POLYNOMIALS

SYNOPSIS -1

Indtroduction :

Algebra is that branch of mathematics which explain the relation among the constants and variables.

Constants and variables :

Generally in Algebra, two types of symbols are used constants and variables (literals)

Constant :

It is a symbol whose value always remains the same, whatever lthe situation be, it is represented by C (or) K

Ex : $\frac{7}{2}, \sqrt{3}, -10, e, \pi$

Variable :

It is a symbol whose value changes according the situation

Ex : $x, y, z,\ bx,\ c+y,\ 9z\ e+c$

Algebraic Expression :

An algebraic expression is a collection of terms separated by + and - symbol.

Ex : $7x+11y,\ 10y-3x,\ 3x+by-dz$

The various parts of an Algebraic expression that are seperated by + (or) - sign are called terms.

Ex :	Algebraic expression	NO.of terms	terms
1)	$-36x$	1	$-36x$
2)	$ax+5y-dz$	3	ax, -5y and dz
3)	$\frac{7}{x}-\frac{6}{y}-\frac{yz}{9}+7$	4	$\frac{7}{x}, \frac{-6}{y}, \frac{-yz}{9}$ *and* 7

i) Monomial :

Mono means one an Algebraic expression baving only one term is called a monomial.

Ex : $8, 7a, -11xyz$ etc

ii) Binomial :

"Bi" means two. An algebraic expression having two terms is called a binomial

for Ex : $ax+by,\ -1+\frac{a}{2},\ 2x^2+4z$ etc

iii) Trinomial :

"Tri" means there an algebraic expression having three terms is called a trinomial

4) Multinomial :

An algebraic expression having two or more terms is called a multinomial

Ex : $3a - 4x + 6y + z$

5) Factors and Coefficents :

Each combination of the constants and varilables which form a term is called factor.

Coefficent :

Any factor of a term is called the coefficent of the remaining term

for Ex : i) In ||x, || is coefficent of x

ii) In $-5x^2y$, 5 is coefficent of $-x^2y$, -5 is coefficent of x^2y

Note :

$-13 = -13 \times 1 \ -13 \times x^0$

coefficent of x^0 is (-13)

6) Definition of polynomial :

A polynomial is an algebraic expression in which each variable involved has power (exponent) a whole number.

Ex : $11x^6 - \sqrt{7}x^5 + 8z$, the power of variables are in $11x^6$ _____6,

$-\sqrt{7}x^5$ _______5

8z ________$8z^1$_____1

7) Polynomial in one variable :

The algebraic expression like i)12x, ii) 13x - 1 $11y^2 - 6y + \frac{1}{3}$ etc

Polynomial in two or more variables :

An algebraic expression, whose terms or more variables (literals) such that the exponent of each variable is a whole number is called a polynomial in two or more variables

Ex : 1) $3x^2 - 6xy + 8y^2$ is a polynomial in two variables x and y

$y + zy^3 - 8xy^2z - 15$ is a polynomial in three variables x,y and z

Degree of a polynomial

The greatest power (exponent) of the terms of a polynomial is called degree of a polynomial

Ex : 1) $5x^3 - 7x^8 + 1$ ____ Degree___8

2) 3x _____$3x^1$_____Degree ___1

3) $2m - 7m^8 + m^{13}$___Degree ___13

8) Zeroes of a polynomial :

If for x = K the value of a polynomial p(x) is ‘0’ i.e p(k) = 0 called zero of the polynomial

a) A zero of polynomial need not be zero

Ex : Let p(x) = 3x +1

$\therefore$ Let p(x) = 0 $\Rightarrow$ 3x+1=0$\Rightarrow$x = $-\frac{1}{3}$

$\therefore\ p\left(-\frac{1}{3}\right)=3\left(-\frac{1}{3}\right)+1=-1+1=0$

$\therefore\ -\frac{1}{3}$ is zero of the polynomial

Let $x^4=0 \Rightarrow x=0$

$\therefore$ '0' is the zero of the polynomial $p(x)=x^4$

(c) Number of zeros of the polynmial is equal to degree of the polynomial

For ex: Let $p(x)=x^2-1$

Let $x^2-1=0 \Rightarrow x^2=1 \Rightarrow x \pm 1$

$\therefore$ Zeroes are -1 & +1

$\therefore$ clearly degree is same as the number of zeroes.

(d)A polynomial having n^{th} degree can have at most 'n' number of zeroes.

Ex : p(x) =x^n -1 can have at most 'n' number of zeroes.

The process of a Quadratic polynomial $p(x)=ax^2+bx+c(a \neq 0)$ are the x - co- ordinates of the points where the graph intersect the x-axis

9) There are three types of graphs

Case I : $p(x)=ax^2+bx+c$

If discriminant = $b^2-4ac>0$ then 'x' has two real and distinct roots it seems graph intersects x axis in two points

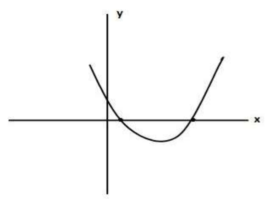


Fig (a)

$p(x)=x^2-7x+12(a>0)$

$=(x-3)(x-4)$

graph intersects x-axis in (3,0) and (4,0)

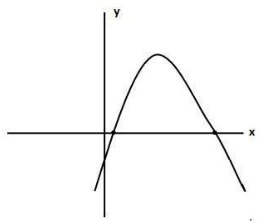


Fig (b)

$p(x)=-x^2-x+12(a<0)$

$=-x^2-4x+3x+12$

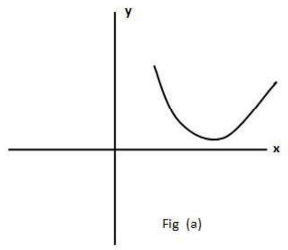
$=-x(x+4)+3(x+4)$

$=(x+4)(3-x)$

graph intersects x-axis at (-4, 0) and (3,0)

Case II : $p(x) = ax^2 + bx + c$

If discriminant $b^2 - 4ac < 0$, then x his two imaginary distinct values it seems graph does n't Intersect x-axis



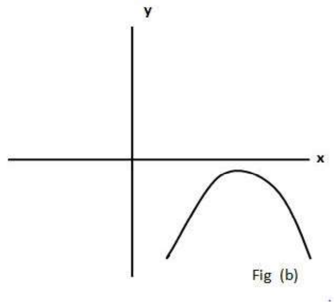
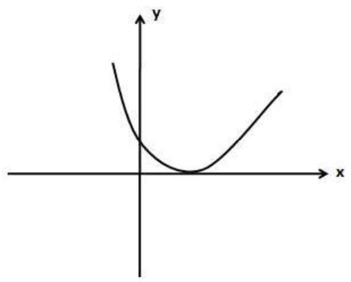


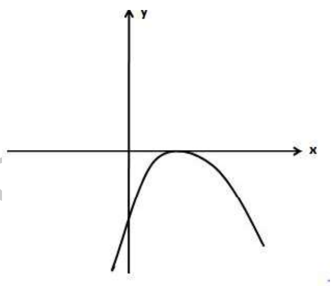
Fig (C) Fig (D)

ex : $x^2 + x - 7$ **ex :** $-x^2 + 2x - 8$

Case III: $p(x) = ax^2 + bx + c$

If discrimant , then x has two real and equal roots (roots will co-incide) it seems graph touches x-axis at only one point





