7. VARIATION OF GRAPHS, PERIODICITY AND EXTREME VALUES ①

2025|26

Class: VIII, MATHEMATICS

FOUNDATION+ SOLUTIONS

TEACHING TASK

JEE MAINS LEVEL

01. $\sin x \cos\left(\frac{\pi}{4} - x\right)$

$= \sin x\left[\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right]$

$= \frac{1}{\sqrt{2}}\sin x\cos x + \frac{1}{\sqrt{2}}\sin^2 x$

$= \frac{1}{2\sqrt{2}}\underbrace{\sin 2x}_{\text{period} = \pi} + \frac{1}{\sqrt{2}}\underbrace{\sin^2 x}_{\text{period } \pi}$

$\therefore$ L.C.M $\{\pi, \pi\} = \pi$

Ans: A

02. $y = \sin^2\theta + \cos^4\theta$

$= \sin^2\theta + \cos^2\theta \cdot \cos^2\theta$

$= \sin^2\theta + (1 - \sin^2\theta)\cos^2\theta$

$= \sin^2\theta + \cos^2\theta - \sin^2\theta \cdot \cos^2\theta$

$= 1 - \frac{1}{4}(2\sin\theta\cos\theta)^2 = 1 - \frac{1}{4}\sin^2 2\theta.$

We know, $0 \le \sin^2 2\theta \le 1$

$$\Rightarrow 0 \ge -\frac{1}{4}\sin^2 2\theta \ge -\frac{1}{4}$$

$$1 \ge 1-\frac{1}{4}\sin^2 2\theta \ge 1-\frac{1}{4}$$

$$\therefore \frac{3}{4} \le 1-\frac{1}{4}\sin^2 2\theta \le 1$$

$$\therefore y \in \left[\frac{3}{4}, 1\right]$$

Ans. D

---

03 $$\frac{\tan\theta - \frac{1}{3}\tan^3\theta}{\frac{1}{3} - \tan^2\theta} = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta} = \tan 3\theta$$

Ans. A

$$\therefore \text{period} = \frac{\pi}{3}$$

---

04 Given $f(x) = \dfrac{\cos nx}{\sin\left(\frac{x}{n}\right)}$

period of $\cos nx = \dfrac{2\pi}{n}$

period of $\sin\left(\frac{x}{n}\right) = \dfrac{2\pi}{\left(\frac{1}{n}\right)} = 2n\pi$

period of $f(x) =$ L.C.M. of $\left\{\dfrac{2\pi}{n}, \dfrac{2n\pi}{1}\right\}$

$$= \frac{\text{LCM}\{2\pi, 2n\pi\}}{\text{H.C.F}\{n, 1\}}$$

$$= \frac{2n\pi}{1} = 2n\pi$$

Given $2n\pi = 4\pi \Rightarrow n = 2$ Ans: C

05 least value of $\left(\dfrac{1}{\text{max. } 5\cos x+12\sin x+15}\right)$

$=$ least value $\left(\dfrac{1}{15+\sqrt{25+144}}\right)$

$=$ least value $\left(\dfrac{1}{28}\right)$ Ans: B

06 Given $\cos^2\theta-6\sin\theta\cos\theta+3\sin^2\theta+2$

$=(\cos^2\theta+\sin^2\theta)-3(2\sin\theta\cos\theta)+2\sin^2\theta+2$

$=3-3\sin 2\theta+1-\cos 2\theta$

$=4-3\sin 2\theta-\cos 2\theta$

least value $=c-\sqrt{a^2+b^2}$

$=4-\sqrt{(-3)^2+(-1)^2}=4-\sqrt{10}$ Ans: B

07. Given $f(x)=\cos^2 x+\sec^2 x$

$=\cancel{\cos^2 x+\frac{1}{\cos^2 x}}$

$=(\cos x-\sec x)^2+2\cos x.\sec x$

$=(\cos x-\sec x)^2+2$

We know $(\cos x-\sec x)^2\geq 0$

$\therefore (\cos x-\sec x)^2+2\geq 2$

$\therefore$ minimum value of $f(x)=2$

$\therefore$ Range $=[2,\infty)$ Ans: C

$\boxed{08}$ Opt A! $f(x) = \frac{1}{2}\sin(3\pi x)$

period $= \frac{2\pi}{|3\pi|} = \frac{2}{3}$ $\boxed{\text{Ans: B}}$

---

09 We know $\sin^2 x + \cos^2 x = 1$ $\forall x \in R$

Also $\sin^2\frac{3x}{2} + \cos^2\frac{3x}{2} = 1$ $\forall \frac{3x}{2} \in R$

$\therefore \frac{\text{minimum value } 1}{\text{maxim value } 1} = \frac{1}{1} = 1$ Ans: A

