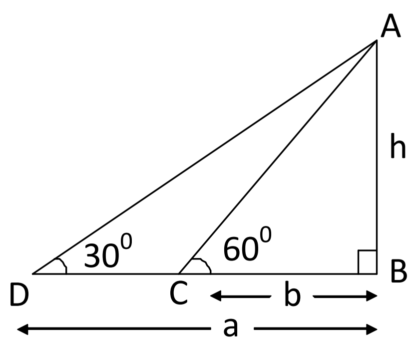
Heights and Distances

TEACHING TASK

1. From the diagram :



$$\text{Tan } 30^0 = \frac{h}{a}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{a}$$

$$\Rightarrow h = \frac{a}{\sqrt{3}} \quad \text{.......... (1)}$$

Again $\text{Tan } 60^0 = \frac{h}{b}$

$$\Rightarrow \sqrt{3} = \frac{h}{b}$$

$$\Rightarrow h = b\sqrt{3} \quad \text{.......... (2)}$$

Now (1) x (2)

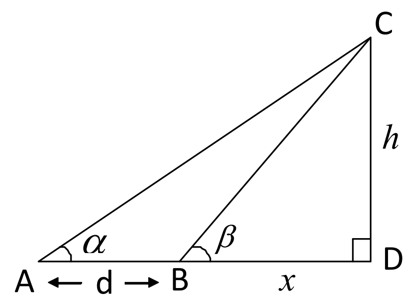
$$\Rightarrow h^2 = \frac{a}{\sqrt{3}} \text{ x } b\sqrt{3}$$

$$\Rightarrow h^2 = ab$$

$$\Rightarrow h = \sqrt{ab}$$

Ans: B

2. From the diagram



$$\text{Tan } \beta = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\tan\beta}$$

$$\Rightarrow x = h\cot\beta \quad \text{.......... (i)}$$

Now, $\tan\alpha = \frac{h}{d+x}$

$$\Rightarrow d + x = h\cot\alpha$$

$$\Rightarrow x = h\cot\alpha - d \quad \text{.......... (ii)}$$

from (i) and (ii)

$$h\cot\beta = h\cot\alpha - d$$

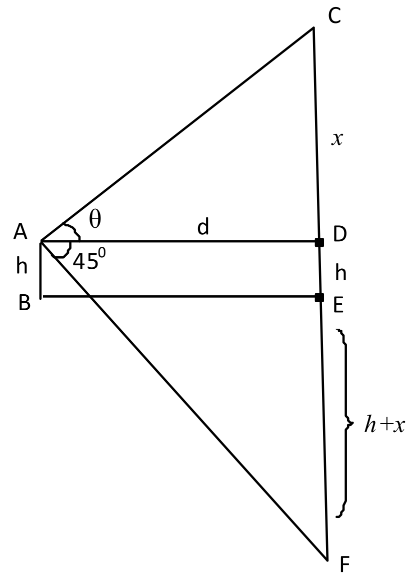
$$\Rightarrow h\cot\alpha - h\cot\beta = d$$

$$\Rightarrow h(\cot\alpha - \cot\beta) = d$$

$$\Rightarrow h = \frac{d}{\cot\alpha - \cot\beta}$$

Ans: A

3. From diagram



$\text{Tan}\,\theta = \frac{x}{d}$ .... (1)

$\text{Tan}\,45^0 = \frac{2h+x}{d}$

$\Rightarrow d = 2h + x$ .... (2)

Now

$\text{Tan}\left(45^0+\theta\right) = \frac{1+\text{Tan}\,\theta}{1-\text{Tan}\,\theta}$

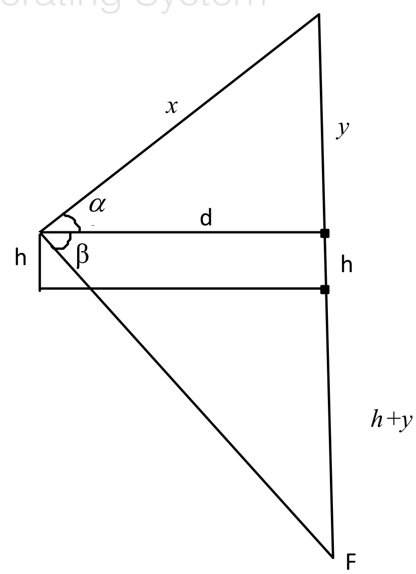
$= \frac{1+\frac{x}{2h+x}}{1-\frac{x}{2h+x}} = \frac{2\left(h+x\right)}{2h} = \frac{h+x}{h}$

$\therefore h + x = h\left(\text{Tan}\,45^0+\theta\right)$

$\therefore$ The hight of the cloud is $h\left(\text{Tan}\,45^0+\theta\right)$

Ans : A

4. From diagram



$\text{Tan}\,\alpha = \frac{y}{d}$

$\text{Tan}\,\beta = \frac{2h+y}{d}$

$\therefore d = \frac{y}{\text{Tan}\,\alpha}$ *and* $d = \frac{2h+y}{\text{Tan}\,\beta}$

$\therefore \frac{y}{\text{Tan}\,\alpha} = \frac{2h+y}{\text{Tan}\,\beta}$

$\Rightarrow \frac{y}{\text{Tan}\,\alpha} = \frac{2h}{\text{Tan}\,\beta} + \frac{y}{\text{Tan}\,\beta}$

$\Rightarrow \frac{y}{\text{Tan}\,\alpha} - \frac{y}{\text{Tan}\,\beta} = \frac{2h}{\text{Tan}\,\beta}$

$\Rightarrow y\left(\frac{\text{Tan}\,\beta - \text{Tan}\,\alpha}{\text{Tan}\,\alpha.\text{Tan}\,\beta}\right) = \frac{2h}{\text{Tan}\,\beta}$

$\Rightarrow y\left(\frac{\text{Tan}\,\beta - \text{Tan}\,\alpha}{\text{Tan}\,\alpha}\right) = 2h$

$\Rightarrow \frac{y}{\text{Tan}\,\alpha} = \frac{2h}{\text{Tan}\,\beta - \text{Tan}\,\alpha}$

$$\Rightarrow d = \frac{2h}{\text{Tan}\,\beta - \text{Tan}\,\alpha} \qquad \text{............ (i)}$$

Now $\sec\alpha = \frac{x}{d}$

$\Rightarrow x = d\sec\alpha$

$$\Rightarrow x = \frac{2h\sec\alpha}{\text{Tan}\,\beta - \text{Tan}\,\alpha} \qquad (\because from\,(i))$$

Hence, the distance between the cloud and the observer is $\frac{2h\sec\alpha}{\text{Tan}\,\beta - \text{Tan}\,\alpha}$

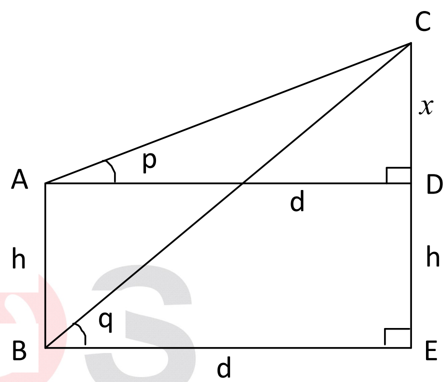
Ans: C

5. From the diagram

AB = Building

CE = Hill

A,B = Position of the observers.



Now, $\text{Tan}\,p = \frac{x}{d}$ and $\text{Tan}\,q = \frac{x+h}{d}$

$\Rightarrow d = x\cot p \qquad \Rightarrow d = (x+h)\cot q$

$\therefore\; x\cot p = (x+h)\cot q$

$\Rightarrow x\cot p - x\cot q = h\cot q$

$\Rightarrow x(\cot p - \cot q) = h\cot q$

$$\Rightarrow x = \frac{h\cot q}{\cot p - \cot q}$$

Now, height of the hill

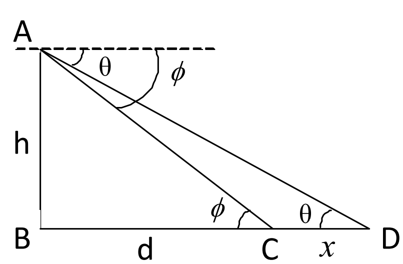
$$x + h = \frac{h\cot q}{\cot p - \cot q} + h$$

$$= \frac{h\cot q + h\cot p - h\cot q}{\cot p - \cot q}$$

$$= \frac{h\cot p}{\cot p - \cot q}$$

Ans: C

6. In the diagram



C,D = Position of the boats

A = Position of the observer

AB = Light house

Now, $\text{Tan}\,\phi = \frac{h}{d}$ and $\text{Tan}\,\theta = \frac{h}{d+x}$

$$\Rightarrow d = \frac{h}{\text{Tan}\,\phi} \qquad \Rightarrow d + x = \frac{h}{\text{Tan}\,\theta}$$

$$\therefore \frac{h}{\text{Tan}\,\phi} + x = \frac{h}{\text{Tan}\,\theta}$$

$$\Rightarrow x = \frac{h}{\text{Tan}\,\theta} - \frac{h}{\text{Tan}\,\phi}$$

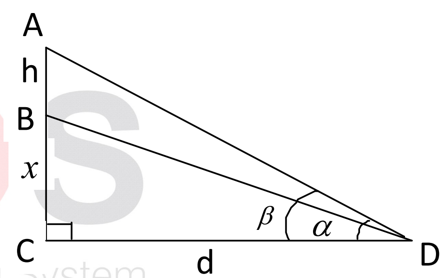
$$\therefore x = h\left(\frac{\text{Tan}\,\phi - \text{Tan}\,\theta}{\text{Tan}\,\theta.\text{Tan}\,\phi}\right)$$

