WS-14. Equilibrium ~~of~~ & energy

Thank

①

Given $m_b = 100\ gm = 10^{-1}\ kg$

$u_b = 50\ m/s$

$K.E = \frac{1}{2} m_b u_b^2 = \frac{1}{2} \times 10^{-1} \times (50)^2 = \frac{2500 \times 10^{-1}}{2}$

$= 125 J \rightarrow B$

②

Let $v_1 = v,\ v_2 = 3v$ also $m_1 = m_2$ are masses of two bodies

$K.E = \frac{1}{2} m v^2 \Rightarrow K.E \propto v^2$

$\Rightarrow \frac{k_1}{k_2} = \left[\frac{v_1}{v_2}\right]^2 = \left(\frac{v}{3v}\right)^2 = \frac{1}{9} \rightarrow A$

③

Given P.E $U(x) = x^2 - 4x$

$F = -\frac{du}{dx} = -\frac{d}{dx}[x^2 - 4x] = -\left[\frac{d}{dx} x^2 - 4\frac{dx}{dx}\right]$

$= -[2x - 1]$ at $x = 2$

$F = -[2(2) - 1] = -3N$

and $U(x) = (2)^2 - 4(2) = 4 - 8 = -4J$ P.E minimum

F is -ve so the particle is in stable equilibrium

(4)

at $x = x_1$ and $x = x_3$ are not equilibrium

because $\frac{du}{dx} \neq 0$ at these points

at $x = x_2$ is unstable. as U is maximum at this

Point

$\therefore$ at $x = x_3$ $\frac{du}{dx} \neq 0$ $\rightarrow D$

(5)

At point C, P.E is minimum and F = − slope of U-x curve

$F = 0$

Hence ~~at~~ it is a point of stable equilibrium.

At B and D → slope of U-x-curve, $\frac{du}{dx} \neq 0$, hence they are not equilibrium points

From A to C:

$F = -\frac{du}{dx} = +ve$ $\because$ slope $\frac{du}{dx} = -ve$

A +ve force between interacting particles indicates the repulsion between the particles and instability in that region of the curve.

Also from B → C. → $\frac{du}{dx} = +ve$: $F = -\frac{du}{dx} = -ve$.

Between E and F ⇒ $\frac{du}{dx} = -ve$ i.e. $F = +ve$

→ Repulsive

(6)

At $x = 2m$, $U_F = 6J$; $U_I = 2J$; $m = 2\,kg$

we know $\Delta U = \Delta k.E$

$\therefore \frac{1}{2}kx^2 = k.E_F - k.E_I$ (or) $\frac{1}{2}mv^2 = U_F - U_I$

$= \frac{1}{2}(2)v^2 = 6-2$

$\Rightarrow v^2 = 4 \Rightarrow V = 2\,m/s \rightarrow C$

(7)

Given $m = 2\,kg$, $U = 6x - 8y$, velocity $\vec{u} = -5\hat{i} + 2\hat{j}$

$t = 2\,sec$

We know $F = -\frac{dU}{dx}\hat{i} - \frac{dU}{dy}\hat{j} = -\frac{d}{dx}(6x-8y)\hat{i} - \frac{d}{dy}(6x-8y)\hat{j}$

$\Rightarrow -6\hat{i} + 8\hat{j}$

~~After~~ $a = \frac{F}{m} = \frac{-6\hat{i}+8\hat{j}}{2} = -3\hat{i} + 8\hat{j}$

From $S = ut + \frac{1}{2}at^2$

$\Rightarrow S = (-5\hat{i}+2\hat{j})2 + \frac{1}{\cancel{2}}(-3\hat{i}+8\hat{j})2^{\cancel{2}}$

$\Rightarrow -10\hat{i} + 4\hat{j} + (-3\hat{i}+8\hat{j})2$

$= -10\hat{i} + 8\hat{j} - 6\hat{i} + 16\hat{j} = -16\hat{i} + 24\hat{j}$

$|\vec{S}| = \sqrt{(-16)^2 + 24^2} =$.