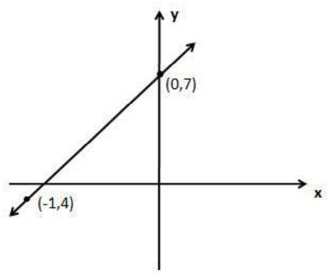
ex : $x^2 - 6x + 9$ **ex :** $-x^2 + 6x - 9$

10. Geometric meaning of the zeroes of polynomial

Case : I :

Let us take a linear polynomial $y = p(x) = 3x + 7$

x	0	-1
y=3x+7	7	4
points	(0,7)	(-1,4)

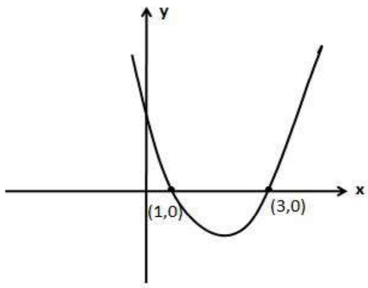


$\therefore$ Graph of a polynomial is straight line passing through the point (-1, 4), (0, 7)

Case II: Let us consider a Quadratic polynomial

$p(x) = x^2 - 3x + 4$

x	1	2	3	4
$y=x^2-4x+3$	0	-1	0	3
points	(1,0)	(2,-1)	(3, 0)	(4, 3)



$\therefore$ Graph of a polynomial is a parabola

Case III : Let us consider a cubic polynomial

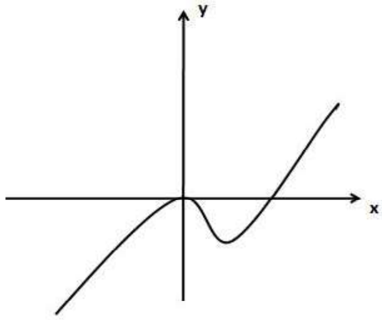
$p(x) = x^3 - x^2$

Let $p(x) = 0 \Rightarrow x^3 - x^2 = 0$

$\Rightarrow x^2(x-1) = 0$

$\Rightarrow x = 0, x = 1$

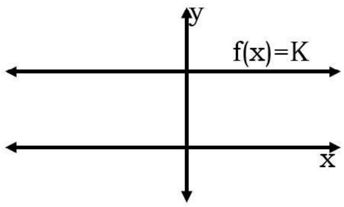
$\therefore$ The curve $p(x) = x^3 - x^2$ intersects x-axis at (0, 0) and (1, 0)



11) Types of polynomials :

i) Constant polynomial : A polynomial having degree '0' is called constant polynomial

For ex: $\sqrt{11}, 3, -7/8, 1$ and so on



Degree of constant polynomial '0' because $-7/8 = -7/8 \times 1 = \frac{-7}{8} x^{o}$

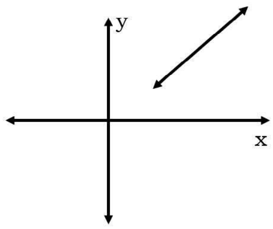
ii)**Zero polynomial :** '0' is called Zero polynomial. We can't define degree of zero polynomial

$\because \quad 0 = 0.x^n + 0.x^{n-1} + ...$

as 'n' may be any number. We can't say degree

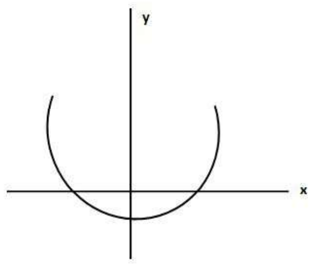
$\therefore$ Degree is not defined

iii)**Linear polynomial :** A polynomial with degree one is called Linear polynomial, Graph of a linear polynomial is a straight line



For ex : $11x, \frac{3x}{2}, \frac{\sqrt{2}}{x^{-1}}$ $3x+1, 5p-3, 6q-\frac{13}{2}$ and so on

iv) **Quadratic polynomial** :A polynomial of degree '2' is called Quadratic polynomial



For ex: $3x^2 + 5x + 7, x^2 - 5x + 6, ax^2 + bx + c (a \neq 0)$

Graph of Quadratic polynomial is parabola

v) **Cubic polynomial:** A polynomial of degree '3' is called cubic polynomial

For ex : $3x^3+7x^2+x+1, 9x^3+11x^2+20x-5, ax^3+bx^2+cx+d(a\neq 0)$ and so on.

v) **Biquadratic polynomial :** A polynomial of degree '4' is called as a Quadratic polynomial. It is also known as a Quadratic polynomial

For ex : $5x^4+6x^3+7x^2-11x+1, \sqrt{20}x^4+11x^3-6x+1,$

$$ax^4+bx^3+cx^2+dx+e(a\neq 0)$$

11. **Value of a polynomial :** If $p(x)$ is a polynomial at $x = K$. The Value of polynomial is defined as $p(K)$, it may be zero or any non-zero real number

For ex : 1) For a polynomial $p(x)=x^2-5x+6$ at $x = 2$, $p(2)=2^2-5.2+6=0$

$$\therefore p(2)=0$$

2) For a polynomial $p(x)=8x^3-7x^2+6x-5$ at $x = 1$

$$p(1)=8.1^3-7.1^2+6.1-5$$
$$=8-7+6-5=2$$
$$p(1)=2\neq 0$$

12. **Remainder Theorem**

Let $p(x)$ be a polynomial of degree greater than or equal to one, if $p(x)$ is divided by $(x - a)$. Then remainder $R = p(a)$

$\therefore$ $p(a)$ may be zero (or) non-zero real number

Proof : If $p(x)$ is divided by (x-a). Let we have obtained quotient $Q(x)$ and remainder $r(x)$

$\therefore$ by divisior Algorithm

$p(x)=(x-a)q(x)+r$

if $x=a \Rightarrow p(a)=r$

For ex : Let $p(x)=x^2-3x+5$, if $p(x)$ is divided by x - 2 $x-2 = 0$

$\therefore r=p(2)=2^2-3.2+5=3$ $x = 2$

$\therefore r=3$

Remarks :

Divisor	Remainder
$x-a$	$f(a)$
$x+a$	$f(-a)$
$ax+b$	$f(-b/a)$
$ax-b$	$f(b/a)$

Some Algebraic Identities

1. $(a+b)^2 = a^2 + 2ab + b^2$
2. $(a-b)^2 = a^2 - 2ab + b^2$
3. $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$
4. $(a+b)^2 - (a-b)^2 = 4ab$
5. $(a+b)(a-b) = a^2 - b^2$
6. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 = a^3 + b^3 + 3ab(a+b)$
7. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3 = a^3 - b^3 - 3ab(a-b)$
8. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
9. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
10. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + cbc + ca)$
11. $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$
12. If $a+b+c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$
13. $a^k + b^k = (a^{k-1} + b^{k-1})(a+b) - (a^{k-2} + b^{k-2})ab$

Special Products

1. $(x+a)(x+b) = x^2 + (a+b)xab$
2. $(ax+b)(cx+d) = acx^2 + (ad+bc)x + bd$
3. $(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$

Factor Theorem : Let $p(x)$ be a polynomial of any degree if $R = p(K) = 0$, then $(x-k)$ is said to be factor of $p(x)$

ex : Let $p(x) = x^2 - 5x + 6$

consider $p(2) = 2^2 - 5.2 + 6$

$= 4 - 10 + 6$

$p(2) = 0$

According factor theorem $x = 2 \Rightarrow (x-2)$ is a factor of $p(x)$

Reamarks:

1) If (x - 1) is a factor of $f(x) = a_0.x^n + a_1x^{n-1} + ... + a_n$ then sum of the coefficient is equal to zero.

i.e., $a_1 + a_2 + a_3 + ... + a_n = 0$

Explanetion : $\because (x-1)$ is a factor of $f(x)$

$f(1) = 0$

$\Rightarrow a_0.1^n + a_1.1^{n-1} + ... + a_n = 0$

$\Rightarrow a_0 + a_1 + ... + a_n = 0$

$\Rightarrow$ sum of the coefficents = 0

2) If $(x+1)$ is a factor of $f(x)$, then sum of the coefficients of even powers of x = sum of the coefficients of odd powers of x

i.e., $a_0 + a_2 + a_4 + ... = a_1 + a_3 + a_5 + ...$

Explanetion : $\because (x+1)$ is a factor of $f(x)$

$f(-1) = 0$

i.e., $a_0(-1)^n a_1(-1)^{n-1} + a_2(-1)^{n-2} + ... + a_n = 0$

$a_0 - a_1 + a_2 - a_3 + ... + a_n = 0$

$a_0 + a_2 + a_4 + ... = a_1 + a_3 + a_5 + ...$

Note: Constant is taken as coefficient of even power of *x*.

3) $x^n - y^n$ is divisible by *x - y* for every positive integers '*n*'

For ex : 1) $x^2 - y^2$ is divisible by $(x-y)$ 2) $x^3 - y^3$ is divisible by $(x-y)$

4) $x^n - y^n$ is divisible by (*x*+*y*) for every positive even Integer *n*

For ex: $x^4 - y^4$ is divisible by (*x*+*y*)

5) $x^n + y^n$ is divisible by (*x*+*y*) for every odd positive Integer '*n*'

For ex : $x^3 + y^3$ is divisible by (*x*+*y*)

Synthetic division of Horner's method

Horner'S Method of synthetic deivision :

We sheall explain the method with the following examples :

To divide $x^4 + 4x^3 + 3x^2 - 4x - 4$ by (x-1)

$$
(\text{Multiplier} =)\ 1 \left|\begin{array}{ccccc} 1 & 4 & 3 & -4 & -4 \\ 0 & 1 & 5 & 8 & 4 \\ \hline 1 & 5 & 8 & 4 & 0 \end{array}\right.
$$

The quotient is $x^3 + 5x^2 + 8x + 4$

Explanation :

First horizontal row contains the multiplier which is obtained by the zero of x - 1, which is 1.

The remaining elements in the first horizontal row are the coefficients of descending power of x(The coefficinet of missing power of x, if any, that should be taken as zero).

To form the second horizontal row start with zero right under the second element of first row and add, the result is 1 which is first entry in the third row. Multiply this 1 with the multiplier 1 put the result under 4 (the third entry of the first row). Thus we get the second entry 1 in the second row. Then add 4 and 1 to get the second entry in the third row which is 5. Now multiply this 5 with the multiplier, put the product right under 3(the fourth entry of the first row). Add 3 amd 5 to get third entry in the third row. Thus the third entry in the third row is 8. Now multiply 8 with the multiplier 1, put the product under - 4 (the fifth entry of th first row). Add 8 and -4 to get the fourth entry of the third row. Repeat the same procedure to get zero as the fifth entry of the third row. The last entry in the third row stands for the remainder, while the first four figures stand for the coefficients of descending powers of x of quotient.

Thus the quotient is $x^3 + 5x^2 + 8x + 4$. We shall return to our problem.

$f(x) = x^4 + 4x^3 + 3x^2 - 4x - 4 = (x-1)(x^3 + 5x^2 +\_8x + 4)$

Now if we write $g(x) = x^3 - 5x^2 + 8x + 4$

$g(-1) = (-1)^3 + 5(-1)^2 + 8(-1) + 4 = 0$

$\therefore$ (x+1) is a factor of g(x). Hence a factor of f(x).

Now to divide g(x) by (x+1), the multiplier is -1, the zero of x + 1.

Let us once again apply synthetic division, to g(x).

$$\begin{array}{r|rrrr} & 1 & 5 & 8 & 4 \\ -1 & 0 & -1 & -4 & -4 \\ \hline & 1 & 4 & 4 & 0 \end{array}$$

The quotient is $x^2 + 4x + 4 = 0$

$\therefore$ $f(x) = (x-1)\,g(x) = (x-1)(x+1)(x^2 + 4x + 4) = (x-1)(x+1)(x+2)^2$

H.C.F and L.C.M of Polynomials :

If a polynomial $p(x)$ is a product of two polynomials $h(x)$ and $g(x)$ i.e.,

$f(x) = g(x) \times h(x)$ then $g(x)$ and $h(x)$ are said to be factor of *x*.

Ex : Let $f(x) = x^2 - 7x + 10$

$f(x) = (x-2)(x-5)$

Note : If $h(x)$ is a factor of $f(x)$

$\therefore$ $-h(x)$ is a factor of $f(x)$.

Highest common factor (H,C,F) or Greatest common divisior (G.C.D):

The product of the least powers of the common factors is said to be H.C.F of the given polynomials.

Let $f(x) = (x-1)^2(x-2).(x-3)^3 = (x-1)^2.(x-2)^1.(x-3)^3(x-4)^0$

$g(x) = (x-1)^3(x-4) = (x-1)^3.(x-2)^0.(x-3)^0.(x-4)^1$

$\therefore$ H.C.F = $(x-1)^2.(x-2)^0.(x-3)^0.(x-4)^0 = (x-1)^2 \times 1 \times 1 \times 1 = (x-1)^2$

Least common multiple of polynomials (L.C.M)

The product of the highest powers of common factors is said to be L.C.M of the given polynomials.

Let $f(x) = (x-1)^1.(x-2)^2.(x-4)^1$

$g(x) = (x-1)^3.(x-2)^1.(x-4)^3$

$\therefore\ L.C.M = (x-1)^3.(x-2)^2.(x-4)^3$

WORK SHEET - I

1. Which of the following is not a polynomial

   A) $x^3 - 2x + \sqrt{3}$ B) $\frac{1}{\sqrt[3]{x}} + x^2 - x + 1$

   C) $\frac{1}{\sqrt[3]{x}} + x^2$ D) $\frac{1}{x^{-2}} + 5$

2. Which of the following is correct

   A)All Algebraic expressions are Polynomials

   B) All Polynomials are Algebraic expressions

   C)All Polynomials are not Algebraic expressions

   D)None of these

3. The degree of $x^{-2}\left(x^5 + \frac{3}{x^{-4}} + x^{11}\right)$

   A) 7 B) 10 C) 9 D) -11

4. The degree of $x^3 + 2x^4y^2 - \frac{x^5.y^3}{z^{-1}} + 13$

   A) 8 B) 11 C) -10 D) 9

5. If $x^2 - x + \frac{1}{2} = Ax^2 + Bx + c$, then what is $\frac{A}{B}$

   A) 1 B) -1 C) D) 0

6. If $p(x) = x^2 - 3x + 2$, clearly $a + b + c = 0$, then

   A) $a^3 + b^3 + c^3 = 3abc$ B) $a^3b^3c^3 = 3(a + b + c)$

   C) $a^3 + b^3 + c^3 = 3(a + b + c)$ D) All

7. The degree of the product cubic polynomial and a bioquadratic polynomial

   A) 9 B) 7 C) 14 D) 12

8. Degree of constant polynomial

   A) constant B) 1 C) 0 D) any real number

9. Zero of the polynomial 2x+1

   A) -2 B) -1 C) $-\frac{1}{2}$ D) none of these

10. If $ax^2 + bx + c$ is a quadratic polynomial

    A) a = 0 B) $a \neq 0$ C) a > 0 D) a < 0

JEE Mains

MCQ's with single correct answers type

1. Factors of are $x^2+2xy+y^2-z^2$

A) $(x+y+z)$ $(x-y+z)$ B) $(x+y+z)$ $(x-z+y)$ C) $(x+y+z)$ $(x-y+z)$ D) $(y+z-x)$ $(y+z+x)$