$\therefore$ The distance between the two boats is

$$\frac{h(\text{Tan}\,\phi - \text{Tan}\,\theta)}{\text{Tan}\,\theta.\text{Tan}\,\phi}$$

Ans: B

7. In the diagram D = position of the observer
AB = Flag staff BC = Vertical tower



Now, $\text{Tan}\,\alpha = \frac{x}{d}$ and $\text{Tan}\,\beta = \frac{h+x}{d}$

$$\Rightarrow d = \frac{x}{\text{Tan}\,\alpha} \qquad \Rightarrow d = \frac{h+x}{\text{Tan}\,\beta}$$

$$\therefore \frac{x}{\text{Tan}\,\alpha} = \frac{h+x}{\text{Tan}\,\beta}$$

$$\Rightarrow \frac{x}{\text{Tan}\,\alpha} = \frac{h}{\text{Tan}\,\beta} + \frac{x}{\text{Tan}\,\beta}$$

$$\Rightarrow \frac{x}{\text{Tan}\,\alpha} - \frac{x}{\text{Tan}\,\beta} = \frac{h}{\text{Tan}\,\beta}$$

$$x\left(\frac{1}{\text{Tan}\,\alpha} - \frac{1}{\text{Tan}\,\beta}\right) = \frac{h}{\text{Tan}\,\beta}$$

$$\Rightarrow x\left(\frac{\text{Tan}\,\beta - \text{Tan}\,\alpha}{\text{Tan}\,\alpha.\text{Tan}\,\beta}\right) = \frac{h}{\text{Tan}\,\beta}$$

$$\Rightarrow x = \left(\frac{h\,\text{Tan}\,\alpha}{\text{Tan}\,\beta - \text{Tan}\,\alpha}\right)$$

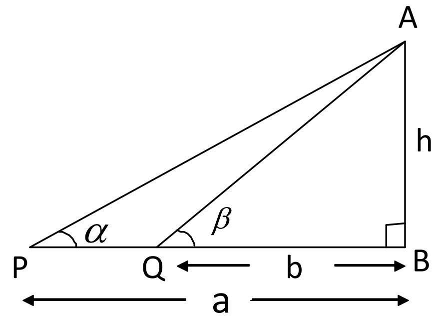
$\therefore$ Height of the tower is $\frac{h\,\text{Tan}\,\alpha}{\text{Tan}\,\beta - \text{Tan}\,\alpha}$

Ans : B

8. In the diagram AB = Tower

P,Q = Position of the observer

Given $\alpha+\beta=90^0$



$\Rightarrow \text{Tan}(\alpha+\beta)=\text{Tan}\,90^0$

$\Rightarrow \dfrac{\text{Tan}\,\alpha+\text{Tan}\,\beta}{(1-\text{Tan}\,\alpha.\text{Tan}\,\beta)}=$ Not defined

$\Rightarrow 1-\text{Tan}\,\alpha.\text{Tan}\,\beta=0$

$\Rightarrow \text{Tan}\,\alpha.\text{Tan}\,\beta=1$

$\Rightarrow \dfrac{h}{a}.\dfrac{h}{b}=1$

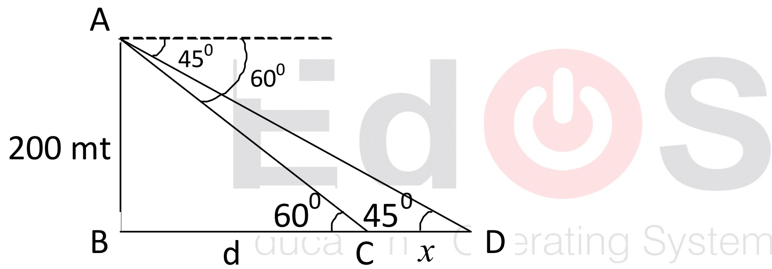
$\Rightarrow h^2=ab$

$\Rightarrow h=\sqrt{ab}$

$\therefore$ Heigth of the tower is $\sqrt{ab}$

Ans: A

9.



In the above diagram AB = Altitude of the aeroplane C,D = Opposite sides of the banks of a river

$CD=x$ = width of the river.

Now, $\text{Tan}\,60^0=\dfrac{200}{d}$

$\Rightarrow \sqrt{3}=\dfrac{200}{d}$

$\Rightarrow d=\dfrac{200}{\sqrt{3}}$ and $\text{Tan}\,45^0=\dfrac{200}{d+x}$

$\Rightarrow d+x=200$

$\therefore \dfrac{200}{\sqrt{3}}+x=200$

$\Rightarrow x=200-\dfrac{200}{\sqrt{3}}$

$\Rightarrow x=200\left(\dfrac{\sqrt{3}-1}{\sqrt{3}}\right)$

Ans : B

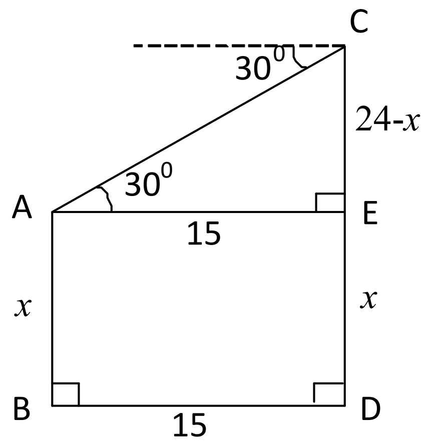
10. In the diagram AB = First pole

CD = Second pole

BD = 15mt; CD=24mt

Let AB = x mt

$\therefore CE = (24 - x)\, mt$



In ΔAEC, $\text{Tan}\, 30^0 = \dfrac{24 - x}{15}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{24 - x}{15}$$

$$\Rightarrow 24 - x = \frac{15}{\sqrt{3}}$$

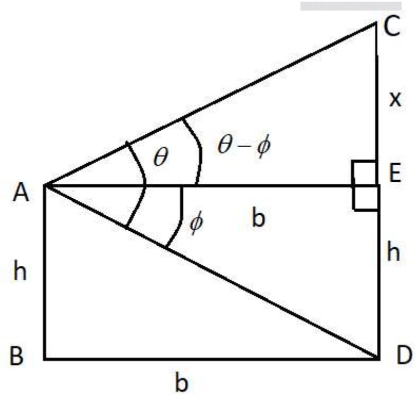
$$\Rightarrow 24 - x = 8.67$$

$$\Rightarrow x = 24 - 8.66$$

$$\Rightarrow x = 15.34 \text{ (opprex)}$$

Ans : C

11. In the figure AB = Window, CD = Building, BD = Road, given AB = h



Now, $\tan \phi = \dfrac{h}{b} \rightarrow (i)$

$$\tan(\theta - \phi) = \frac{x}{b}$$

$$\Rightarrow \frac{\tan\theta - \tan\phi}{1 + \tan\theta \cdot \tan\phi} = \frac{x}{b}$$

$$\Rightarrow \frac{\dfrac{\sin\theta}{\cos\theta} - \dfrac{h}{b}}{1 + \dfrac{\sin\theta}{\cos\theta} \cdot \dfrac{h}{b}} = \frac{x}{b} \quad (\because from(i))$$

$$\Rightarrow \frac{b\sin\theta - h\cos\theta}{b\cos\theta + h\sin\theta} = \frac{x}{b}$$

$$\Rightarrow x = \frac{b(b\sin\theta - h\cos\theta)}{b\cos\theta + h\sin\theta}$$