$\boxed{10}$ $\tan(x + 4x + 9x + \cdots + n^2 x)$

$= \tan(1 + 2^2 + 3^2 + \cdots + n^2)x$

$= \tan\left[\frac{n(n+1)(2n+1)}{6}\right]x$

$\therefore$ period $= \frac{\pi}{\left[\frac{n(n+1)(2n+1)}{6}\right]} = \frac{6\pi}{n(n+1)(2n+1)}$

$\boxed{\text{Ans: C}}$

JEE ADVANCED LEVEL

---

01 A) $f(x) = \sin x + |\sin x|$

$f(2\pi + x) = \sin(2\pi + x) + |\sin(2\pi + x)|$

$= \sin x + |\sin x| = f(x)$

Hence, $f(x)$ is periodic with period 2π

b) $g(x) = \sin x + \cos x$

$g\left(\frac{\pi}{2}+x\right) = \sin\left(\frac{\pi}{2}+x\right) + \cos\left(\frac{\pi}{2}+x\right)$

$= \cos x + \sin x = f(x)$

Hence, $g(a)$ is periodic with period $\frac{\pi}{2}$

c) $h(x) = \tan x$

$h(x)$ is periodic with period π

d) $p(x) = \sin^4 x + \cos^4 x$

$p\left(\frac{\pi}{2}+x\right) = \sin^4\left(\frac{\pi}{2}+x\right) + \cos^4\left(\frac{\pi}{2}+x\right)$

$= \cos^4 x + \sin^4 x = f(x)$

Hence, $p(a)$ is periodic with period $\frac{\pi}{2}$

Ans. A, B, C, D

02 max of $\sin\theta = 1$, min of $\sin\theta = -1$

Ans: A, B

03 Given $U = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}$

$\Rightarrow U^2 = (a^2\cos^2\theta + b^2\sin^2\theta) + (a^2\sin^2\theta + b^2\cos^2\theta) + 2\sqrt{(a^2\cos^2\theta + b^2\sin^2\theta)(a^2\sin^2\theta + b^2\cos^2\theta)}$

$\Rightarrow U^2 = (a^2+b^2) + 2\sqrt{(a^4\cos^2\theta\sin^2\theta + a^2b^2\cos^4\theta + b^2a^2\sin^4\theta + b^4\sin^2\theta\cos^2\theta)}$

$\Rightarrow U^2 = (a^2+b^2) + 2\sqrt{(a^4+b^4)\cos^2\theta\sin^2\theta + a^2b^2(\cos^4\theta + \sin^4\theta)}$

$$=(a^2+b^2)+2\sqrt{(a^4+b^4)\cos^2\theta\sin^2\theta+a^2b^2\left[(\cos^2\theta+\sin^2\theta)^2-2\cos^2\theta\sin^2\theta\right]}$$

$$=(a^2+b^2)+2\sqrt{(a^4+b^4)\cos^2\theta\sin^2\theta+a^2b^2-2a^2b^2\cos^2\theta\sin^2\theta}$$

