -\frac{1}{2} \cdot \log_x 2 = -\frac{1}{3}$

Ans: 2

$\Rightarrow \log_x 2 = 1 \Rightarrow 2 = x$

12 $\log_2 \{ \log_3 (\log_2 x) \} = 1$

$\Rightarrow \log_3 (\log_2 x) = 2$

$\Rightarrow \log_2 x = 3^2 = 9$

$\Rightarrow x = 2^9 = 512$

Ans: 512

13 a) $\log_x \sqrt[3]{x} = a \Rightarrow \sqrt[3]{x} = x^a$

$\Rightarrow x^{1/3} = x^a \Rightarrow a = \frac{1}{3}$

$\therefore \sqrt{a} = \sqrt{\frac{1}{3}} (r)$

Ans: ~~1/3~~

b) $\sqrt[4]{a} = a^x \Rightarrow a^{1/4} = a^x \Rightarrow x = \frac{1}{4}$

$\sqrt[3]{x} = \sqrt[3]{\frac{1}{4}} (t)$

$$c) \sqrt[m]{a} = a^m$$

(6)

$$\Rightarrow m = \frac{1}{3} \Rightarrow m^2 = \frac{1}{9} (S)$$

$$d) \sqrt[5]{x} = x^{2a} \quad \left| \quad a^2 = \frac{1}{100} (P) \right.$$

$$\Rightarrow 2a = \frac{1}{5}$$

$$\Rightarrow a = \frac{1}{10}$$

Ans: S, T, S, P

14 a) $\log_{(0.2)} (0.2)^3 = 3 (S)$

b) $\log_{2^3} 2 = \frac{1}{3} (P)$

c) $\frac{\log 10 \times \log 2^4}{\log 2} = 4 \log 10 (C)$

d) $\log_{\sqrt[6]{6}} 216 = \log_{6^{1/2}} 6^3 = \frac{(3)}{(\frac{1}{2})} \log_6 6 = 3 \times 2 = 6 (S)$

Ans: S, P, F, S

LEARNERS TASK

QUR'S

01 $a = 2^2 = 4 \Rightarrow \log_4 4 = \log_4 4 = 1$

Ans: B

02 $3^{-2} = \frac{1}{9}$

$\Rightarrow -2 = \log_3 \left(\frac{1}{9} \right)$

Ans: D

03 $\left(\frac{1}{x} \right) = \left(\frac{1}{y} \right)^{\left(\frac{1}{z} \right)} \Rightarrow x^{-1} = \left(\frac{1}{y} \right)^{z^{-1}} \Rightarrow \frac{1}{x} = \left(\frac{1}{y} \right)^{z^{-1}}$

Ans: B, C, D

Ans: B, C, D

$$04 \quad \log_5 5^2 = 2 \cdot \log_5 5 = 2$$

Ans. B (9)

$$05 \quad 7^{\log_7 6} = \frac{1}{6}$$

Ans. B

$$06 \quad \log_a 1 = 0 \Rightarrow a \neq 1$$

Ans. C

07 Conceptual

Ans. B

$$08 \quad -6 + 0.325$$

Ans. A

09 All the options correct

Ans. D

$$10. \quad \log_{\sqrt[1/3]{6}} \sqrt[1/5]{6} = \frac{3}{5} \log_6 6$$

Ans. D

JEE MAINS LEVEL

$$01. \quad 6^x = 7^{x+4}$$

$$\Rightarrow x \log 6 = (x+4) \log 7$$

$$\Rightarrow x(\log 6 - \log 7) = 4 \log 7$$

$$\Rightarrow x = \frac{4 \log 7}{\log 6 - \log 7} = \frac{4 \log 7}{\log 2 + \log 3 - \log 7} = \frac{4C}{0+B-C}$$

Ans. B

$$02 \quad (\log_3 4 \times \log_4 5) \times (\log_5 6 \times \log_6 7) \times (\log_7 8 \times \log_8 9)$$

$$= \log_3 5 \times \log_5 7 \times \log_7 9$$

$$= \log_3 9 = 2$$

Ans. A

03

$$\text{let } N = x^x \cdot y^y \cdot z^z$$

(8)

$$\log N = x \log x + y \log y + z \log z$$

$$= x K(y-x) + y K(z-x) + z \cdot K(x-y)$$

$$= 0 \quad \therefore N = 1$$

Ans: A

04

$$a = \log_{12} 18 = \frac{\log_2 18}{\log_2 12} = \frac{\log_2 2 + \log_2 9}{\log_2 4 + \log_2 3}$$