$\therefore$ Height of the building = $h + x$

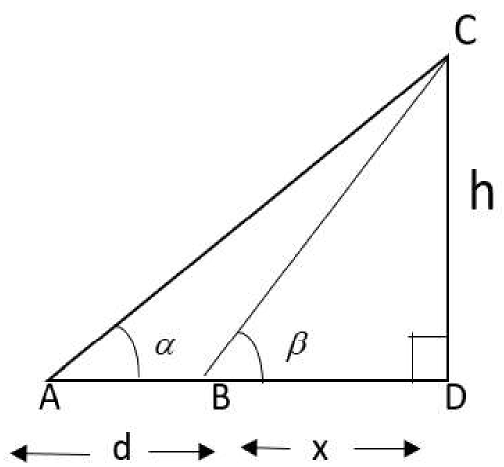
$$\therefore \; h + x = \frac{b(b\sin\theta - h\cos\theta)}{b\cos\theta + h\sin\theta} + h$$

$$= \frac{b^2\sin\theta - bh\cos\theta + bh\cos\theta + h^2\sin\theta}{b\cos\theta + h\sin\theta}$$

$$= \frac{(b^2 + h^2)\sin\theta}{b\cos\theta + h\sin\theta}$$

ANS: C

12. In the figure A, B = positions of the observers, CD = Tower, AB = d



Let BD = x, CD = h

Now, $Tan\beta = \frac{h}{x}$ and $Tan\alpha = \frac{h}{d + x}$

$\Rightarrow x = h\cot\beta$ and $d + x = h\cot\alpha$

$\therefore d + h\cot\beta = h\cot\alpha$

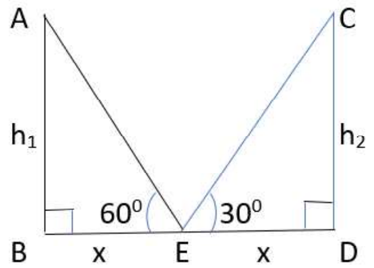
$\Rightarrow h(\cot\alpha - \cot\beta) = d$

$$\Rightarrow h = \frac{d}{\cot\alpha - \cot\beta}$$

$$\therefore Height \text{ of the tower} = \frac{d}{\cot\alpha - \cot\beta}$$

ANS : B

13. Statement I



In the figure $Tan60^0 = \frac{h_1}{x}$ $Tan30^0 = \frac{h_2}{x}$

$\Rightarrow \sqrt{3} = \frac{h_1}{x}$ $\Rightarrow \frac{1}{\sqrt{3}} = \frac{h_2}{x}$

$\Rightarrow x = \frac{h_1}{\sqrt{3}} \rightarrow (i)$ $\Rightarrow x = \sqrt{3}h_2 \rightarrow (ii)$

From (i) and (ii)

$\frac{h_1}{\sqrt{3}} = \sqrt{3}h_2$

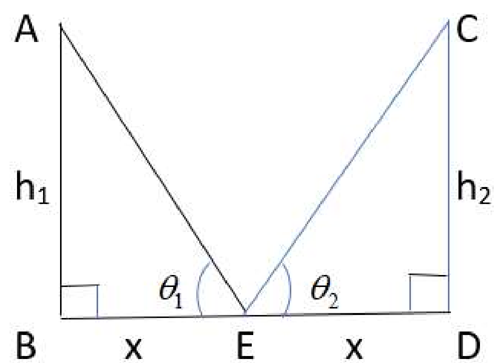
$\Rightarrow h_1 = 3h_2$

$\Rightarrow h_1 : h_2 = 3:1$

Hence, Statement I is correct

Statement II

In the figure



$\tan\theta_1 = \frac{h_1}{x}$ and $\tan\theta_2 = \frac{h_2}{x}$

$\Rightarrow x = \frac{h_1}{\tan\theta_1}$ and $x = \frac{h_2}{\tan\theta_2}$

$\Rightarrow \frac{h_1}{\tan\theta_1} = \frac{h_2}{\tan\theta_2}$

$\Rightarrow \frac{h_1}{h_2} = \frac{\tan\theta_1}{\tan\theta_2}$

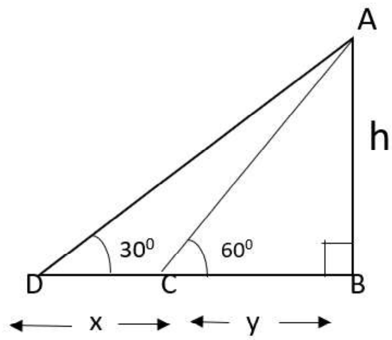
$\Rightarrow h_1 : h_2 = \tan\theta_1 : \tan\theta_2$

Hence, Statement II is correct.

Also, Statement II is the correct explanation of Statement I

ANS : A

14. Statement I



From the figure $Tan30^0 = \frac{h}{x+y}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+y}$

$\Rightarrow x+y = \sqrt{3}h \rightarrow 1$

Again $Tan60^0 = \frac{h}{y}$

$\Rightarrow \sqrt{3} = \frac{h}{y}$

$\Rightarrow y = \frac{h}{\sqrt{3}}$ .... (ii)

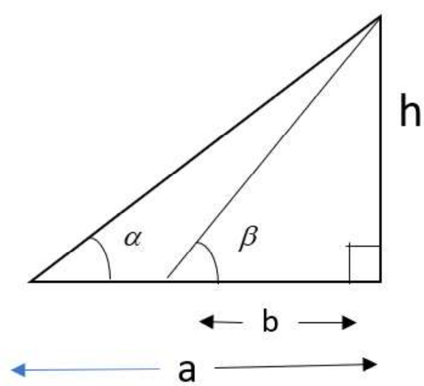
Now, from (i), $x + \frac{h}{\sqrt{3}} = \sqrt{3}h$

It is not possible to find h value unless we know x value.

$\therefore$ Statement I is incorrect.
Statement II : See the answer given for question number 12.
$\therefore$ Statement II is correct.

ANS : D

15. Statement I



From figure $Tan\beta = \frac{h}{b}$

$\Rightarrow h = bTan\beta$ .... (i)

Also, $Tan\alpha = \frac{h}{a}$

$\Rightarrow h = a\ Tan\alpha$ .... (ii)

Now, $(i)\, x\, (ii) \Rightarrow h^2 = abTan\alpha \cdot Tan\beta$

$\Rightarrow h = \sqrt{abTan\alpha \cdot Tan\beta}$

Statement I is incorrect.
Statement II : Given $a = 9mt$, $b = 4mt$

$\alpha = 30^0$, $\beta = 60^0$

Now, $h = \sqrt{ab\ \text{Tan}\alpha \cdot \text{Tan}\beta}$

$= \sqrt{9 \times 4 \times Tan30^0 \cdot Tan60^0}$

$$= \sqrt{36 \times \frac{1}{\sqrt{3}} \times \sqrt{3}}$$

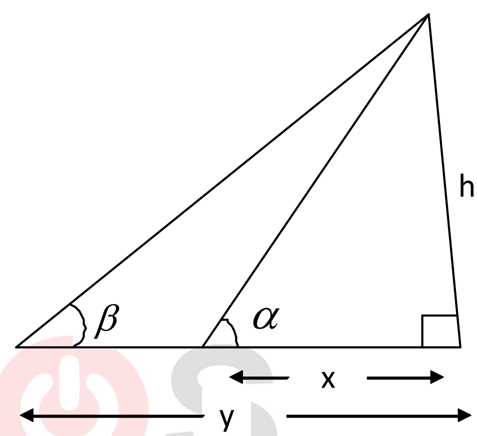
$$= \sqrt{36}$$

$$= 6mt$$

Hence, Statement II is correct.

ANS : D

16. Statement I



From figure $Tan\alpha = \frac{h}{x}$ and $Tan\beta = \frac{h}{y}$

Also, $\alpha + \beta = 90^0$

$$\Rightarrow Tan(\alpha + \beta) = Tan90^0$$

$$\Rightarrow \frac{Tan\alpha + Tan\beta}{1 - Tan\alpha \cdot Tan\beta} = Not \text{ defined}$$

$$\Rightarrow 1 - Tan\alpha \cdot Tan\beta = 0$$

$$\Rightarrow Tan\alpha \cdot Tan\beta = 1$$

$$\Rightarrow \frac{h}{x} \cdot \frac{h}{y} = 1$$

$$\Rightarrow h^2 = xy$$

$$\Rightarrow h = \sqrt{xy}$$

Hence, Statement I is correct.

Statement II

Given $x = a^2 + b^2, \quad y = 2ab$

We have, $h = \sqrt{xy}$

$$h = \sqrt{(a^2 + b^2)2ab} \neq a + b$$

Statement II is incorrect.