(8)

Given

$W_A = 300\,N$; $W_B = 50\,N$

$m_A = 30\,kg$ $m_B = 5\,kg$

$u_A = 0$ $v_A = 4\,m/s$.

as A descends by x, then B moves up by a distance of $2x$

$x_A = x$; $x_B = 2x$. $v_A = 4$ $v_B = 2v_A = 8\,m/s$

From W-E theorem

$\Rightarrow W_A + W_B = K.E_A + K.E_B$

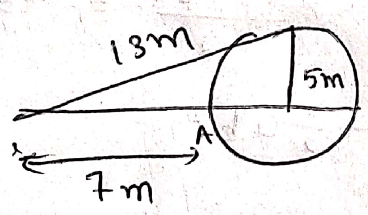
$\Rightarrow m_A g x_1 + m_B g x_2 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$

$\Rightarrow 300x - 50(2x) = \frac{1}{2}\, 30 \times 4^2 + \frac{1}{2} \times 5 \times 8^2$

$\Rightarrow 200x = 240 + 160$

$\Rightarrow 200x = 400 \Rightarrow x = \underline{2m} \rightarrow B$

(9)



$x = 13 - 7 = 6\,m$

$K.E = \frac{1}{2} m v^2 = \frac{1}{2} \times 200 \times 6^2$

$= 3600\,J$

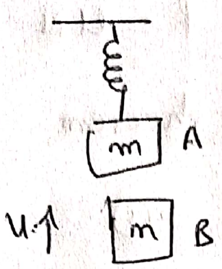
$\Rightarrow mv^2 = 7200\,J$

centripetal force $= \frac{mv^2}{R} = \frac{7200}{5} = 1440\,N$

Here Normal reaction provides necessary centripetal force

'x' is elongation in the spring as collar starts sliding from B

(10) (3)



As B collides 'A' let 'x' be compression

From $F = kx$

$\Rightarrow mg = kx \Rightarrow x = \frac{mg}{k}$

Acc to law of conservation of linear momentum

$P_i = P_f$ (After collision A & B stick and move with same velocity)

$\Rightarrow mv = 2mu$

$\Rightarrow u = \frac{v}{2}$

$\therefore$ According to W-E theorem

$m = m_A + m_B = 2m$

$K_i + U_i = K_f + U_f$

$\Rightarrow \frac{1}{2}m u^2 + \frac{1}{2}kx^2 = 0 + mgx$

$\Rightarrow \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 + \frac{1}{2}kx^2 = 2mgx$

$\Rightarrow \frac{mu^2}{4} + \frac{1}{2}\left[k\left(\frac{mg}{k}\right)^2\right] = 2mg\frac{mg}{k}$

$\Rightarrow \frac{mv^2}{4} + \frac{m^2g^2}{2k} = \frac{2m^2g^2}{k}$

$\Rightarrow \frac{mv^2}{4} = \frac{2m^2g^2}{k} - \frac{m^2g^2}{2k} = \frac{3m^2g^2}{2k}$

$\Rightarrow \frac{v^2}{4} = \frac{3}{2}\frac{mg^2}{k}$

$\Rightarrow v = \sqrt{\frac{6mg^2}{k}} \rightarrow B$

(i)

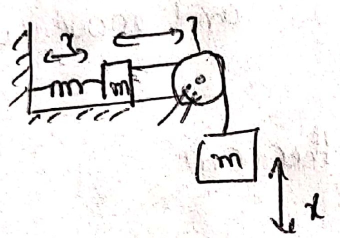
we know frictional force $f = \mu N = \mu mg$

According to work energy theorem

$$w_f + w_s + w_g = \Delta K.E \rightarrow (1)$$

let 'x' be the distance moved by m. (suspended body)