$$=(a^2+b^2)+2\sqrt{(a^4+b^4-2a^2b^2)\cos^2\theta\sin^2\theta+a^2b^2}$$

$$=(a^2+b^2)+2\sqrt{(a^2-b^2)^2\frac{1}{4}\cdot\sin^2 2\theta+a^2b^2}$$

maximum value will be obtained when $\sin^2 2\theta = 1.$

$$\therefore v^2_{max}=(a^2+b^2)+2\sqrt{\frac{[a^2-b^2]^2}{4}+a^2b^2}$$

$$=(a^2+b^2)+\sqrt{(a^2-b^2)^2+4a^2b^2}$$

$$=(a^2+b^2)+\sqrt{(a^2+b^2)^2}=2(a^2+b^2)$$

minimum value will be obtained when $\sin^2 2\theta=0$

$$\therefore v^2_{min}=(a^2+b^2)+2\sqrt{a^2b^2}$$

$$=a^2+b^2+2ab=(a+b)^2$$

Difference $=2(a^2+b^2)-(a+b)^2$

$$=a^2+b^2-2ab=(a-b)^2$$

Ans: B

04. Statement I: The period of $\sin 8x + \cos 2x$

period of $\sin 8x = \frac{2\pi}{8} = \frac{\pi}{4}$

period of $\cos 2x = \frac{2\pi}{2} = \pi$

period of $f(x) = L.C.M\left\{\frac{\pi}{4}, \frac{\pi}{1}\right\}$

$= \frac{LCM\{\pi, \pi\}}{H.C.F\{4, 1\}}$

$= \frac{\pi}{1} = \pi$ (True)

Statement II: Conceptual (True) Ans: A

05 Statement I: $f(x) = \text{Cosec}\left(x + \frac{x}{2} + \frac{x}{4} + \dots \infty\right)$

$= \text{Cosec}\left(1 + \frac{1}{2} + \frac{1}{4} + \dots \infty\right)x$

$\boxed{S_\infty = \frac{a}{1-r}}$

$= \text{Cosec}\left(\frac{1}{1-\frac{1}{2}}\right)x$

$= \text{Cosec } 2x$

period $= \frac{2\pi}{2} = \pi$ (True)

Statement II: Conceptual (True) Ans.. A

06 Assertion: Conceptual (True)

Reason: Conceptual (True) Ans. A

07. Assertion: Conceptual (True)

Reason: Conceptual (True) Ans. A

08 max. value $= 5 + \sqrt{3^2 + 4^2}$

$= 5 + 5$

$= 10$ Ans. C

09 $f(\theta) = 5\sin^2\left(\frac{\theta}{2}\right) + 4\cos^2\left(\frac{\theta}{2}\right)$

$= 5\left(\frac{1-\cos\theta}{2}\right) + 4\left(\frac{1+\cos\theta}{2}\right)$

$= \frac{9}{2} + \left(\frac{4}{2} - \frac{5}{2}\right)\cos\theta$

$= \frac{9}{2} - \frac{1}{2}\cos\theta$

We know $-1 \leq \cos\theta \leq 1$

$-\frac{1}{2} \leq \frac{\cos\theta}{2} \leq \frac{1}{2}$

$\frac{1}{2} \geq -\frac{\cos\theta}{2} \geq -\frac{1}{2}$

$\frac{9}{2} + \frac{1}{2} \geq \frac{9}{2} - \frac{1}{2}\cos\theta \geq -\frac{1}{2} + \frac{9}{2}$

$5 \geq f(\theta) \geq 4$

$\therefore 4 \leq f(\theta) \leq 5$

$\therefore$ minimum value $= 4$ Ans. D

Scanned with OKEN Scanner

10. $5|\sec x| - 7|\operatorname{cosec} x|$

$a = 5,\ b = -7 \Rightarrow a \neq b$

$\therefore$ The period $= \pi$

Ans: A

---

11. $29|\tan x| + \sqrt{841}\,|\cot x|$

$= 29|\tan x| + 29|\cot x|$

here $a = b \longrightarrow$ period $= \frac{\pi}{2}$

$\boxed{\text{Ans. B}}$

Ans.- D

12 $a = b = \sqrt{13} \longrightarrow$ Period $= \frac{\pi}{2}$

---

13. Let $f(\theta) = \dfrac{1}{\sin^2\theta + 5\sin\theta\cos\theta + 5\cos^2\theta}$

We have to find max. value of $f(\theta)$.

Now, find min. of $\sin^2\theta + 5\sin\theta\cos\theta + 5\cos^2\theta$

$= \sin^2\theta + 5\sin\theta\cos\theta + \cos^2\theta + 4\cos^2\theta$

$= 1 + \frac{5}{2}\sin 2\theta + 2(2\cos^2\theta)$

$= 1 + \frac{5}{2}\sin 2\theta + 2(1 + \cos 2\theta)$

$= \frac{5}{2}\sin 2\theta + 2\cos 2\theta + 3$

min. value $= 3 - \sqrt{\left(\frac{5}{2}\right)^2 + (2)^2}$

$= 3 - \frac{\sqrt{41}}{2}$

$= 3 - \frac{6.403}{2}$

$= 3 - 3.2015 = -0.2015$ (Approx)

max. $f(0) = \frac{-1}{0.2015} = \frac{-10000}{2015} = -4.963$ (10)