$$= \frac{1 + 2\log_2 3}{2 + \log_2 3}$$

$$\text{let } \log_2 3 = t$$

$$\therefore a = \frac{1+2t}{2+t}$$

$$\Rightarrow t = \frac{2a-1}{2-a} \rightarrow (1)$$

$$\text{Now, } b = \log_{24} 54 = \frac{\log_2 54}{\log_2 24}$$

$$= \frac{1 + 3\log_2 3}{3 + \log_2 3}$$

$$b = \frac{1+3t}{3+t}$$

$$\Rightarrow t = \frac{3b-1}{3-b} \rightarrow (2)$$

$$\therefore \frac{2a-1}{2-a} = \frac{3b-1}{3-b}$$

$$\Rightarrow ab + 5(a-b) = 1$$

Ans: C

05

$$a^2 + b^2 = 7ab$$

$$\Rightarrow a^2 + b^2 + 2ab = 9ab$$

$$\Rightarrow (a+b)^2 = 9ab$$



$$\Rightarrow 2 \log(a+b) = \log 9 + \log ab$$

(9)

$$\Rightarrow 2 \log(a+b) = 2 \log 3 + \log ab$$

$$\Rightarrow 2 \left[\log \left(\frac{a+b}{3} \right) \right] = \log ab$$

$$\Rightarrow \log \left(\frac{a+b}{3} \right) = \frac{1}{2} \cdot \log ab$$

Ans. B

06 $a = b^x, b = c^y, c = a^z$

~~xyz~~ $\log_a b \cdot \log_b c \cdot \log_c a$

$$\Rightarrow xyz = 1$$

Ans. A

07

$$\log_{49} \sqrt{7} = p$$

$$\Rightarrow \frac{\left(\frac{1}{7} \right)}{2} = p \Rightarrow p = \frac{1}{14}$$

$$\Rightarrow \log_{7^2} 7^{\frac{1}{2}} = p$$

Ans. B

08

$$\log_{10} x = a \Rightarrow x = 10^a$$

$$10^{3a-2} = \frac{(10^a)^3}{10^2} = \frac{x^3}{100}$$

Ans. B

09

$$m+n = \log_{100} n + \log_{100} m = \log_{100} mn$$

Ans. D

10

$$\log_{\sqrt{y^2}} z^2 = \log_{x^2} z^2$$

$$\Rightarrow y = x$$

(logarithmic form)

$$\begin{aligned} x^2 + y^2 + xy \\ = y^2 + y^2 + y^2 \\ = 3y^2 \end{aligned}$$

Ans. B

JEE ADVANCED LEVEL

(10)

01. $(a+b)^2 = a+b$ | $a^3 + b^3 = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$
 $\Rightarrow a+b=1$ | $= \sqrt[3]{\frac{1}{64}}$
 $\Rightarrow a=\frac{1}{2}, b=\frac{1}{2}$ |
 ans: B, D

02 Conceptual and mathematically.
 options: A, B, C are false
 Ans: A, B, C

03 Statement - I: Conceptual (True)
Statement - II: Conceptual (True)
 Ans: A

04 Statement I: $\log_{10} 10^3 = 3$ (True)
Statement II: characteristic of
 $\log_{10}(1000) = 3$ (~~True~~) (False) | Ans: C

05 Assertion: mathematically (True)
Reason: Conceptually (True)
 Ans: A

06 Assertion:
 $\log_{10} 4 = 2 \cdot \log_{10} 2 = 2 \times (0.3010) = 0.6020$
 (True)
Reason: mathematically (True)
 Ans: A

07 characteristic $= (n-5) - 1 = n-6$
 Ans: C

08 characteristic $= (n+1) - 1 = n$
 Ans: A

09. $\log_{10}(45-72) \rightarrow \text{Characteristic} = 1$

Ans: A

(11)

10. 4712

Ans: C

11. $\log_{2\sqrt{3}} 1728 = \log_{2\sqrt{3}} 2^6 \times 3^3 = \log_{(2\sqrt{3})} (2\sqrt{3})^6 = 6$

Ans: 6

12. 9085

Ans: 9085

13 a) $x \log a = y \log b$

$\Rightarrow \frac{x}{y} = \log_a^b(t)$

b) $\log x^a = \log y^b$

$\Rightarrow a \log x = b \log y$

$\Rightarrow \frac{a}{b} = \log_x^y(q)$

c) $a \log x = y \log b$

$\Rightarrow \frac{a}{y} = \log_x^b(s)$

d) $x \log a = b \log y$

$\Rightarrow \frac{x}{b} = \log_a^y(r)$

Ans: t, q, s, r

14 a) $\log_{10} 10^4 = 4 \cdot \log_{10} 10 = 4 \cdot 1 = 4$

(~~Power rule~~) (r)

b) $\log_{10} 10000 \rightarrow \text{Characteristic} = 4 (P)$

c) Number $< 0.01 (q)$

d) Power Rule (r)

e) Product Rule of Logarithm (s)

Ans: P, q, r, s

$\Rightarrow \text{THE END} \Leftarrow$