$\therefore$ m on surface moves by x, spring is stretched by x



From (1)

$$f(-x) - \frac{kx^2}{2} + mgx = 0$$

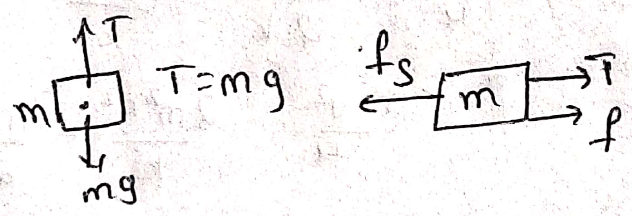
$$\Rightarrow f + \frac{kx}{2} = mg \rightarrow (2)$$

$$\Rightarrow \mu mg + \frac{kx}{2} = mg \quad \Rightarrow \frac{kx}{2} = mg(1-\mu) = mg\left(1-\frac{1}{4}\right)$$

$$= \frac{3mg}{4}$$

$$\Rightarrow x = \frac{3mg}{2k}$$

(ii) The minimum value of μ for which the system is at rest



$f_s \rightarrow$ spring force $= -kx$

From (2) $kx = 2mg - 2f$

$= 2mg - 2\mu mg$

$$\because f + T = f_s$$

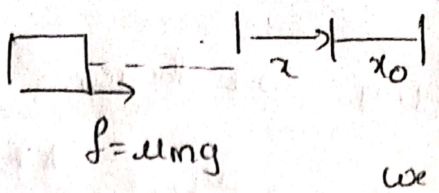
$$\Rightarrow f + mg = 2mg - 2\mu mg$$

$$\Rightarrow f = mg - 2\mu mg$$

$$\Rightarrow \mu mg = mg - 2\mu mg$$

$$\Rightarrow 3\mu mg = mg$$

$$\mu = \frac{1}{3}$$



we know initial stretch

$$x = \frac{3mg}{2k}$$

here let it be $x_0 = \frac{3mg}{2k}$

Acc to WET

$$W_f + W_s + W_g = \Delta K.E$$

$$\left[W_f = -f(x+x_0) = -\mu mg(x+x_0)\right]$$

$$= -\mu mg(x+x_0) + \frac{1}{2}kx_0^2 - \frac{1}{2}kx^2 + 0 = 0$$

$$\Rightarrow -\mu mg\left[x + \frac{3mg}{2k}\right] = \frac{1}{2}kx^2 - \frac{1}{2}k\frac{9m^2g^2}{4k^2}$$

$$\Rightarrow -x - \frac{3mg}{2k} = \frac{kx^2}{2\mu mg} - \frac{9mg}{8k\mu}$$

By solving & sub $[\mu = \frac{1}{4}]$

we get $\Rightarrow x = \cancel{\frac{mg}{4}}\ \frac{mg}{k}$

A, B, C correct

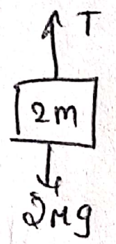
(12)

Maximum extension will be at the moment when both masses stop momentarily after going down. Apply W-E theorem

$$W_{spring} + W_{ten} = \Delta K.E$$

$$\Rightarrow 2Mgx + \left(-\frac{1}{2}kx^2\right) = 0 \Rightarrow x = \frac{4Mg}{k}$$

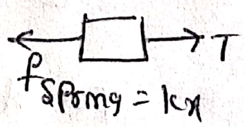
For K.E = Max Net force = 0.



i.e. $2Mg = T$ and $T = kx$

$$\Rightarrow 2Mg = kx$$

$$\Rightarrow x = \frac{2mg}{k}$$



Hence K.E is maximum when 2M has gone down by $\frac{2mg}{k}$

Applying W-E theorem

$$W_{spring} + W_{ten.} = \Delta K.E$$

$$\Rightarrow 2mg(x) - \frac{1}{2}kx^2 = k_f - k_i$$

$$\Rightarrow 2mg\left(\frac{2mg}{k}\right) - \frac{1}{2}k\left(\frac{2mg}{k}\right)^2 = k_f$$