$\therefore f(0) = \cancel{\frac{1}{(2)}}$ $\cancel{2}$

Ans: -4

14 $f(x) = \cos 3\pi x + \tan 5\pi x - 3\sin 4\pi x$

period of $\cos 3\pi x = \frac{2\pi}{3\pi} = \frac{2}{3}$

" " $\tan 5\pi x = \frac{2\pi}{5\pi} = \frac{2}{5}$

" " $\sin 4\pi x = \frac{2\pi}{4\pi} = \frac{1}{2}$

$\therefore$ Period of $f(x) = \text{L.c.m}\left\{\frac{2}{3}, \frac{2}{5}, \frac{1}{2}\right\}$

$$= \frac{\text{Lcm}\{2,2,1\}}{\text{H.C.F}\{3,5,2\}}$$

$$= \frac{2}{1} = 2$$

Ans: 2

15 a) $\frac{7+6\tan\theta-\tan^2\theta}{1+\tan^2\theta}$

$$= \frac{7+6\tan\theta-\tan^2\theta}{\sec^2\theta}$$

$= 7\cos^2\theta + 6\sin\theta\cos\theta - \sin^2\theta$

$= 6\cos^2\theta + 3\sin 2\theta + (\cos^2\theta - \sin^2\theta)$

$= 3(1+\cos 2\theta) + 3\sin 2\theta + \cos 2\theta$

$= 3\sin 2\theta + 4\cos 2\theta + 3$

max. value $\lambda = 3 + \sqrt{9+16} = 3+5 = 8$

min. value $\mu = 3 - \sqrt{9+16} = 3-5 = -2$

$\therefore \lambda + \mu = 8 - 2 = 6$ (r)

b) $f(\theta) = 5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$

$= 5\cos\theta + 3\left[\cos\theta \cdot \frac{1}{2} - \sin\theta \cdot \frac{\sqrt{3}}{2}\right] + 3$

$= \left(5 + \frac{3}{2}\right)\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

$= \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

$\max = \lambda = 3 + \sqrt{\left(\frac{-3\sqrt{3}}{2}\right)^2 + \left(\frac{13}{2}\right)^2}$

$= 3 + 7 = 10$

$\min = \mu = 3 - 7 = -4$

Now $\lambda + \mu = 10 - 4 = 6$ (r)

c) $1 + \sin\left(\frac{\pi}{4} + \theta\right) + 2\cos\left(\frac{\pi}{4} - \theta\right)$

$= 1 + \left(\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta\right) + 2\left(\frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta\right)$

$= 1 + \left(\frac{1}{\sqrt{2}} + \sqrt{2}\right)\cos\theta + \left(\frac{1}{\sqrt{2}} - \sqrt{2}\right)\sin\theta$

$\max = \lambda = 1 + \sqrt{\left(\frac{1}{\sqrt{2}} + \sqrt{2}\right)^2 + \left(\frac{1}{\sqrt{2}} - \sqrt{2}\right)^2} = 1 + \sqrt{5}$

$\min = \mu = 1 - \sqrt{5}$ $\quad \therefore \lambda + \mu = 2$ (p)

(12)

d) $\sin^2(120^\circ+\theta)+\sin^2(120^\circ-\theta)$

$= [\sin(120^\circ+\theta)+\sin(120^\circ-\theta)]^2 - 2\sin(120^\circ+\theta)\cdot\sin(120^\circ-\theta)$

$= [2\sin 120^\circ\cdot\cos\theta]^2 - 2[\sin^2 120^\circ - \sin^2\theta]$

$= \left[2\cdot\frac{\sqrt3}{2}\cdot\cos\theta\right]^2 - 2\left[\frac{3}{4}-\sin^2\theta\right]$