2. If $P(x) = 3x^2-5x$, then the zero of this polynomial

A) $0, 3/5$ B) $0, 5/3$ C) $0, -3/5$ D) $0, -5/3$

3. Then number of zero of the polynomial having degree 'n'

A) Atmost 'n' number of zeroes B) Atleast 'n' number of zeroes.

C) $(n+1)$ number of zeroes. D) $(n-1)$ number of zeros

4. The remainder when x^3-px^2+6x-p is divided by $x - p$

A) $5+p$ B) p^3 C) $5\ p$ D) $5 - p$

5. The remainder when $f(x)=ax^3+bx^2+cx+d$ is divided $(x - 1)$

A) $a+b+c-d$ B) $a+b = c+d$ C) $a+b+c+d$ D) $a+c = b+d$

6. If both $(x - 2)$ and $\left(x-\frac{1}{2}\right)$ are the factors of px^2+5x+r, then

A) $p^2 - r^2 = 0$ B) $p^2 + r^2 = 0$ C) $p^2.r^3 = 0$ D) $p^2 + r^2 = 0$

7. If $a^{1/3}+b^{1/3}+c^{1/3}=0$, then $(a+b+C)^3$

A) 54 abc B) 81 abc C) 27 abc D) 72 abc

8. If $a+b+c = 9$ and $ab+bc+ca = 2b$, then $a^2+b^2+c^2$.

A) 28 B) 27 C) 29 D) 31

9. Factors of $\left(5x-\frac{1}{x}\right)^2+4\left(5x-\frac{1}{x}\right)+4$ are

A) $(2x+5y)^2$ B) $(5y-2x)^2$ C) $(5y+2x)^2$ D) $(5x-2y)^2$

10. Factors of $a^{12}x^4-a^4x^{12}$ are

A) $a^4x^4(a^4+x^4)(a^2+x^2)(a+x)(a-x)$ B) $a^4x^4(x^4-a^4)(a^2+x^2)(a+x)(a-x)$

C) $a^4x^4(a^4+x^4)(x^2-a^2)(a+x)(a-x)$ D) $a^4x^4(a^6-x^6)(a+x)(a-x)$

11. Factors of $1-2cb - (a^2-b^2)$ are

A) $(1+a+b)$ $(1-a+b)$ B) $(1-a+b)$ $(1-a-b)$ C) $(1+ab)$ $(1+a+b)$ D) $(1-a-b)$ $(a+b)$

12. If the length and breadth are the factors of a rectangle whose area is given by $p(y)$ $35y^2+13y-12$, then l and b are

A) $7y - 3$, $5y +4$ B) $7y + 3$, $5y - 4$ C) $7y + 4$, $5y + 3$ D) $7y - 4$, $5y+3$

13. The number of rational factors of $x^{12} - y^{12}$

A) 7 B) 8 C) 6 D) 10

14. One of the factors of $a^3-b^3+1+3ab$

A) $(a+b+1)$ B) $(a-b+1)$ C) $(a+b-1)$ D) $(a-b-1)$

15. One of the factors of $p^3(q-r)^3+q^3(r-p)^3+r^3(p-q)^3$

A) $-pqr$ B) $(p+q+r)$ C) $(p+q)(q+r)(r-p)$ D) pqr

16. If $3x = a+b+c$, then the value of $(x - a)^3 + (x - b)^3 + (x - c)^3$.
A) $3(x-a)(x-b)(x-c)$ B) $3(x+a)(x+b)(x+c)$
C) $3(a-x)(x-b)(x-c)$ D) $3(x-a)(x-b)(c-x)$

17. The factors of $\dfrac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$
A) $(a+b)\ (b-c)\ (c+a)$ B) $(a+b)\ (b+c)\ (c+a)$
C) $(b-a)\ (c-b)\ (a-c)$ D) $(abc+ab+bc+ca$

18. Which of the following expression value can be foud by using, if $a+b+c = 0$, then $a^3+b^3+c^3 = 3abc$
A) $30^3+20^3+50^3$ B) $-30^3+20^3+50^3$
C) $30^3+20^3-50^3$ D) $30^3+20^3+50^3$

19. If $f(x)= x^4-2x^3+3x^2-ax-b$, when divided by $x - 1$, the remainder is 6 then $a+b$
(A) 4 B) 5 C) -4 D) 0

20. If $x^{140}+2.x^{151}+k$ is divisible by $x+1$, then the value of k is
A) 1 B) -3 C) 2 D) -2

21. If $(3x-1)^7= a_7.x^7+a_6.x^6+a_5.x^5+.........+a_1x+a_0$, then $a_0+a_1+a_2+a_3+....+a_7$ =
A) 0 B) 1 C) 128 D) 64

22. The polynomials ax^3+3x^2-13 and $2x^3-5x+a$ are divided by $x+2$ if the remainder in each case is the same what is a
(A) $9/5$ B) $-9/5$ C) $-5/9$ D) $5/9$

23. If a/b = b/c then product of the factors $(a+b+C)\ (a-b+C)$
A) a^2+c^2-ac B) $a^2 +c^2+ac$ C) $a^2 - c^2-ac$ D) c^2-a^2+ac

24. The L.C.M of $xy+yz+zx+y^2$ and $x^2+xy+yz+zx$ is
A) $(x+y)\ (y+z)$ B) $(x+y)\ (y+z)\ (z+x)$ C) $(y+z)\ (z+x)$ D) $(x+y)\ (z+x)$

25. The factors of $(x+y)(1-z) - (y+z)\ (1-x)$ =
A) $(x-z)\ (1-y)$ B) $(x-z)\ (1-z)$ C) $(x+y)\ (1-y)$ D) $(x-z)\ (1+y)$

26. If x^4+x^3 is divide by $x+9$, then the degree of the remainder
A) 1 B) 0 C) 2 D) 3

27. The remainder when x^3+3x^2+3x+1 is exactly divisible by $x-\pi$, if $\left(\pi = 22/7\right)$
A) $\left(\frac{27}{7}\right)^3$ B) $\left(\frac{30}{7}\right)^3$ C) $\left(\frac{29}{7}\right)^3$ D) $\left(\frac{32}{7}\right)^3$

28. What must be added to $3x^3+x^2-2x+9$ so that the result is exactly divisible by $3x^2+7x-6$.
A) $2x-3$ B) $2+3x$ C) $3x-2$ D) $2x+3$