$$\Rightarrow \frac{4m^2g^2}{k} - \frac{2m^2g^2}{k} = k_f$$

$$\Rightarrow k_f = \frac{2m^2g^2}{k}$$

Maximum Energy of spring $= \frac{1}{2}kx_{max}^2$

$$= \frac{1}{2}k\left(\frac{4mg}{k}\right)^2 = \frac{8m^2g^2}{k}$$

$\therefore$ Maximum Spring Energy $= 4 \times K.E_{max}$

Spring Energy $= \frac{1}{2}kx^2$ $\left[x = \frac{2mg}{k}\right]$

$$= \frac{1}{2}k\frac{4m^2g^2}{k^2} = \frac{2m^2g^2}{k^2}$$

D is wrong, A, B, C are correct

(14), (15), (16)

We know $F = -\frac{du}{dx}$ $\quad \frac{du}{dx} > 0$ or $-\frac{du}{dx} > 0$

$F < 0$ $\therefore$ Force in -ve x-direction

(15)

at $x = 2$ P.E $U = 10J$

For this T.E = constant $\quad k_i + U_i = k_f + U_f$

$\Rightarrow 0 + 10 = \frac{1}{2}mv^2 - 15$

$\Rightarrow \frac{1}{2}mv^2 = 25$ Given $m = 2$ kg

$\Rightarrow \frac{1}{2}(2)v^2 = 25$

$v = 5$ m/s

(16)

At $x = -5 \rightarrow$ stable position because $U = -ve$

$x = 10m$ unstable position because $U = +ve$

$\rightarrow$ D is correct

(17)

Given $x = \frac{t^4}{4}$; $m = 1kg$

$\therefore$ velocity $v = \frac{dx}{dt} = \frac{4t^3}{4} = t^3$ $\left[\frac{d}{dx}x^n = nx^{n-1}\right.$ Here $x = t$ $\left.n = 4\right]$

As per work-Energy theorem

$W = \Delta KE$

$W = \frac{1}{2}m(v^2 - u^2)$ At $t = 1sec$ $v = t(1)^3 = 1$ $u = 0$

$W = \frac{1}{2} \times 1 (1) = 0.5J$

(18)

Block posses P.E at A $= mgh$

$= 2 \times 10 \times 1 = 20J$

(19)

~~At B&C F=0~~ $\rightarrow$ S

(19) At J:

F & x are changing continuously, so it is not a state of equilibrium

At K:-

we see that slope $= \frac{dF}{dx} = +ve$

(i-e) $\frac{du}{dx} = +ve$ ⇒ unstable equilibrium

At L:- we see that slope $\frac{dF}{dx} = -ve$ ⇒ $\frac{du}{dx} = -ve$

⇒ stable equilibrium

At M → $F=0$: Neutral equilibrium

L Task

Jee main level

(1) Given $m = 2\,kg$; $h = 10\,m$

$P.E = mgh = 2 \times 10 \times 10 = 200\,J$

(2) Given $P.E = 490\,J$: $m = 5\,kg$

∴ $mgh = 490$

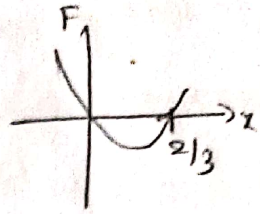
⇒ $5 \times 9.8 \times h = 490$

⇒ $h = 10\,m$ → A

(4)

Given

$F = x(3x-2) = 3x^2 - 2x$



If $F=0$ $\quad x=0$ (or) $(3x-2)=0$

$x = \frac{2}{3}$

$F = \frac{du}{dx}$

$\Rightarrow \frac{d^2u}{dx^2} = -\frac{dF}{dx} = -\frac{d}{dx}(3x^2-2x)$

$= -(6x-2)$

$\Rightarrow +2-6x$

$= 2 - 6\times\frac{2}{3} = -2$

$\frac{d^2u}{dx^2} < 0$ $\quad$ i.e. $x=0$ stable

$x = \frac{2}{3}$ unstable

Acc to work energy theorem $\quad W = \Delta$ k.E

$\Rightarrow -\int F\,dx = \frac{1}{2}mv^2$

$\Rightarrow -\int_{\frac{2}{3}}^{4}(3x^2-2x)\,dx = \frac{1}{2}mv^2$

$\Rightarrow -\cancel{3}\left[\frac{x^3}{\cancel{3}}\right]_{2/3}^{4} + \cancel{2}\left[\frac{x^2}{\cancel{2}}\right]_{2/3}^{4} = \frac{1}{2}mv^2$