ANS: C

17. We have $d = \frac{h(Tan\alpha + Tan\beta)}{Tan\alpha \cdot Tan\beta}$

Given $h = 10mts$, $\alpha = 45^0$, $\beta = 30^0$

$$\therefore d = \frac{10(Tan45^0 + Tan30^0)}{Tan45^0 \cdot Tan30^0}$$

$$= \frac{10\left(1+\frac{1}{\sqrt{3}}\right)}{1.\frac{1}{\sqrt{3}}} = 10\left(\sqrt{3}+1\right)mt$$

ANS: A

18. Given $d = 20\sqrt{3}, \alpha = 15^0, \beta = 30^0$

We have $20\sqrt{3} = \dfrac{h\left(Tan15^0 + Tan30^0\right)}{Tan15^0 \cdot Tan30^0}$

$$\Rightarrow 20\sqrt{3} = \frac{h\left(2-\sqrt{3}+\frac{1}{\sqrt{3}}\right)}{\left(2-\sqrt{3}\right)\times\frac{1}{\sqrt{3}}}$$

$$\Rightarrow 20\sqrt{3} = \frac{h\left(2\sqrt{3}-2\right)}{2-\sqrt{3}}$$

$$\Rightarrow \left(20\sqrt{3}\right)\left(2-\sqrt{3}\right) = h\left(2\sqrt{3}-2\right)$$

$$\Rightarrow 10\sqrt{3}\left(2-\sqrt{3}\right) = h\left(\sqrt{3}-1\right)$$

$$\Rightarrow h = \frac{10\sqrt{3}\left(2-\sqrt{3}\right)}{\sqrt{3}-1}$$

ANS: B

19. Given $\alpha = 45^0$, $d = 30\sqrt{2}mt$, $h = 2mt$ then $\tan\beta$

We have $30\sqrt{2} = \dfrac{2\left(Tan45^0 + Tan\beta\right)}{Tan45^0 \cdot Tan\beta}$

$$\Rightarrow 30\sqrt{2} = \frac{2\left(1+Tan\beta\right)}{Tan\beta}$$

$$\Rightarrow 30\sqrt{2}\ Tan\beta = 2 + 2Tan\beta$$

$$\Rightarrow \left(30\sqrt{2}-2\right)Tan\beta = 2$$

$$\Rightarrow Tan\beta = \frac{2}{30\sqrt{2}-2}$$

$$\Rightarrow Tan\beta = \frac{1}{15\sqrt{2}-1}$$

ANS: B

20. We have $H = \frac{h\left(Tan\beta + Tan\alpha\right)}{Tan\beta - Tan\alpha}$

Given $\alpha = 30^0$, $\beta = 60^0$

$$\therefore H = \frac{h\left(Tan60^0 + Tan30^0\right)}{Tan60^0 - Tan30^0}$$

$$\Rightarrow H = \frac{h\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)}{\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)}$$

$$\Rightarrow H = \frac{h\left(4\right)}{2}$$

$$\Rightarrow H = 2h$$

$$\Rightarrow h : H = 1 : 2$$

ANS: A

21. We have

$$H = \frac{h\left(Tan\beta + Tan\alpha\right)}{Tan\beta - Tan\alpha}$$

$$\Rightarrow h = \frac{H\left(Tan\beta - Tan\alpha\right)}{Tan\beta + Tan\alpha}$$

$$\Rightarrow h = \frac{H\left(\frac{1}{\cot\beta} - \frac{1}{\cot\alpha}\right)}{\left(\frac{1}{\cot\beta} + \frac{1}{\cot\alpha}\right)}$$

$$\Rightarrow h = \frac{H\left(\cot\alpha - \cot\beta\right)}{\left(\cot\alpha + \cot\beta\right)}$$

ANS : A

22. Given $\alpha = 0^0$

Now $H = \frac{h\left(Tan\beta + \tan 0^0\right)}{\left(Tan\beta - \tan 0^0\right)}$

$$\Rightarrow H = \frac{h\left(Tan\beta + Tan - 0^0\right)}{\left(Tan\beta - Tan0^0\right)}$$

$$\Rightarrow H = h$$

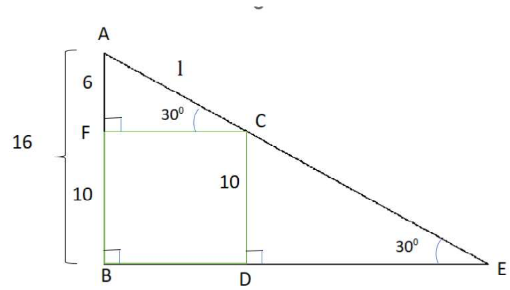
Now, $h^2 + H^2 = 2h^2$ or $2H^2$

$\therefore$ The possible answer is 2.

ANS: B

23.

In the above figure



AB and CD are the poles.

$\therefore$ AB = 16 mt, CD = 10 mt

$\therefore$ AF = 6 mt

AC = Length of the wire.

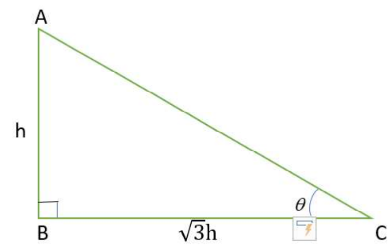
Now in $\triangle AFC, \sin 30^0 = \frac{6}{l}$

$$\Rightarrow \frac{1}{2} = \frac{6}{l}$$

$$\Rightarrow l = 12cm$$

24.

a) AB = Tower, BC = Shadow



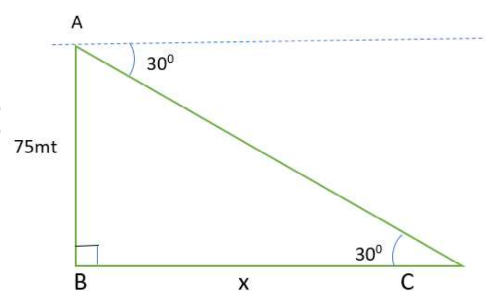
Now $Tan\theta = \frac{h}{\sqrt{3}h}$

$$\Rightarrow Tan\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = 30^0$$

b) A= Position of the observer

C = Position of the car

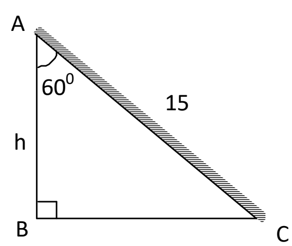


Now $Tan30^0 = \frac{75}{x}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{x}$$

$$\Rightarrow x = 75\sqrt{3}$$

c) $\cos 60^0 = \frac{h}{15}$

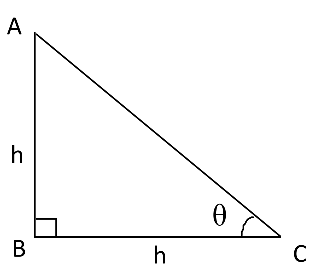


$$\Rightarrow \frac{1}{2} = \frac{h}{15}$$

$$\Rightarrow h = \frac{15}{2} mt$$

d) Here AB = Pole, BC = Shadow of the pole

Given AB = BC



$$\therefore Tan\theta = \frac{AB}{BC}$$

$$\Rightarrow Tan\theta = 1$$

$$\Rightarrow \theta = 45^0$$

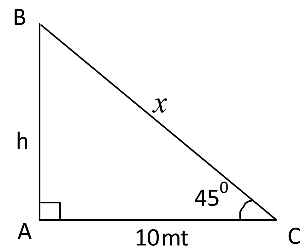
ANS: $a \to q, b \to t, c \to r, d \to s$

25. a) From figure, $Tan45^0 = \frac{h}{10}$ $\quad\quad \sin 45^0 = \frac{h}{x}$

α

$$\Rightarrow 1 = \frac{h}{10} \quad\quad \Rightarrow \frac{1}{\sqrt{2}} = \frac{10}{x}$$

$$\Rightarrow h = 10mt \quad\quad \Rightarrow x = 10\sqrt{2}mt$$



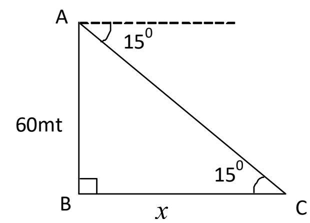
The length of the entire tree = AB + BC

= h + x

$= 10 + 10\sqrt{2}$

$= 10\left(\sqrt{2}+1\right)mt$

b) From figure $Tan15^0 = \frac{60}{x}$



$$\Rightarrow \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{60}{x}$$

$$\Rightarrow x = 60\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$$

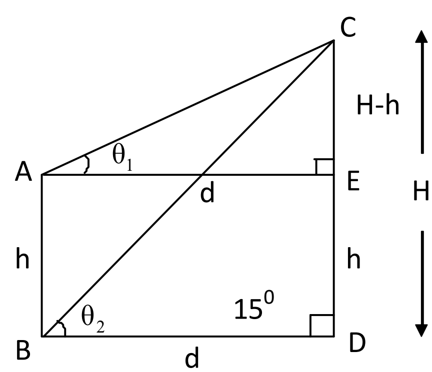
c) Hence, AB = Building, CD = Hill A, B = Positions of the observer