29. If x^2-1 is a factor of $ax^4+bx^3+cx^2+dx+e$ then
A) $a+c+e = 0$ B) $b+d = 0$ C) $a+b+c+d+e = 0$ D) All

30. If $f(x) = cx^2+d^2$ then, then zero of $f(x)$
A) $\left(-\frac{d}{c}\right)^2$ B) $-\left(\frac{d}{c}\right)^2$ C) $\frac{d^2}{c^2}$ D) $\frac{c^2}{d^2}$

JEE Advanced

Multi correct answers type

1. If (x-a) (x-b) is a factor of a polynomial $p(x)$

A) $p(a) = 0$ B) $p(ab) = 0$ C) $p(b) = 0$ D) both b & c

2. If $p = r$ which are the factors of $p(x) = px^2+5x+r$

A) $x+2$ B) $x - \frac{1}{2}$ C) $x + \frac{1}{2}$ D) $x - 2$

3. If (x^2-1) is a factor of $p(x)=a_0.x^n + a_1.x^{n-1} + a_2.x^{n-2} + ... + a_n$ then

A) $a_0+a_1+......a_y = 0$ B) $a_0+a_2+.......a_n = 0$

C) $a_0+a_3+a_5...+a_{n-1} = 0$ D) $a_0+a_2+a_4+......= a_1+a_3+a_5+..........$

4. If x=2 and x = 0 are the roots of the polynomial $f(x) = 2x^3-5x^2+ax+b$ then the values of a & b

A) $a = 2$ B) b = 0 C) $b = -2$ D) $ab = -4$

5. If (x^2-1) is a factor of $ax^4+bx^3+cx^2+dx+e$ then

A) $a+c+e = b+d$ B) $a+b+e+ = c+d$ C) $a+b+c+d+e=0$ D) $b+c+d = a+e$

Reasoning type

A) both statement I & II are true.
B)both statement I & II are false
C)Statement I is true but statement II is false
D) Statement I is false but statement II is true

6. **Statement I:** (x - 2) is the factor of the expression x^3+ax^2+bx+6 when this expression is divided by x-3, it leaves remainder 3 and a^2+b^2 is 10.

Statement II : Let $p(x)$ be a polynomial of degree greater than or equal to 1 and a be a real number such that $p(A) = 0$, then (x -A) is a factor of $p(x)$

7. **Statement -I:** The factors of $p(x)$ $x^3-6x^2+11x-6$ are (x-1), (x-2) & (x-3)

Statement-II: If α, β, γ are the zeroes of $p(x) = ax^3+bx^2+cx+d$ then $(x-\alpha), (x-\beta)$ and $(x-\gamma)$ are the zeroes.

8. **Statement I :** If $(3x-1)^7 = a_7x^7+a_6.x^6+a_5.x^5+.....+a_1x+a_0$ then sum of coefficient is 128.

Statement II: By substituting x =-1, we get the sum of the coefficient.

Comprehension

Paragraph I : If ($ax+B$)is a factor of $p(x)$, then its remainder

9. If 3x-1 is a factor of $p(x)= x^2+ax$, then value of ‘a’

A) $1/3$ B) 3 C) $-1/3$ D) $-1/2$

10. If x-1 is a factor of $p(x)= x^3-3x^2+3x-1$ then $p(1)+p(-1)$ =

A) 0 B) 8 C)-8 D) none of these

11. If (x-1) and (x-2) are the factors of $p(x)= ax+b$ then $a+b$ =

A)3 B) -1 C)0 D) 1

Paragraph II :

If $p(A) = 0$, $p(B) = 0$, $p(C) = 0$, then the polynomial is $p(x)=(x-A)(x-B)(x-C)$

12. If $p\left(\frac{1}{2}\right)=0, p(\sqrt{2})=0$ then the polynomial $p(x)$

A) $x^2+\left(\sqrt{2}+\frac{1}{2}\right)x+\frac{1}{\sqrt{2}}$ B) $x^2+\left(2+\frac{1}{\sqrt{2}}\right)x+\frac{1}{\sqrt{2}}$

C) $x^2-\left(\sqrt{2}+\frac{1}{2}\right)x+\frac{1}{\sqrt{2}}$ D) $x^2-\frac{5}{2}x+\frac{1}{2}$

13. If $p\left(\frac{m}{n}\right)=0$ and $p\left(\frac{-m}{n}\right)=0$, then $p(x)$ =

A) $n^2x^2+m^2$ B) $m^2x^2-n^2$ C) $n^2x^2-m^2$ D) $m^2n^2+x^2$

14. If $p(\sqrt{2})=0$ and $p(\sqrt{3})=0$ then product of the zeroes of the polynomial

A) $\sqrt{6}$ B) $-\sqrt{6}$ C) $\sqrt{2/3}$ D) $\sqrt{3/2}$

Paragraph III : If $a+b+c = 0$ then $a^3+b^3+c^3 = 3abc$

15. If $x-y+z = 0$, then x^3+z^3

A) $3xyz-y^3$ B) y^3+3xyz C) $3y^3+xyz$ D) y^3-3xyz

16. If $x^{1/3}+y^{1/3}+z^{1/3}=0$, then $(x+y+z)^3$

A) $9xyz$ B) $27xyz$ C) $81xyz$ D) $3xyz$

17. If $l-m-n = 0$ then $l^3=$

A) $3lmn-m^3-n^3$ B) $3lmn+m^3n^3$ C) $3lmn - m^3+n^3$ D) m^3+n^3-3lmn

Integer Answer Type :

18. If $+b+c = 8$, $ab+bc+ca = 26$ then the value of $a^2+b^2+c^2-20$ = ______
19. If $(x-1)$ and $(x+2)$ are both the factors of expression $x^3-ax^2+bx-10$, then a/b = ______
20. The zero of the polynomial $x^3-23x^2+142x-120$ =__________
21. $\sum_{x,y,z} x^2(y^2-z^2)$ ___________

Matrix Match Type

22. **Match the following with list I to list II**

List - I	List -II
A) $x^2y^2+2xyz^2-x^2z^2-y^2z^2$	p) $x^3+xyz+(y^3-z^3)$
B) $x^3+y^3-z^3+3xyz$	q) $x^2(y-z)+(y^2+z^2+yz)x-y^2z-z^2y$
C) $(y-z)x^3+(z-x)y^3+(x-y)z^3$	r) $(y^2-z^2)x^2+2yz^2x-y^2z^2$
D) $x^2(y-z)+y^2(x-z)+z^2(x-y)+xyz$	s) $(y-z)x^3-y^3x+z^3x+yz^3-yz^3$
	t) $(y-z)x^3-y^3x+z^3x-yz^3+yz^3$

23. Matrix matching type

List - I	List -II
(A) The remainder when x^3-px^2+6x-p is divided by $x-p$	p) $-p^3-6p+2$
(B) The remainder when x^3+px^2+5x-p is divided by $x+p$	q) p^2-1
(C) The remainder when $x^3-x^2p+p^2x-1$ is divided by $x-1$	r) $5p$
(D) The remainder when $p^3x^3-3x^2p+3px+2$ is divided by $x+1$	s) $-3p(p^2+2)$
	t) $-2p(p^2+3)$