$\Rightarrow -[x^3]_{2/3}^{4} + (x^2)_{2/3}^{4} = \frac{1}{2}mv^2$

$\Rightarrow \frac{1}{2}mv^2 = -(4^3-(\frac{2}{3})^3) + 4^2 - (\frac{2}{3})^2$

$= \frac{8}{27} - 64 + 16 - \frac{4}{9}$

$\Rightarrow \frac{1}{2}(1)v^2 = \frac{1300}{27} \Rightarrow v^2 = \frac{2600}{27} \Rightarrow v = \sqrt{\frac{2600}{27}} \rightarrow A$

(6)

we know $W_{ext} = \Delta U$ and Given $U = k(x+y)$

$= U_{(2,3)} - U_{(1,1)}$

$= k(2+3) - k(1+1)$

$\Rightarrow 5k - 2k$

$= 3k \rightarrow A$

(7)

we know $U = \frac{1}{2}kx^2$ for $x <= 0$

$U = 0$ for $x \geq 0$.

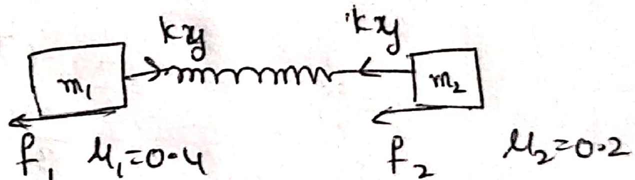
$T.E = k + U \Rightarrow k = T.E - U$

For $x = -\sqrt{\frac{2E}{k}} \Rightarrow U = \frac{1}{2}k\left(-\sqrt{\frac{2E}{k}}\right)^2 = E$

$\therefore k = E - U = E - E = 0 \rightarrow$ (A)

(8)

For given arrangement



Block m_1 will shift

$ky \geq \mu_1 mg \rightarrow$ (1) Acc to W-E theorem

$k_i + W_N + W_{spring} + W_f + W_g = k_f$

$\Rightarrow \frac{m_1 u^2}{2} + 0 + 0 - f_s y - \frac{ky^2}{2} = 0$

$\Rightarrow \frac{mu^2}{2} - \mu m g y - \frac{ky^2}{2} = 0 \rightarrow$ (2) By sub k, m_1, μ_1

For ~~u min~~ From (1) $y = \frac{\mu_1 m_1 g}{k} = 0.4 \times 5 \times \frac{10}{100} = 0.2\ m$

From (2) $\therefore u = \sqrt{2.8}\ m/s^2$ [sub all values in (2) to get u]

(9)

we know $W_{ext} = \Delta(M.E)$ ME = Mechanical energy

M.E will keep on increasing up to the instant that $W_{ext} = +ve$, which will happen till there is no compression in the spring. First the spring gets extended to a maximum and after which the extension decreases upto natural length. After that there is a compression in the spring, results in a -ve external work. (so as to move the end of spring at constant speed v.)

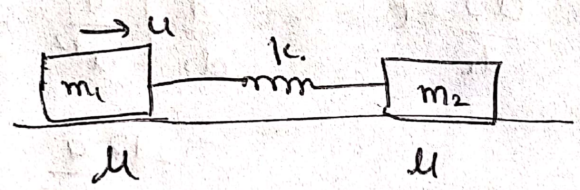
Hence maximum energy stored at natural length

$$\Delta M.E_{max} = \frac{1}{2} m v^2$$

$v = 2u$ since the block is moving at this instant at a speed u w.r.t other end of the spring

$$M.E_{max} = \frac{1}{2} m (2u)^2 = 2mu^2$$

(10)



The frictional force on $m_2 = f_2 = \mu m_2 g = \frac{1}{2}(2) \times 10 = 10N$

If the spring force exceeds this value, then m_2 will move.