$= 3\cos^2\theta - \frac{3}{2} + 2\sin^2\theta$

$= 3\cos^2\theta - \frac{3}{2} + 2(1-\cos^2\theta)$

$= \frac{1}{2}+\cos^2\theta$

We know, $0 \le \cos^2\theta \le 1$

$\Rightarrow \frac{1}{2} \le \frac{1}{2}+\cos^2\theta \le \frac{3}{2}$

$\therefore$ max $= \lambda = \frac{3}{2}$ | $\lambda+\mu = \frac{3}{2}+\frac{1}{2} = 2$ (p)

min $= \mu = \frac{1}{2}$

Ans: r, r, p, p

---

16 a) $f(x) = 2\sin^2 x + \cos^2 x$

$f(\pi+x) = 2\sin^2(\pi+x)+\cos^2(\pi+x) = f(x)$

period $= \pi$ (s)

b) $f(\theta) = \sin(\pi\sin\theta)$

$f(2\pi+\theta) = \sin[\pi\sin(2\pi+\theta)] = \sin(\pi\sin\theta) = f(\theta)$

period $= 2\pi$ (p)

c) $f(x) = \sin x\cdot\sin(120^\circ+x)\cdot\sin(120^\circ-x) = \frac{1}{4}\sin 3x$

period $= \frac{2\pi}{3}$ (q)

d) $f(x) = |\sin x|\cdot|\cos x|$

$f(\frac{\pi}{2}+x) = |\sin(\frac{\pi}{2}+x)|\cdot|\cos(\frac{\pi}{2}+x)| = f(x)$

Period $\pi/2$ Ans: s, p, q, r

LEARNERS TASK (CUQ's)

01.	period $= \frac{2\pi}{\lvert a \rvert}$	Ans: C
02	period $= \frac{2\pi}{\lvert 2 \rvert} = \pi$	Ans. C
03	Conceptual	Ans: C
04	Conceptual	Ans. D
05	Conceptual	Ans: B
06	Conceptual	Ans: A
07	Conceptual	Ans: B
08	$a^2\sin^2 x + b^2\operatorname{cosec}^2 x$ $= (a\sin x - b\operatorname{cosec} x)^2 + 2ab\sin x\operatorname{cosec} x$ $= (a\sin x - b\operatorname{cosec} x)^2 + 2ab \geq 2ab$ $\therefore$ minimum value $= 2ab$	Ans. A
09	Conceptual	Ans: A
10	$f(x) = \tan 3x + \tan x$ $\tan 3x \Downarrow \frac{\pi}{3}$, $\tan x \Downarrow \pi$ $\therefore$ period of $f(x) = \text{L.C.M}\{\frac{\pi}{3}, \pi\}$ $= \frac{\text{L.C.M}\{\pi, \pi\}}{\text{H.C.F}\{3, 1\}} = \frac{\pi}{1} = \pi$	Ans: D

JEE MAINS LEVEL

01. $f(x) = \sin\left(\frac{\pi x}{2}\right) + \cos\left(\frac{\pi x}{3}\right)$

period of $\sin\left(\frac{\pi x}{2}\right) = \frac{2\pi}{\left(\frac{\pi}{2}\right)} = 4$

period of $\cos\left(\frac{\pi x}{3}\right) = \frac{2\pi}{\left(\frac{\pi}{3}\right)} = 6$

$\therefore$ Period of $f(x) = \text{L.C.M}\{4, 6\} = 12$ Ans: C

02. $f(x) = \sin x \cdot \cos x$

$= \frac{1}{2}(2\sin x \cdot \cos x)$

$= \frac{1}{2}\sin 2x$

$-1 \le \sin 2x \le 1$

$-\frac{1}{2} \le \frac{1}{2}\sin 2x \le \frac{1}{2}$

max. value $= \frac{1}{2}$

Ans: C

03. $f(\theta) = \cos\theta + \cos 2\theta$

$= \cos\theta + 2\cos^2\theta - 1$

$= 2\cos^2\theta + \cos\theta - 1$

$= 2\left[\cos^2\theta + \frac{1}{2}\cos\theta\right] - 1$

$= 2\left[\cos^2\theta + 2\cdot\frac{1}{4}\cdot\cos\theta + \frac{1}{16} - \frac{1}{16}\right] - 1$

$= 2\left[\left(\cos\theta + \frac{1}{4}\right)^2\right] - \frac{1}{8} - 1$

$= 2\left(\cos\theta + \frac{1}{4}\right)^2 - \frac{9}{8}$

We know, $0 \le \left(\cos\theta + \frac{1}{4}\right)^2 \le 1$

$0 \le 2\left(\cos\theta + \frac{1}{4}\right)^2 \le 2$

$$-\frac{9}{8} \le 2\left(\cos\theta + \frac{1}{4}\right)^2 - \frac{9}{8} \le 2 - \frac{9}{8}$$