ALGEBRA SYNOPSIS -2

Factorisation of Quadratic polynomial by splitting the middle term

Solving a Quadratic equation $ax^2+bx+c=0$

Step I : Splitting of middle term

i) First multiply a and c i.e., ac (or) $-ac$

ii) Split 'b' into factors as their product is ac (or) $-ac$

Step II : Let the factors of $ax^2+bx+c=(px+q)(rx+s)$

$$\Rightarrow px+q=0 \text{ (or) } rx+s=0$$

$$\Rightarrow x=\frac{-q}{p} \text{ (or) } x={}^{-s}\!/_{r}$$

Ex : factorize $2x^2+9x+10$

Here $ac=2\times 10=20$

$b=9=4+5$

$ac=4\times 5=20$

Factorisation of the difference of two squares

1) $(a+b)^2-(a-b)^2=4ab$

2) $ab=\left(\frac{a+b}{2}\right)^2-\left(\frac{a-b}{2}\right)^2$

Ex : $(x+y+2z)(x+y)$ as the difference of 2 squares

$$(x+y+2z)(x+y)=\left(\frac{(x+y+2z)+(x+y)}{2}\right)^2-\left(\frac{(x+y+2z)-(x+y)}{2}\right)^2$$

$$=\left(\frac{2x+2y+2z}{2}\right)^2-\left(\frac{2z}{2}\right)^2$$

$(x+y+2z)(x+y)=(x+y+z)^2-z^2$

Factorisation of sum and difference of cubes

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

Ex : $64a^3+27=(4a)^3+3^3$

$=(4a+3)[(4a)^2-4a.3+3^2)$

$=(4a+3)(16a^2-12a+9)$

Ex : $a^6-b^6=(a^3)^2-(b^3)^2 \quad (x^2-y^2=(x+y)(x-y))$

$=(a^3-b^3)(a^3+b^3)$

$a^6-b^6=(a-b)(a^2+ab+b^2)$

$(a+b)(a^2-ab+b^2)$

Factorisation of $a^3+b^3+c^3-3abc$ on Simplifying :

$(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$=a^3+ab^2+ac^2-a^2b-abc-a^2c+a^2b+b^3+c^2b-ab^2-b^2c-abc$
$+ca^2+cb^2+c^3-abc-bc^2-c^2a$

$=a^3+b^3+c^3-3abc$

$\therefore\ a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

Factorisation of $a^3+b^3+c^3=0$, if $a+b+c=0$

$a+b+c=0$

$\Rightarrow a+b=-c$

$\Rightarrow C.O.B.S$

$\Rightarrow (a+b)^3=(-c)^3$

$\Rightarrow a^3+b^3+3ab(a+b)=-c^3$

$\Rightarrow a^3+b^3+3ab(-c)=-c^3$

$\Rightarrow a^3+b^3-3abc=-c^3$

$\Rightarrow a^3+b^3+c^3=3abc$

Ex : $5+2-7=0 \Rightarrow 5^3+2^3+(-7)^3=3.5.(-7)^3=3.5.(-7).2=-210$

It is better to write from geethanjali material page no. 44, 45, 46 (upto the line drawn by pen)

$\sum_{a,b,c} a=a+b+c$

$\sum_{a,b,c} a^2=a^2+b^2+c^2$

$$\sum_{a,b,c} ab = ab + bc + ca$$

$$\sum_{a,b,c} (a+b) = (a+b)(b+c)(c+a)$$

$$\sum_{a,b,c} a4 = a^4 + b^4 + c^4$$

$$\prod_{a,b,c} a = abc$$

$$\prod_{a,b,c} a^2 = a^2b^2c^2$$

1. General form of Quadratic polynomial $p(x) = ax^2 + bx + c, (a \neq 0)$

   Ex : $p(x) = \sqrt{3}x^2 - 2x + 5$

2. If α, β are the zeroes of $p(x) = ax^2 + bx + c,$ then $ax^2 + bx + c = a(x-\alpha)(x-\beta)$

   Ex : $p(x) = 2x^2 + 9x + 10 = 2\left(x^2 + \frac{9}{2}x + \frac{10}{2}\right)$

$$= 2\left(x^2 - \left(-\frac{9}{2}\right)x + \frac{10}{2}\right)$$

$$= 2\left(x^2 - (\alpha+\beta)x + \alpha\beta\right) = 2(x-\alpha)(x-\beta)$$

3. If α, β are the zeroes of $p(x) = ax^2 + bx + c,$

$$S = \alpha + \beta = \frac{-\text{coefficient of } x}{\text{coefficient of } x^2} = {}^{-b}\!/_{a}$$

$$P = \alpha\beta = \frac{\text{constant}}{\text{coefficient of } x^2} = {}^{c}\!/_{a}$$

Ex : $p(x) = 3x^2 + 11x + 10$

$$\therefore\ S = \alpha + \beta = {}^{-b}\!/_{a} = \frac{-11}{3}$$

$$p = \alpha\beta = {}^{c}\!/_{a} = {}^{10}\!/_{3}$$

4. If α, β are the zeroes of quadratic polynomial then

$$p(x) = K(x^2 - (\alpha+\beta)x + \alpha\beta], K \in R$$

Ex : If 2 & 3are the zeroes of a Quadratic polynomial then

$$p(x) = K[x^2 - (2+3)x + 2.3], K \in R$$

$$p(x) = K[x^2 - 5x + 6], K \in R$$

5. General form of cubic polynomial

$$p(x) = ax^3 + bx^2 + cx + d$$

Ex : $p(x) = 5x^3 - 6x^2 + 3x + 4$

6. If α, β, γ are the zeroes of a cubic polynomial

$$p(x) = ax^3 + bx^2 + cx + d$$

$$S_1 = \alpha + \beta + \gamma = \frac{-\text{coefficient of } x}{\text{coefficient of } x^3} = -b/a$$

$$S_2 = \alpha\beta + \beta\gamma + \gamma\alpha = \frac{\text{coefficient of } x}{\text{coefficient of } x^3} = c/a$$

$$S_3 = \alpha\beta\gamma = \frac{\text{-constant}}{\text{coefficient of } x^3} = -d/a$$

Ex : $5x^3 + 6x^2 + 7x + 11 \Rightarrow a = 5, b = 6, c = 7, d = 11$

$$S_1 = \alpha + \beta + \gamma = -b/a = -6/5$$

$$S_2 = \alpha\beta + \beta\gamma + \gamma\alpha = c/a = 7/5$$

$$S_3 = \alpha\beta\gamma = -d/a = -11/5$$

7. If α, β, γ are the zeroes of $p(x) = ax^3 + bx^2 + cx + d$

$$p(x) = ax^3 + bx^2 + cx + d = a(x-\alpha)(x-\beta)(x-\gamma)$$

$$= a[x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\beta\gamma+\gamma\alpha)x + \alpha\beta\gamma]$$

$$= a[x^3 - S_1x^2 + S_2x + S_3]$$

S_1 = Sum taken one at a time
S_2 = Sum taken 2 at a time
S_3 = Sum taken 3 at a time

WORKSHEET-2

C.U.Q's

1. The of a factors $(x-y)^3 + (y-z)^3 + (z-x)^3$
   A) $3(x-y)(y-z)(z-x)$ B) $3(x+y)(y+z)(z+x)$
   C) $3(x+y)(y+z)(z+x)$ D) (x-y) (y-z) (z-x)
2. $x^2 + y^2 + z^2 - xy - yz - zx$ this is an expression what is another form of this
   A) $\frac{1}{2}\left[(x-y)^2 - (y-z)^2 - (z-x)^2\right]$ B) $\frac{1}{2}\left[(x-y)^2 + (y+z)^2 + (z-x)^2\right]$
   C) $\frac{1}{2}\left[(x+y)^2 - (y+z)^2 - (z+x)^2\right]$ D) $\frac{1}{2}\left[(x-y)^2 + (y-z)^2 + (z+x)^2\right]$