Now, $Tan\theta_1 = \frac{H-h}{d}$ $\quad\quad Tan\theta_2 = \frac{H}{d}$

$$\Rightarrow d = \frac{H-h}{Tan\theta_1} \text{ ...... } (i) \quad\quad \Rightarrow d = \frac{H}{Tan\theta_2} \text{ ...... } (ii)$$

from (i) & (ii)

$$\frac{H-h}{Tan\theta_1} = \frac{H}{Tan\theta_2}$$



$$\Rightarrow \frac{H}{Tan\theta_1} - \frac{H}{Tan\theta_2} = \frac{h}{Tan\theta_1}$$

$$\Rightarrow H\left(\frac{Tan\theta_2 - Tan\theta_1}{Tan\theta_1 . Tan\theta_2}\right) = \frac{h}{Tan\theta_1}$$

$$\Rightarrow H = \frac{h.Tan\theta_2}{Tan\theta_2 - Tan\theta_1}$$

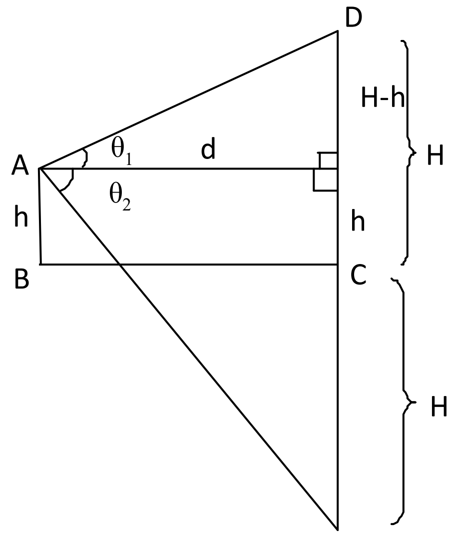
d) From the figure

$Tan\theta_1 = \frac{H-h}{d}$ $\quad\quad Tan\theta_2 = \frac{H+h}{d}$

$$\Rightarrow d = \frac{H-h}{Tan\theta_1} \text{ ...... } (i) \quad\quad \Rightarrow d = \frac{H+h}{Tan\theta_2} \text{ ...... } (ii)$$

from (i) & (ii)

$$\therefore \frac{H-h}{Tan\theta_1} = \frac{H+h}{Tan\theta_2}$$



$$\Rightarrow \frac{H}{Tan\theta 1} - \frac{H}{Tan\theta_2} = \frac{h}{Tan\theta_2} + \frac{h}{Tan\theta_1}$$

$$\Rightarrow H\left(Tan\theta_2 - Tan\theta_1\right) = h\left(Tan\theta_1 + Tan\theta_2\right)$$

$$\Rightarrow H = \frac{h\left(Tan\theta_1 + Tan\theta_2\right)}{\left(Tan\theta_2 - Tan\theta_1\right)}$$

$$\Rightarrow H = \frac{h\left(\frac{\sin\theta_1}{\cos\theta_1} + \frac{\sin\theta_2}{\cos\theta_2}\right)}{\left(\frac{\sin\theta_2}{\cos\theta_2} - \frac{\sin\theta_1}{\cos\theta_1}\right)}$$

$$\Rightarrow H = h\left(\frac{\sin\theta_1 \cdot \cos\theta_2 + \cos\theta_1 \cdot \sin\theta_2}{\sin\theta_2 \cdot \cos\theta_2 - \cos\theta_2 \cdot \sin\theta_1}\right)$$

$$\Rightarrow H = \frac{h\sin\left(\theta_1 + \theta_2\right)}{\sin\left(\theta_2 - \theta 1\right)}$$

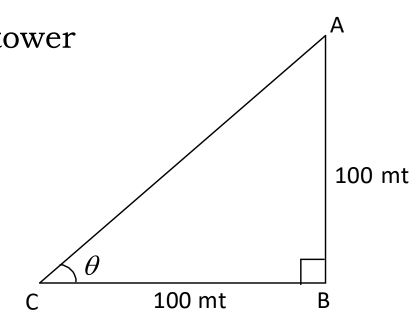
ANS: $a \rightarrow t, b \rightarrow q, c \rightarrow r, d \rightarrow s$

LEARNERS TASK

1. AB = Tower

θ = Angle of elevation of top of the tower

Given AB = BC = 10 unit



$$\therefore Tan\theta = \frac{AB}{BC}$$

$$\Rightarrow Tan\theta = \frac{100}{100}$$

$$\Rightarrow Tan\theta = 1$$

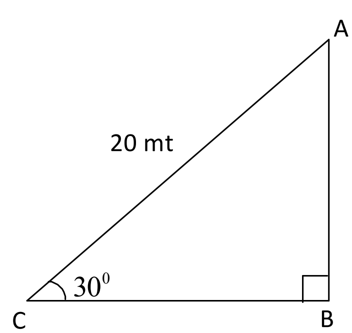
$$\Rightarrow \theta = 45^0$$

Ans: C

2. From tigure

AC = Rope

AB = Pole



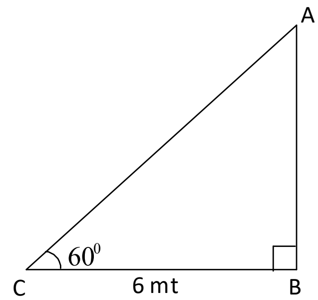
$$\therefore \sin 30^0 = \frac{AB}{AC}$$

$$\Rightarrow \frac{1}{2} = \frac{AB}{20}$$

$\Rightarrow \quad AB = 10mt$

Ans : C

3. From figure



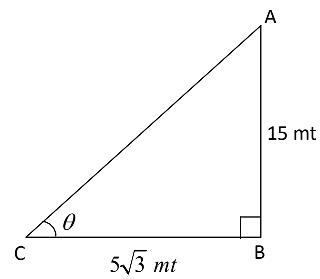
$$Tan60^0 = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{AB}{6}$$

$$\Rightarrow AB = 6\sqrt{3}\, mt$$

Ans : B

4. AB = Pole



BC = Shadow

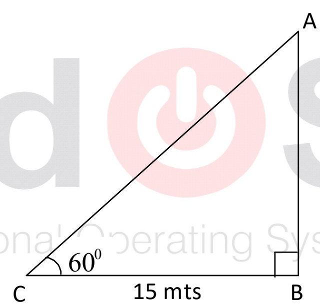
$$Tan\theta = \frac{15}{5\sqrt{3}}$$

$$\Rightarrow Tan\theta = \sqrt{3}$$

$$\Rightarrow \theta = 60^0$$

Ans : A

5. AB = tower



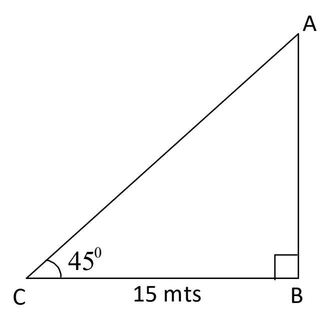
$$Tan60^0 = \frac{AB}{BC}$$

$$\Rightarrow \sqrt{3} = \frac{AB}{15}$$

$$\Rightarrow AB = 15\sqrt{3}$$

Ans : A

6. $$Tan45^0 = \frac{AB}{15}$$



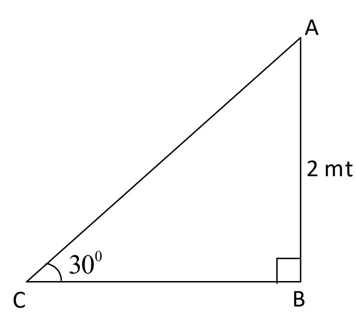
$$\Rightarrow 1 = \frac{AB}{15}$$

$$\Rightarrow AB = 15mt$$

Ans : D

7. In the diagram

AC = slide



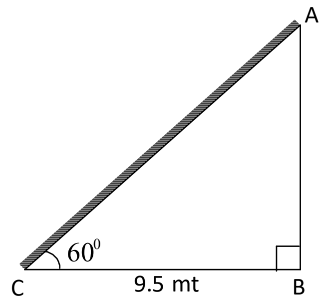
$$\therefore \sin 30^0 = \frac{AB}{AC}$$

$$\Rightarrow \frac{1}{2} = \frac{2}{AC}$$

$$\Rightarrow AC = 4mt$$

Ans : C

8. AC = Ladder



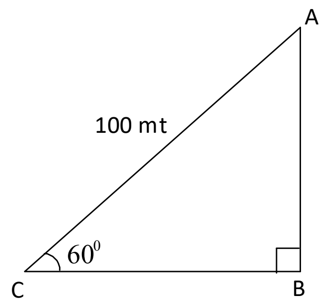
$$\cos 60^0 = \frac{BC}{AC}$$

$$\Rightarrow \frac{1}{2} = \frac{9.5}{AC}$$

$$\Rightarrow AC = 19\,mt$$

9. AC = string

AB = Height of the kite



$$\sin 60^0 = \frac{AB}{AC}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AB}{100}$$

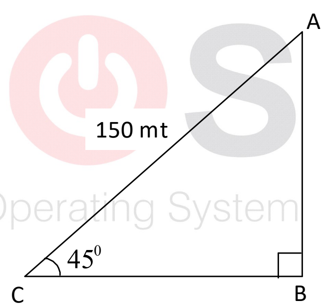
$$\Rightarrow AB = 50\sqrt{3}$$

Ans : C

10. In the figure

AC = Bridge

BC = width of the river



Now, $\cos 45^0 = \frac{BC}{AC}$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{BC}{150}$$