$$-\frac{9}{8} \le f(\theta) \le \frac{7}{8}$$

$\therefore$ minimum value $= -\frac{9}{8}$ Ans.. A

---

04 $f(x) = 118\sin 2x - 143 . \cot 4x$

period of $\sin 2x = \frac{2\pi}{|2|} = \pi$

" " $\cot 4x = \frac{\pi}{4}$

$\therefore$ The period of $f(x) =$ L.c.m $\left\{\frac{\pi}{1}, \frac{\pi}{4}\right\}$

$$= \frac{\text{L.c.m}\{\pi, \pi\}}{\text{H.C.F}\{1, 4\}} = \frac{\pi}{1}$$

$$= \pi$$

Ans: B

---

05 $f(x) = \tan[x + 2x + 3x + \cdots + nx]$

$$= \tan(1+2+3+\cdots+n)x$$

$$= \tan\left[\frac{n(n+1)}{2}\right]x$$

period of $f(x) = \frac{\pi}{\left[\frac{n(n+1)}{2}\right]} = \frac{2\pi}{n(n+1)}$ Ans. B

---

06 $f(x) = \frac{1}{5\sin x - 6}$

$$-1 \le \sin x \le 1$$

$$-5 \le 5\sin x \le 5$$

$$-5-6 \le 5\sin x - 6 \le 5-6$$

$$-11 \le 5\sin x - 6 \le -1$$

$$-\frac{1}{11} \ge \frac{1}{5\sin x - 6} \ge -1$$

$$-1 \le \frac{1}{5\sin x - 6} \le \frac{-1}{11}$$

Ans: B

07. option Verification:

A) $\tan\frac{\pi x}{5} \rightarrow$ period $= \frac{\pi}{\left(\frac{\pi}{5}\right)} = 5$ Ans: A

08 $f(\theta) = \cos(\pi\cos\theta)$

$f(\pi+\theta) = \cos(\pi\cos(\pi+\theta)) = \cos(-\pi\cos\theta) = \cos(\pi\cos\theta)$

period $= \pi$ Ans: A

09 Conceptual Ans.. A

10 $f(x) = 3\sin x + 4\cos x + 2$

max $= 2 + \sqrt{9+16} = 2+5 = 7$

min $= 2 - 5 = -3$

$\therefore$ Range $= \left[\frac{-1}{3}, \frac{1}{7}\right]$ Ans: B

JEE ADVANCED LEVEL

01 $f(x) = \tan(x + 2x + 3x + \cdots + nx)$

$= \tan(1+2+3+\cdots+n)x$

$= \tan\left[\frac{n(n+1)}{2}\right]x$

$\therefore$ period $= \frac{\pi}{\left(\frac{n(n+1)}{2}\right)} = \frac{2\pi}{n(n+1)}$ (or) $\frac{\pi}{\sum n}$

Ans: B, C

02 $f(x) = \operatorname{cosec} x - 2\cot 2x \cdot \cos x$

$= \frac{1}{\sin x} - \frac{2\cos 2x}{\sin 2x} \cdot \cos x$

$$= \frac{1}{\sin x} - \frac{2\cos 2x}{2\sin x \cos x} \cdot \cos x$$

$$= \frac{1-\cos 2x}{\sin x} = \frac{2\sin^2 x}{\sin x} = 2\sin x$$

$$-1 \le \sin x \le 1$$

$$-2 \le 2\sin x \le 2$$

min. value $= -2$; max. value $= 2$ $\boxed{\text{Ans: A, B}}$

03. Statement I: $f(\theta) = 2\cos^2\theta + \sqrt{5}\sin\theta\cos\theta + 4\sin^2\theta$

$f(\theta) = a\cos^2\theta + b\sin\theta\cos\theta + c\sin^2\theta$

max. value $= \frac{1}{2}(a+c) + \frac{1}{2}\sqrt{(a-c)^2 + b^2}$

$= \frac{1}{2}(2+4) + \frac{1}{2}\sqrt{(2-4)^2 + (\sqrt{5})^2}$

$= 3 + \frac{3}{2} = \frac{9}{2}$ (True)