3. One of the factors of $(x-y)^3+64$

A) *(x-y-4)* B) *(x+y+4)* C) $(x-y)^2-8(x-y)+16$ D) $(x-y)^2+8(x-y)-16$

4. If $x+\frac{1}{x}=2,$ then $x-\frac{1}{x}=$

A)0 B)+1 C)-2 D)+2

5. The vemainder when x^{5051} + a is divided by (x-1) is

A)a-1 B) 1-a C) 1+a D)-1-a

6. From the following which is the homogeneous first degree in x.y and z

A) 4x+7y-z B) $z^2+y^2+z^2$ C) x -y D) x^2xy-y^2

7. $\sum_{a,b,c} a^2 =$

A) $a^2 b^2 c^2$ B) $(ab+c)^2$ C) $(abc)^2$ D) $a^2 +b^2+c^2$

8. If $f(x) = ax^2 + bxy$ then it is

A) complete homogeneous B) homogeneous
C) sysmmetric D) cyclic

9. $\sum_{i-1}^{n} 1 =$

A) n B) $\frac{n(n+1)}{2}$ C) 1 D)None

10. The vemainder when $4x^3 - 5x^2 +3x - 4$ is divided by 'x'

A) +4 B) 0 C) -4 D) -2

JEE MAINS

1. If a+b =7 and ab =5, then the value of $a^3 +b^3$

A) 237 B) 236 C) 239 D)238

2. The remainder and quotient when $x^4 - 4x^3 + 4x^2 - 2$ is divided (x-3)

A) 7, x^3-x^2+x+3 B) -7, x^3-x^2+x+3 C) 7, x^3+x^2+x+3 D)None of these

3. If $x^2 - 4$ is a factor of $ax^4+2x^3-3x^2+bx-4$ then the values of a and b are

A) a=1, b=-8 B) a=3,b=5 C) a=6,b=7 D) a=-1,b=5

4. The H.C.F of the polynomials $x^2-3x +2$ and x^2+x-6 is

A) x+2 B) (x-1) (x-2) (x+3) C) x - 2 D) None

5. The value of $x^{12} - 7. x^6 +2001$ if $x^2 =2$

A) 2009 B) 0 C) 2007 D)2

6. One of the factors of $x^3 +x^2 - 2/x - 38$ is

A) 2x B) x^2+x+19 C) x - 2 D) x+2

7. The Quatient of $a^3+b^3+1-3ab$ when divided by a+b+1

A) $a^2 +b^2 - b -a -ab$ B) $(a+b)(a^2+ab+1)$ C) $a^2 - b^2+b-a$ D) None of these

8. Factors of $9-a^6+2a^3b^3-b^6$ are

A) $(a^3+b^3-3)(-a^3+b^3+3)$ B) $(a^3-b^3+3)(-a^3+b^3+3)$
C) $(a^3-b^3-3)(a^3+b^3-3)$ D) None of these

9. The simplest form of $\sqrt{2a^2-2\sqrt{6}ab+3b^2}$

A) $\sqrt{2}a-\sqrt{3}b$ B) $\sqrt{3}a+\sqrt{2}b$ C)2a+3b D) 3a+2b

10. Factors of x^6+y^6 are
    A) (x^2-y^2), $(x^4+x^2y^2+y^4)$ B) x^2-y^2, $x^4 - x^2y^2 + y^4$
    C) $x^2 + y^2$, $x^4+x^2y^2-y^4$ D) x^2+y^2, x^4+xy+y^4
11. If (y-1) is a factor of $p(y) = y^3 -7y +6$. then other two factors are
    A) x - 3, x+2 B) x+3, x-2 C) x-3, x-2 D) None of these
12. The square root factor of $36x^2 +60y+25$
    A) 6x+5 B) -(6x+5) C) $\pm(6x+5)$ D) None of these
13. The value of $x^3 - 8y^3 - 36xy - 216$ if x = 2y+6.
    A) 0 B) $8y^3$ C) $-27y^3$ D) -1
14. Factors of $(x^2-4x)(x^2-4x-1) -20$
    A) (x-5) B) (x+1) (x+2) C) x-2 D) 1 and 2
15. The value of $x^3 +y^3 -12xy +64$ % x+y = -4
    A) -1 B) +1 C)0 D) (x+y+4)
16. Whether g(x) = 3x-4 is a factor of $f(x) =3x^3+x^2-20x+12$ or not
    A)no B)yes C)may be D)maynot be
17. The vemainder when $3x^5 +4x^4+90x^2 - 19x +53$ is divided by x+5
    A) -101 B) -100 C) -102 D) -99
18. The relation between a and b if $2x^4 -7x^3+ax+b$amy be divisiable by x - 3
    A) b+3a = 27 B) b+3a = -27 C) b+3a=54 D) b+3a = 0
19. If 'n' is odd then the factor of $f(x) = x^{mn} +1$
    A) x^m+1 B) x^{m+1} C) x^n+1 D) x^n-1
20. The remainder when $f(x)=x^{1999}$ is divisiable by $x^2 -1$
    A) -x B) +x C) 1-x D) x+1
21. Tthe remainder when x^{100} is divided by x^2-3x+2
    A)$(1-2^{100})x+(-2^{100}+2)$ B)$(2^{100} -1)x+(2-2^{100})$
    C)$(2^{100}+1)x + (2+2^{100})$ D) None of these
22. If α,β are the zerores of $x^2 - 5x +6$ then $(\alpha-\beta)^2$
    A) 1 B) -1 C)0 D)$\frac{25}{24}$
23. If α,β are the zeroes of $p(x) =x^2-p(x)+c$ then $(1+\alpha)(1+\beta)=$
    A) 1-C B) C -1 C) C+1 D) C
24. If one zero of $p(x) =x^2+6x+k$, is double of the other then k =
    A) 8 B) -8 C) 0 D) 1
25. If the roots (zeroes) of $p(x) = x^2-(a^2-3a+2)x+4$ are equal in magnitude and oppsite in sign then a =
    A)1 and 3 B)1 and 2 C) -1 and 2 D)1 and -2
26. If the zeroes of the quadratic polynomial $x^2+(a+1)x+b$ are 2 and -3 then
    A) a = -7, b = -1 B) a = 5, b = -1, C) a =2, b=-6 D) a =0, b=-6
27. One of the factors of $(25x^2-1) +(1+5x)^2$
    A)5+x B) 5-x C) 5x-1 D)10x
28. If ab+bc+ca = 4 and abc=2 then 1/a+1/b+1/c =
    A) 1 B) 3 C) 4 D) 2

29. If $\frac{x}{y}+\frac{y}{x}=-1\ (x,y\neq 0)$ then the value of $x^3 - y^3$ =

A) 1 B) -1 C) 0 D) 1/2

30. $\sum(a+b)(b-c)=$

A) (a+b)(b-c)+(b+c) (c-a) +(c+a)(a-b)
B) (a-b)(b+c)+(b-c) (c+a) +(c-a)(a+b)
C) (a+b) (b+c) +(b+c) (c+a) +(c+a)(a+b) D) All