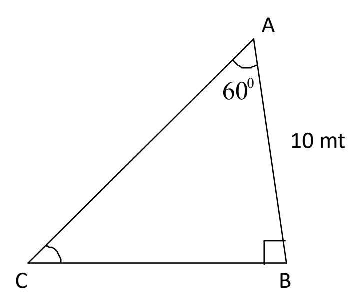
$$\Rightarrow BC = \frac{150}{\sqrt{2}} = 75\sqrt{2}$$

Ans : B

JEE MAINS LEVEL QUESTIONS

1. In the figure AB = Height of the pole

AC = Length of the rope



$$\cos 60^0 = \frac{AB}{AC}$$

$$\Rightarrow \frac{1}{2} = \frac{10}{AC}$$

$$\Rightarrow AC = 20mt$$

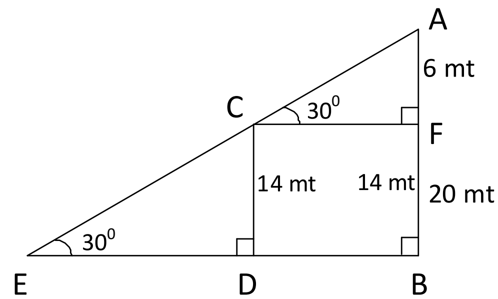
$\therefore$ The length each rope = 20mt

$\therefore$ the length of 3 ropes = 3x20 = 60mt

Ans : D

2. In the figure

AB, CD are poles and AC is rope



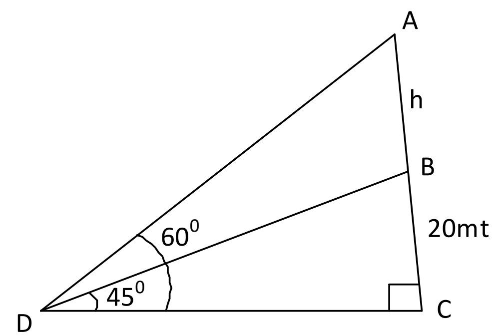
$\therefore \sin 30^0 = \dfrac{AF}{AC}$

$\Rightarrow \dfrac{1}{2} = \dfrac{6}{AC}$

$\Rightarrow AC = 12$

Ans : A

3. In the diagram AB = tower BC = Building



$Tan45^0 = \dfrac{BC}{CD}$

$\Rightarrow 1 = \dfrac{20}{CD}$

$\Rightarrow CD = 20mt$

Now, $Tan60^0 = \dfrac{AC}{CD}$

$\Rightarrow \sqrt{3} = \dfrac{20+h}{20}$

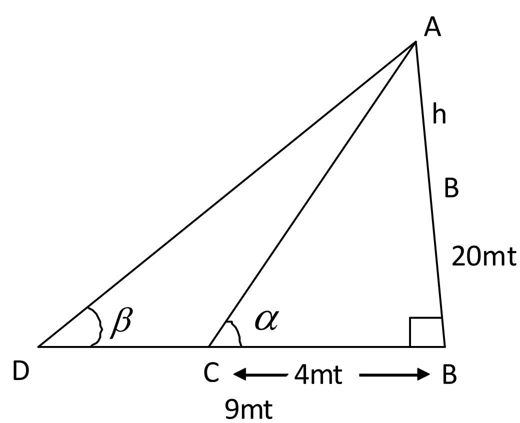
$\Rightarrow 20 + h = 20\sqrt{3}$

$\Rightarrow h = 20\sqrt{3} - 20$

$\Rightarrow h = 20\left(\sqrt{3} - 1\right)$

$\Rightarrow h = 14mt$ (approx)

4. From figure $\alpha + \beta = 90^0$



$\Rightarrow Tan\left(\alpha + \beta\right) = \tan 90^0$

$\Rightarrow \dfrac{Tan\alpha + Tan\beta}{1 - Tan\alpha . Tan\beta} = not\ defind$

$\Rightarrow 1 - Tan\alpha . Tan\beta = 0$

$\Rightarrow Tan\alpha . Tan\beta = 1$

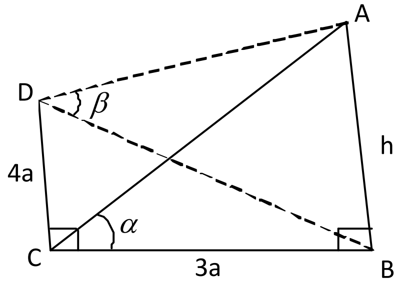
$\Rightarrow \dfrac{h}{4} . \dfrac{h}{9} = 1$

$\Rightarrow h^2 = 36$

$\Rightarrow h = 6mt$

Ans : A

5. In the figure AB = Tower BC = First direction CD = second direction
Given BC = 3a and CD = 4a



Now In ΔBCD

$$\begin{aligned} BD^2 &= BC^2+CD^2 \\ &= (3a)^2+(4a)^2 \\ &= 9a^2+16a^2 \\ &= 25a^2 \end{aligned}$$

$\therefore\ BD = 5a$

Now, $Tan\alpha = \dfrac{h}{3a}$

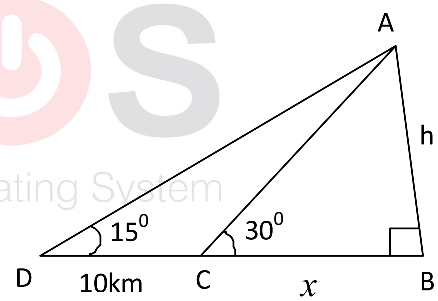
Again, $Tan\beta = \dfrac{h}{5a}$

$\Rightarrow h = 5a\tan\beta\ also\ h = 3a\ \tan\alpha$

Ans : A,B

6. AB = Mountain

Let $BC = x\,km$



Now, $Tan30^0 = \dfrac{h}{x}$ $\quad\quad$ $Tan15^0 = \dfrac{h}{10+x}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{h}{x}$ $\quad\quad$ $\Rightarrow 2-\sqrt{3} = \dfrac{h}{10+x}$

$\Rightarrow x = \sqrt{3}h$ $\quad\quad$ $\Rightarrow 10+x = \dfrac{h}{2-\sqrt{3}}$

Now, $10+\sqrt{3}h = \dfrac{h}{2-\sqrt{3}}$

$\Rightarrow \left(10+\sqrt{3}h\right)\left(2-\sqrt{3}\right) = h$

$\Rightarrow 20-10\sqrt{3}+2\sqrt{3}h-3h = h$

$\therefore\ 20-10\sqrt{3} = h+3h-2\sqrt{3}\,h$

$\Rightarrow\ 20-10\sqrt{3} = 4h-2\sqrt{3}\,h$

$\Rightarrow\ 20-10\sqrt{3} = 2h\left(2-\sqrt{3}\right)$

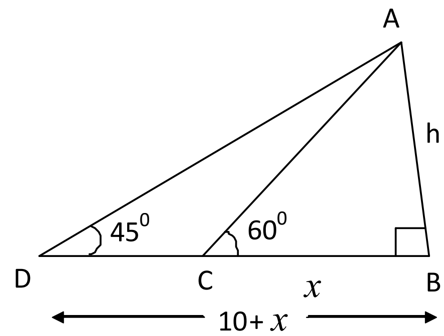
$\Rightarrow\ 10\left(2-\sqrt{3}\right) = 2h\left(2-\sqrt{3}\right)$

$\Rightarrow 2h = 10$

$\Rightarrow h = 5\,km$

Ans : D

7. $Tan60^0 = \frac{h}{x}$



$\Rightarrow \sqrt{3} = \frac{h}{x}$

$\Rightarrow x = \frac{h}{\sqrt{3}}$

Also, $Tan45^0 = \frac{h}{10+x}$

$\Rightarrow 1 = \frac{h}{10+x}$

$\Rightarrow 10 + x = h$

$\Rightarrow 10 + \frac{h}{\sqrt{3}} = h$

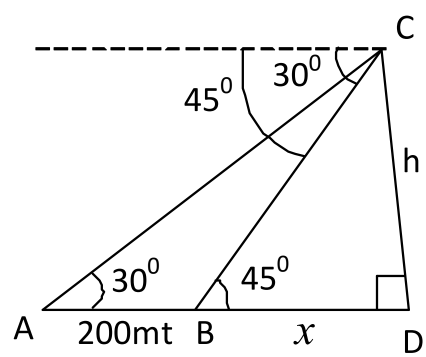
$\Rightarrow 10\sqrt{3} + h = \sqrt{3}\,h$