Statement II: Conceptual (False) Ans: C

04 Statement I: $3\sin\theta + \csc^3\theta$

$= 3\sin\theta + \frac{1}{\sin^3\theta}$

$= (\sin\theta + \sin\theta + \sin\theta) + \frac{1}{\sin^3\theta}$

~~We know A.M $3\sin\theta$ & $\frac{1}{\sin^3\theta}$~~

We know $A.M \ge G.M$

$$\frac{\sin\theta + \sin\theta + \sin\theta + \frac{1}{\sin^3\theta}}{4} \geq \left(\sin\theta \cdot \sin\theta \cdot \sin\theta \cdot \frac{1}{\sin^3\theta}\right)^{\frac{1}{4}}$$

$$\Rightarrow \quad 3\sin\theta + \frac{1}{\sin^3\theta} \geq 4$$

$\therefore$ minimum value $= 4$ (True)

Statement II: $f(x) = \sin\left(x+\frac{\pi}{6}\right) + \cos\left(x+\frac{\pi}{6}\right)$

$$\sin A + \cos A = \sqrt{2}\sin\left(A+\frac{\pi}{4}\right)$$

$$\sin\left(x+\frac{\pi}{6}\right) + \cos\left(x+\frac{\pi}{6}\right) = \sqrt{2}\sin\left(x+\frac{\pi}{6}+\frac{\pi}{4}\right)$$

$$= \sqrt{2}\sin\left(x+\frac{5\pi}{12}\right)$$

The maximum value of $\sqrt{2}\sin\left(x+\frac{5\pi}{12}\right)$ in the interval $\left(0,\frac{\pi}{2}\right)$, will occur when $\sin\left(x+\frac{5\pi}{2}\right)$ is maximum.

~~Let find the critical point~~

$\cancel{x+\frac{5\pi}{12} = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{12}}$

~~at let us find~~ ~~at~~ $\cancel{x = \frac{\pi}{6}}$

$\therefore$ The maximum value of $f(x) = \sqrt{2} = 1.414$

Let $f(x) = \sin\left(x+\frac{\pi}{6}\right) + \cos\left(x+\frac{\pi}{6}\right)$

$$f\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}+\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{6}+\frac{\pi}{6}\right)$$

$$= \sin\frac{\pi}{3} + \cos\frac{\pi}{3} = \frac{\sqrt{3}+1}{2} = 1.366$$

Which is close to 1.414.

Hence maximum value occurs at $\frac{\pi}{6}$

Ans 4

05 Assertion: Conceptual (True)
Reason: Conceptual (True) Ans.: A

06 Assertion: Conceptual (True)
Reason: Conceptual (True) Ans.: A

07 Conceptual Ans.: D

08 min value $= 5 - \sqrt{(3)^2 + (-4)^2}$ Ans: C
$= 5 - 5 = 0$

09 $f = 3\sin^2 x + 5\sin x\cos x + 4\cos^2 x$
$a = 3,\ b = 5,\ c = 4.$
max. value $= \frac{a+c}{2} + \frac{1}{2}\sqrt{b^2 + (a-c)^2}$
$= \frac{3+4}{2} + \frac{1}{2}\sqrt{5^2 + (3-4)^2}$
$= \frac{7}{2} + \frac{\sqrt{26}}{2} = \frac{7+\sqrt{26}}{2}$ Ans. D

10 $f = 7 + 10\sin 2x + 8\sin^2 x$
$f = 7 + 10\sin 2x + 8\left(\frac{1-\cos 2x}{2}\right)$
$f = 7 + 10\sin 2x + 4(1 - \cos 2x)$
$= 11 + 10\sin 2x - 4\cos 2x$
mini value $= 11 - \sqrt{(10)^2 + (-4)^2}$
$= 11 - \sqrt{116}$
$= 11 - 2\sqrt{29}$ Ans. B

11. $11+5\sqrt{1-\cos^2 2x}+10\cos^2 x$

$= 11+5\sqrt{\sin^2 2x}+10\left(\frac{1+\cos 2x}{2}\right)$

$= 11+5\sin 2x+5+5\cos 2x$

$= 16+5\sin 2x+5\cos 2x$

mini value $= 16-\sqrt{(5)^2+(5)^2}$

$= 16-5\sqrt{2}$

Ans: A

$\boxed{12}$ $f(x) = 1-8\sin^2 x\cos^2 x$

$= 1-2(2\sin x\cos x)^2$

$= 1-2\cdot\sin^2 2x$

We know, $0 \le \sin^2 2x \le 1$

$0 \ge -2\sin^2 2x \ge -2$

$1 \ge 1-2\sin^2 2x \ge 1-2$

$\therefore -1 \le f(x) \le 1$

$\therefore$ max value $= 1$

$\boxed{\text{Ans: } 1}$

---

14 a) $f(x) = 2\sin^2 x+\cos^2 x$

$= 2\sin^2 x+1-\sin^2 x$

$= 1+\sin^2 x$

$0 \le \sin^2 x \le 1$

$1 \le 1+\sin^2 x \le 2$

$[1,2]$ (c)