JEE ADVANCED

Multi corrrect answer

1. The coefficents of polynomial $p(z) = z^5 - 2z^2+4$ and product of the coeffficents
A) 1,-2,4 B) 1,0,0, -2, 0, 4 C) 0 D) -8

2. If $f(x) = x^3 + x^2-ax+b$ is divisible by $p(x) =x^2 - x$ then the values of a and b
A) a = 2, b = -1 B) a = -2, b =0 C) a = -1, b =1 D)a = 2, b =0

3. If the zeroes of the polynomial $f(x) = 2x^4 - 3x^3 - 3x^2 +6x - 2$ are $\sqrt{2}$ and $-\sqrt{2}$ then the remains two zeroes are
A) 1 B)-1/2 C) -1 D) 1/2

4. If α, β are the zeroes of $p(x) = x^2-px+q$ then the values of $\alpha^2+\beta^2$ *and* $\frac{1}{\alpha}+\frac{1}{\beta}$
A) pq B) $p^2 -2q$ C) $2q - p^2$ D) p/q

5. Which of the following statements all true
A) every polynomial has a finite number of factors
B) for all values of 'm' then $a^m - b^m$ is divided by a-b
C) if the sum of the coefficents of 'x' in a polynomial is zero then (x-1) is a factor of the polynomial
D) A linear polynomial f(x) = ax+b, $(a\neq 0)$ has umique zero then $x=\frac{-b}{a}$

Reasoning type

A) both statement I & II are true.
B)both statement I & II are false
C)Statement I is true but statement II is false
D) Statement I is false but statement II is true

6. **Statement I** : The product of (a-b-c) and $(a^2+b^2+c^2+ab+ac-bc)$ is $a^3 - b^3 - c^3$
Statement II : $(l+m+n)\ (l^2+m^2-n^2-lm-mn-nl) =l^3+m^3+n^3-3lmn$

7. **Statement I** : $4\sqrt{3}x^2+5x-2\sqrt{3}=\left(\sqrt{3}x+2\right)\left(4x-\sqrt{3}\right)$
Statement II : In order to factorise ax^2+bx+c we find numbers *l and m* such that *l +m =b and lm =ac*

8. **Statement I** : If the polynoimals $ax^3 + 3x^2 - 13$ and $2x^3-5x+9$ are divided by *x-2,* then the value of a is 3
Statement II : If the functions f(x) and g(x) are exactly divisible by7 (x-k) then f(k) = g(k)

COmprehension Type

Paragraph - I: $ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$

9. Express $(x+y+2z)(x+y)$ as the difference of two squares
   A) $(x+y+z)^2 - z^2$ B) $z^2 - (x+y+z)^2$ C) $z^2+(x+y+z)^2$ D) $(x+y+z)^2 - (2z)^2$
10. Express $(x+2a)(x+4a)(x+6a)(x+8a)+7a^4$ as the difference of two squaares.
    A) $(x^2+10ax+20a^2)^2 - (3a^2)^2$ B) $(3a^2)^2 - (x^2+10ax+20a^2)^2$
    C) $(x^2+10ax+20a^2)^2 - (9a^2)^2$ D) $(9a^2)^2 - (x^2+10ax+20a^2)^2$

Paragraph - II : $\sum_{a,b,c} a^2 = a^2+b^2+c^2,\ \pi_{a,b,c}\, a^2 = a^2b^2c^2$

$(\sum$ *stands for addition and* π *stands for multiplication*$)$

11. $\sum_{a,b,c} a^2b =$
    A) $a^2b+b^{23}c+c^2a$ B) $a^2b+bc^2+ab^2$
    C) $ab^2+bc^2+ca^2$ D) $(a^2b)(b^2c)(c^2a)$
12. $\sum_{x,y,z} x^2y+xy^2 =$
    A) $xy(x+y+z)$ B) $xy(x-y)+yz(y-z)+zx(z-x)$
    C) $xy(x+y)+yz(y+z)+zx(z+x)$ D) None of these
13. $\pi_{a,b,c}\left(a^2b^2\right) =$
    A) $a^2b^2+b^2c^2+c^2a^2$ B) $(a^2b^2)(b^2c^2)(c^2a^2)$
    C) $(a^2+b^2)(b^2+c^2)(c^2+a^2)$ D) $a^2b^2c^2$
14. $\sum_{a,b,c} a^2 + 2\sum_{a,b,c} ab =$
    A) $(ab+bc+ca)^2$ B) $(a+b+c)^2$ C) $a^2+b^2+c^2+ab+bc+ca$ D) None of these

Paragraph - III : If $a+b+c=0$ then $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2+ab-bc-ca)$

15. $27x^3 - y^3 - z^3 - 9xyz =$
    A) $(3x-y+z)(9x^2+y^2+z^2-3xy+yz+3zx)$
    B) $(3x+y-z)(9x^2+y^2+z^2+3xy+yz-3zx)$
    C) $(3x-y-z)(9x^2+y^2+z^2+3xy-yz+3zx)$ D) None of these
16. $125+8x^3-27y^3+90xy =$
    A) $(5+2x+3y)(25+4x^2+9y^2+10xy-6xy+15y)$
    B) $(5+2x-3y)(25+4x^2+9y^2-10xy+6xy+15y)$
    C) $(5-2x-3y)(25+4x^2+9y^2-10xy+6xy+15y)$ D) None of these
17. $(3x-5y-4)(9x^2+25y^2+15xy+12x-20y+16]$
    A) $27x^3+125y^3-64-180xy$ B) $27x^3-125y^3+64+180xy$
    C) $27x^3-125y^3-64-180xy$ D) None of these

Integer type questions

18. If α, β are the zeroes of thepolynomial $p(x) = x^2-5x+k$ such that $\alpha - \beta = 1$, then k =
19. If $a^4(b^2-c^2)+b^4(c^2-a^2)+c^4(a^2-b^2) = k(a^2-b^2)(b^2-c^2)(c^2-a^2)$ then k =
20. If one zero of the polynomial $f(x) = (k^2+y)x^2+13x+4k$ is reciprocal of the other, than k =

21. If sum of the squares of the zeroes of quadratic polynomial lp(x) = x^2 - 8x +k is 40, then k =

Matrix Matching

22.

List - I	**List - II**
a) standard form of homogenous expression of 3rd degree in x,y	p) $a(x^2+y^2+z^2) +b(xy+yz+zx) +c(x+y+z)+d$
b) standard form of homogerous expression of second degree in x,y,z	q) $ax^2+by^2+cz^2+dxy+eyz+fzx$
c) standard form of symmetaic expression of degree 2 in x,y,z	r) $a[x^2+y^2z^2) +b(xy+yz+zx)$
d) standard form of homogenous symmetric expression of degree 2 in x,y,z	s) $ax^3 +bx^2y+cxy^2+dy^3$
	t) $a(x^2+y^2) +bxy+c(x+y) +d$

23.

List - I	**List - II**
a) $a^3(b-c)+b^3(c-a)+c^3(a-b)$	p) (x-y) (y-z) (z+x)
b) $a^4(b-c)+b^4(c-a)+c^4(a-b)$	q)(a-b) (b-c) (c-a)
c) $x^3(y-z)+y^3(z-x)+z^3(z-y)$	r) (x-y) (y-z) (z-x)
d) $x^4(y-z)+y^4(z-x)+z^4(z-y)$	s) (a-b) (b-c) (c+a)
	t) (x+y) (y-z) (z-x)