$\Rightarrow \left(\sqrt{3}-1\right)h = 10\sqrt{3}$

$\Rightarrow h = \frac{10\sqrt{3}}{\sqrt{3}-1}$

$\Rightarrow h = 23.66\,mt$

8. From the figure



$Tan45^0 = \frac{h}{x}$

$\Rightarrow 1 = \frac{h}{x}$

$\Rightarrow x = h$ .... (i)

$Tan30^0 = \frac{h}{200+x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{200+h}$ from (i)

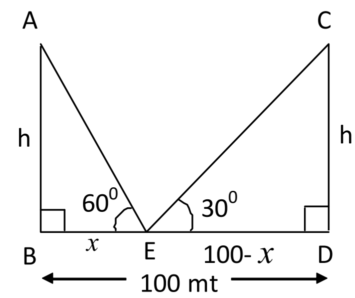
$\Rightarrow 200 + h = \sqrt{3}\,h$

$\Rightarrow \left(\sqrt{3}-1\right)h = 200$

$\Rightarrow h = \frac{200}{\sqrt{3}-1} = 100\left(\sqrt{3}-1\right) = 273.2mt$

Ans : B

9. In the figure AB and CD are two pillars of equal height. BD = 100 mt



Now, $Tan60^0 = \frac{h}{x}$ $\qquad$ $Tan30^0 = \frac{h}{100-x}$

$\Rightarrow \sqrt{3} = \frac{h}{x}$ $\qquad$ $\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{100-x}$

$\Rightarrow x = \frac{h}{\sqrt{3}}$ .... (i)

$\Rightarrow 100 - x = \sqrt{3}\,h$

$\Rightarrow 100 - \frac{h}{\sqrt{3}} = \sqrt{3}\,h$ from (i)

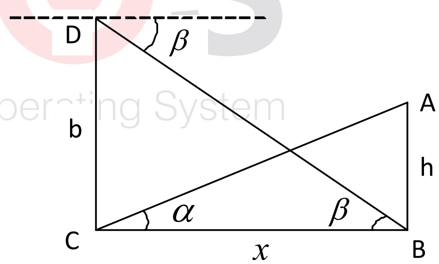
$\Rightarrow 100\sqrt{3} - h = 3h$

$\Rightarrow 4h = 100\sqrt{3}$

$\Rightarrow h = 25\sqrt{3}$

$\Rightarrow h = 43.3\,mt$

10. In the figure AB = Tower



$Tan\alpha = \frac{h}{x}$

$\Rightarrow x = h\cot\alpha$ .... (i)

$Tan\beta = \frac{b}{x}$

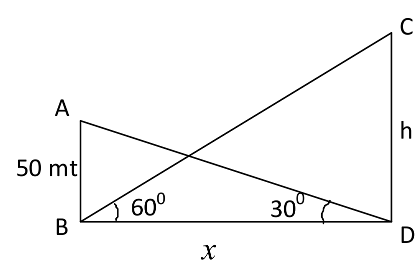
$\Rightarrow x = b\cot\beta$

$\Rightarrow h\cot\alpha = b\cot\beta$ $\quad (\because from\,(i))$

$\Rightarrow h = b\cot\beta\tan\alpha$

Ans : A

11. In the figure AB = Tower CD = Hill



$Tan30^0 = \frac{50}{x}$ $\qquad$ $Tan60^0 = \frac{h}{x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{50}{x}$ $\qquad$ $\Rightarrow \sqrt{3} = \frac{h}{50\sqrt{3}}$ (from (i))

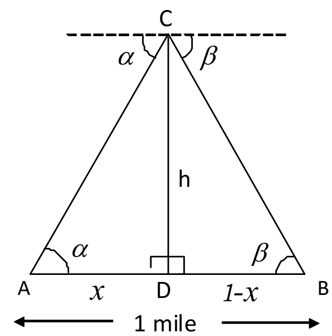
$\Rightarrow x = 50\sqrt{3}$ .... (i) $\qquad$ $\Rightarrow h = 150\,mt$

ADVANCED LEVEL QUESTIONS

12. From the figure

$$\tan\alpha = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\tan\alpha} \quad \ldots\ldots\ldots\ldots\ (1)$$



Again $Tan\beta = \frac{h}{1-x}$

$$\Rightarrow 1-x = \frac{h}{Tan\beta}$$

$$\Rightarrow 1-\frac{h}{Tan\alpha} = \frac{h}{Tan\beta} \qquad (\because\ from\,(i))$$

$$\Rightarrow \frac{h}{Tan\alpha} + \frac{h}{Tan\beta} = 1$$

$$\Rightarrow h\left(\frac{Tan\alpha + Tan\beta}{Tan\alpha.Tan\beta}\right) = 1$$

$$\Rightarrow h = \frac{Tan\alpha.Tan\beta}{Tan\alpha + Tan\beta}$$

$$\Rightarrow h = \frac{\frac{\sin\alpha}{\cos\alpha}.\frac{\sin\beta}{\cos\beta}}{\frac{\sin\alpha}{\cos\alpha}+\frac{\sin\beta}{\cos\beta}}$$

$$\Rightarrow h = \frac{\sin\alpha.\sin\beta}{\sin\alpha\cos\beta + \cos\alpha.\sin\beta}$$

$$\Rightarrow h = \frac{\sin\alpha.\sin\beta}{\sin(\alpha+\beta)}$$

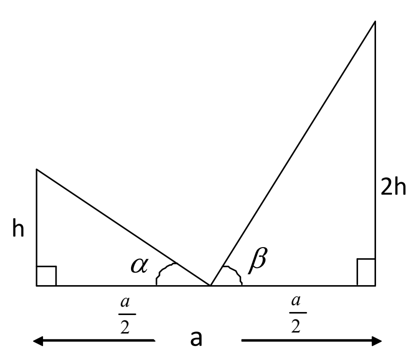
Ans : A,B

13. Given $\alpha + \beta = 90^0$

$$\Rightarrow Tan\alpha.Tan\beta = 1$$

$$\Rightarrow \frac{h}{\left(\frac{a}{2}\right)}.\frac{2h}{\left(\frac{a}{2}\right)} = 1$$

$$\Rightarrow \frac{8h^2}{a^2} = 1$$



$$\Rightarrow h^2 = \frac{a^2}{8}$$

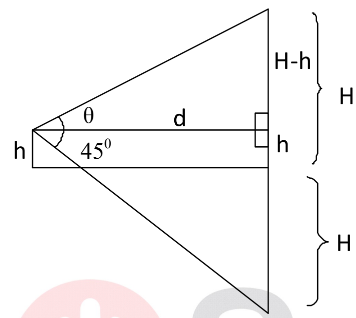
$$\Rightarrow h = \frac{a}{2\sqrt{2}}$$

$\therefore$ The height of the smaller pole $= \frac{a}{2\sqrt{2}}$ mt

The height of the larger pole $= 2h = \frac{a}{\sqrt{2}} mt$

Ans : B,C

14. $Tan 45^0 = \frac{H+h}{d}$



$$\Rightarrow 1 = \frac{H+h}{d}$$

$$\Rightarrow d = H + h \qquad \text{............} \ (i)$$

Again $Tan\theta = \frac{H-h}{d}$

$$\Rightarrow Tan\theta = \frac{H-h}{H+h} \qquad (from\ (i))$$

$$\Rightarrow H\ Tan\theta + h\tan\theta = H - h$$

$$\Rightarrow H\ (1 - Tan\theta) = h(1 + Tan\theta)$$

$$\Rightarrow H = h\left(\frac{1+Tan\theta}{1-Tan\theta}\right)$$

$$\Rightarrow H = h\left(\frac{\cot\theta+1}{\cot\theta-1}\right)$$

$$\Rightarrow H = h\cot\left(45^0 - \theta\right)$$

Ans : B

15. We have $\frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$

Given $\alpha + \beta = 105^0$

Let $\alpha = 60^0\ and\ \beta = 45^0$

$$\therefore \frac{a}{b} = \frac{\cos 60^0 - \cos 45^0}{\sin 45^0 - \sin 60^0}$$

$$\Rightarrow \frac{a}{b} = \frac{\frac{1}{2} - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}}$$

$$\Rightarrow \frac{a}{b} = \frac{\sqrt{2}-2}{2-\sqrt{6}}$$

$$\Rightarrow \frac{a}{b} = \frac{\sqrt{2}\left(1-\sqrt{2}\right)}{\sqrt{2}\left(\sqrt{2}-\sqrt{3}\right)} = \frac{\sqrt{2}-1}{\sqrt{3}-\sqrt{2}}$$