$\therefore$ Maximum spring force = 10 N.

$$k x_{max} = 10 \Rightarrow x_{max} = \frac{10}{2} = 5m \quad [\because k = 2 N/m]$$

The maximum compression in the spring = 5 m

m_1 should stop after compressing the spring by 5m

Applying W-E Theorem for the motion of m_1

$$k_i + U_i + W_f = k_f + U_f$$

$\Rightarrow \frac{1}{2} m_1 u_e^2 + 0 - \mu m_1 g x_{max} = 0 + \frac{1}{2} k (x_{max})^2$

$\Rightarrow \frac{1}{2} m_1 u^2 - \mu m_1 g x_{max} = \frac{1}{2} (2) (5)^2$

$\Rightarrow \frac{1}{2}(1) u^2 - \frac{1}{2} \times \cancel{1 \times 10 \times 5} = \frac{1}{2} 5^2$

$\Rightarrow \cancel{\frac{u^2}{2} \frac{5}{2}} = 25$

$\Rightarrow \frac{1}{2}(1) u^2 - \frac{1}{2} \times 1 \times 10 \times 5 = \frac{1}{2}(2)(5)^2$

$\Rightarrow \frac{u^2}{2} - 25 = 25 \Rightarrow u^2 = 100 \Rightarrow u = 10$ m/s $\rightarrow$ B

(11) Given $U = \frac{a}{x^{12}} - \frac{b}{x^6}$

let us assume that the stable equilibrium seperation between the atoms is x_0, where the P.E is minimum

to find x_0, we take derivative of U and equate it to 0

that means $\frac{dU}{dx} = 0 \Rightarrow \frac{d}{dx}\left[\frac{a}{x^{12}} - \frac{b}{x^6}\right] = 0$

$\Rightarrow -\frac{12a}{x_0^{13}} + \frac{6b}{x_0^7} = 0$

$\Rightarrow x_0 = \left(\frac{2a}{b}\right)^{1/6}$

$F = -\frac{dU}{dx} = -\frac{d}{dx}\left[\frac{a}{x^{12}} - \frac{b}{x^6}\right] = \frac{12a}{x^{13}} - \frac{6b}{x^7}$

A, D correct

(12) Given $U = \frac{A}{r^2} - \frac{B}{r}$ At equilibrium $\frac{dU}{dr} = 0$

$\frac{d}{dr}\left(\frac{A}{r^2} - \frac{B}{r}\right] = 0 \Rightarrow -\frac{2A}{r^3} + \frac{B}{r} = 0 \Rightarrow r = \frac{2A}{B}$

work done $= \int F\,dx$

$= -U = \left[\frac{A}{r^2} - \frac{B}{r}\right] = \left[\frac{AB^2}{4A^2} - \frac{B^2}{4A}\right]$

$\Rightarrow -\left(\frac{AB^2 - 4AB^2}{4A^2}\right) = -\frac{3AB^2}{4A^2} = \frac{3B^2}{4A}$

A & B correct

17, 18

Given

$U = 20 + (x-2)^2$ $m = 1\,kg$

Acc to law of conservation of Energy, if no dissipative forces are present the Total Energy = conserved

At $x = 5m$ $U_2 = 20 + (5-2)^2 = 20 + 9 = 29J$

T.E at $x = 5m$ $K.E = 20J$

$T.E = U + K = 29 + 20 = 49J$

From $U = 20 + (x-2)^2 \Rightarrow 20 + x^2 - 4x - 4 \Rightarrow x^2 - 4x + 16$

$\frac{du}{dx} = 2x - 4$. For $\frac{du}{dx} = 0$

$2x - 4 = 0$

$\Rightarrow x = 2m$

so for $x = 2m$ the particle as max Energy

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