Ans : A

16. Given $\frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$

$$\Rightarrow \frac{a}{b} = Tan\left(\frac{\alpha+\beta}{2}\right)$$

$$\Rightarrow \frac{a}{b} = Tan\left(\frac{30^0}{2}\right) \qquad \sin ce \;\; \alpha+\beta = 30^0$$

$$\Rightarrow \frac{a}{b} = Tan15^0$$

$$\Rightarrow \frac{a}{b} = 2-\sqrt{3}$$

$$\Rightarrow \frac{a}{b} = \frac{1}{2-\sqrt{3}} = 2+\sqrt{3}$$

$$\Rightarrow \frac{b}{a} + 1 = 2+\sqrt{3}+1$$

$$\Rightarrow a+b = a\left(3+\sqrt{3}\right)$$

Ans : B

17. Given $\frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$

$$\Rightarrow \frac{a}{b} = Tan\left(\frac{\alpha+\beta}{2}\right)$$

$$\Rightarrow \frac{a}{b} = Tan\left(\frac{0^0}{2}\right) \quad \sin ce\, \alpha+\beta = 0^0$$

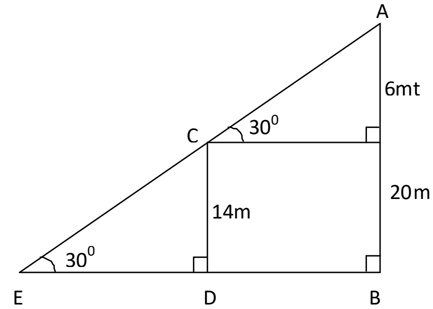
$$\Rightarrow \frac{a}{b} = 0$$

$$\Rightarrow a = 0$$

Ans : C

18. In the figure AC = The length of the wire

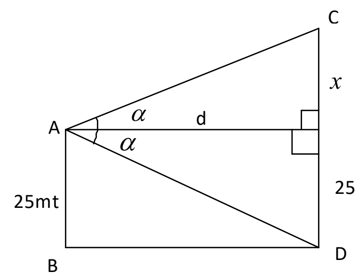
AB, CD are the poles.



Now, $\sin 30^0 = \frac{6}{AC}$ $\Rightarrow AC = 12mt$

$\Rightarrow \frac{1}{2} = \frac{6}{AC}$

19. In the diagram AB = Height of the cliff
CD = Height of the tower



Now $Tan\alpha = \frac{25}{d} = \frac{x}{d}$

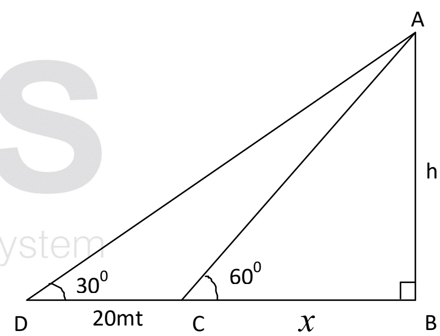
$\Rightarrow x = 25$

$\therefore$ The height of the tower = 25+25 = 50mt

20. In the figure

a) AB = Height of the tree

BC = Width of the river



Now, $Tan60^0 = \frac{h}{x}$ $Tan30^0 = \frac{h}{20+x}$

$\Rightarrow \sqrt{3} = \frac{h}{x}$ $\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{20+x}$

$\Rightarrow x = \frac{h}{\sqrt{3}}$ $\Rightarrow 20 + x = \sqrt{3}h$

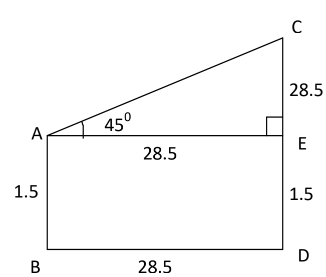
$\Rightarrow 20 + \frac{h}{\sqrt{3}} = \sqrt{3}h$

$\Rightarrow \sqrt{3}h - \frac{h}{\sqrt{3}} = 20$

$\Rightarrow 2h = 20\sqrt{3}$

$\Rightarrow h = 10\sqrt{3}$

b) In the figure AB = observer CD = Tower



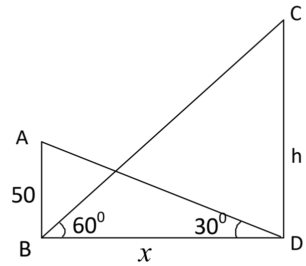
$Tan45^0 = \frac{CE}{AE}$

$\Rightarrow 1 = \frac{CE}{28.5}$

$\Rightarrow CE = 28.5$

$\therefore$ The height of the observer = 28.5+1.5 = 30mt

c) In the figure AB = tower CD = Hill



$$Tan30^0 = \frac{50}{x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{50}{x}$$

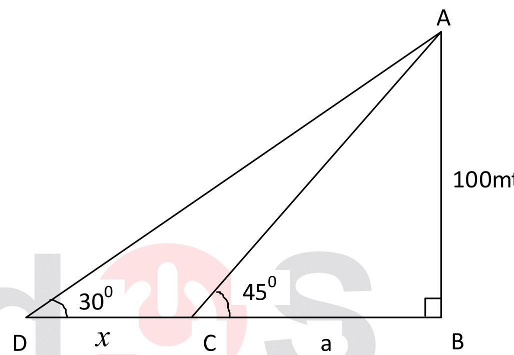
$$\Rightarrow x = 50\sqrt{3}$$

Again $Tan60^0 = \frac{h}{x}$

$$\Rightarrow \sqrt{3} = \frac{h}{50\sqrt{3}}$$

$$\Rightarrow h = 150mt$$

d) AB = Tower



$$Tan45^0 = \frac{100}{a}$$

$$\Rightarrow 1 = \frac{100}{a}$$

$\Rightarrow$ a = 100

Now, $Tan30^0 = \frac{100}{x+a}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100}{x+100}$$

$$\Rightarrow x+100 = 100\sqrt{3}$$

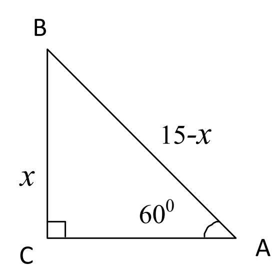
$$\Rightarrow x = 100\sqrt{3} - 100$$

$$\Rightarrow x = 100\left(\sqrt{3}-1\right)$$

**ADDITIONAL PRACTICE QUESTINS
FOR STUDENDTS**

1. See the answer of Q.N0.20, bit no:C

2. ABC is the height of the tree before it was broken



Given ABC = 15mt
Let BC = xmt
$\therefore$ AC = (15-x)mt

Now, $\sin 60^0 = \frac{x}{15-x}$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{x}{15-x}$$

$$\Rightarrow 15\sqrt{3} - \sqrt{3}x = 2x$$

$$\Rightarrow (2+\sqrt{3})x = 15\sqrt{3}$$

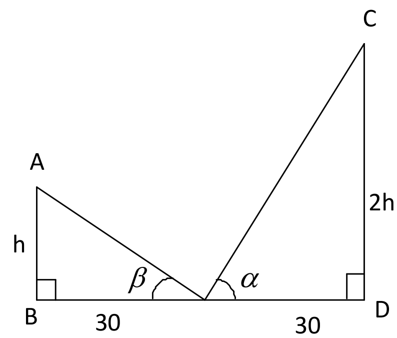
$\Rightarrow x = \frac{15\sqrt{3}}{2+\sqrt{3}} \text{x} \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$\Rightarrow x = 15\sqrt{3}\left(2-\sqrt{3}\right)$

Ans : B

3. In the figure AB, CD are two vertial poles. Let AB = hmts then CD = 2h mts

Given $\alpha + \beta = 90^0$



$\Rightarrow Tan\alpha.Tan\beta = 1$

$\Rightarrow \frac{2h}{30}.\frac{h}{30} = 1$

$\Rightarrow 2h^2 = 900$

$\Rightarrow h^2 = 450$

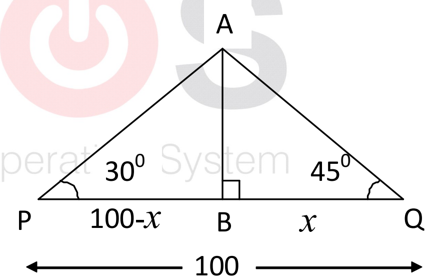
$\Rightarrow h = \sqrt{450}$

$\Rightarrow h = 21.21$

$\therefore$ The heights of the towers are 21.21mt and 42.42mt

4. In the diagram PQ = width of the river AB = Height of the tree.

Now, $Tan45^0 = \frac{AB}{x}$



$\Rightarrow 1 = \frac{AB}{x}$

$\Rightarrow x = AB$

Again $Tan30^0 = \frac{AB}{100-x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{x}{100-x}$

$\Rightarrow 100 - x = \sqrt{3}x$

$\Rightarrow 100 = \sqrt{3}x + x$

$\Rightarrow 100 = \left(\sqrt{3}+1\right)x$

$\Rightarrow x = \frac{100}{\sqrt{3}+1}$

$\Rightarrow x = \frac{100}{\sqrt{3}+1} \text{x} \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$\Rightarrow x = 50\left(\sqrt{3}-1\right)$

$\Rightarrow x = 36.5$

Ans : D