b) $f(x) = 1 + 8\sin^2 x^2 \cdot \cos^2 x^2$

$= 1 + 2\left[2\sin x^2 \cdot \cos x^2\right]^2$

$= 1 + 2\left(\sin 2x^2\right)^2$

$= 1 + 2\sin^2 2x^2$

We know $0 \leq \sin^2 2x^2 \leq 1$

$0 \leq 2\sin^2 2x^2 \leq 2$

$\Rightarrow 1 \leq 1 + 2\sin^2 2x^2 \leq 3$

$f(x) \in [1, 3]$ (P)

c) Range $f(x) = \cos^2 x + \sin^4 x$

$= \cos^2 x + \sin^2 x \cdot \sin^2 x$

$= \cos^2 x + (1 - \cos^2 x) \cdot \sin^2 x$

$= \cos^2 x + \sin^2 x - \cos^2 x \cdot \sin^2 x$

$= 1 - \cos^2 x \cdot \sin^2 x$

$= 1 - \frac{1}{4}\left(2\sin x \cos x\right)^2$

$= 1 - \frac{1}{4}\sin^2 2x$

$0 \leq \sin^2 2x \leq 1$

$0 \geq -\frac{1}{4}\sin^2 2x \geq -\frac{1}{4}$

$1 \geq 1 - \frac{1}{4}\sin^2 2x \geq 1 - \frac{1}{4}$

$$\frac{3}{4} \le f(x) \le 1 \Rightarrow \left[\frac{3}{4}, 1\right] (q)$$

d) $f(x) = 3\sin^2 x + 4\cos^2 x$

$= 3\sin^2 x + 3\cos^2 x + \cos^2 x$

$= 3(\sin^2 x + \cos^2 x) + \cos^2 x$

$= 3 + \cos^2 x$

$\therefore 0 \le \cos^2 x \le 1$

$3 \le 3 + \cos^2 x \le 1 + 3$

$3 \le f(x) \le 4 \Rightarrow [3, 4] (r)$

Ans: s, p, q, r

15 a) $f(x) = 3\sin\sqrt{\frac{\pi^2}{9} - x}$

We know, $-1 \le \sin\sqrt{\frac{\pi^2}{9} - x} \le 1$

$-3 \le 3\sin\sqrt{\frac{\pi^2}{9} - x} \le 3 \rightarrow [-3, 3] (r)$

b) $f(\theta) = \sin\theta \cdot \sin(60^\circ + \theta)\sin(60^\circ - \theta)$

$= \sin\theta\left[\sin^2 60^\circ - \sin^2\theta\right]$

$= \sin\theta\left[\frac{3}{4} - \sin^2\theta\right]$

$= \frac{3\sin\theta - 4\sin^3\theta}{4} = \frac{1}{4}\sin 3\theta$

We know, $-1 \le \sin 3\theta \le 1$

$-\frac{1}{4} \le \frac{1}{4}\sin 3\theta \le \frac{1}{4} \rightarrow \left[-\frac{1}{4}, \frac{1}{4}\right]$

max. value $= \frac{1}{4}$ (p)

c) $f(x) = \cos^2\left(\frac{\pi}{3} - x\right) - \cos^2\left(\frac{\pi}{3} + x\right)$

$= \left(\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right)^2 - \left(\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x\right)^2$

$= \cancel{4} \cdot \frac{1}{2}\cos x \cdot \frac{\sqrt{3}}{2}\sin x$ $\quad (a+b)^2 - (a-b)^2 = 4ab$

$= \sqrt{3}\sin x \cdot \cos x$

$= \frac{\sqrt{3}}{2}\sin 2x$

max. value $= \frac{\sqrt{3}}{2}$, min. value $= \frac{-\sqrt{3}}{2}$

Sum $= \left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right) = 0$ (q)

d) $f(x) = |2\sin x + 3\cos x|$

$f(2\pi + x) = |2\sin(2\pi + x) + 3\cos(2\pi + x)|$

$= |2\sin x + 3\cos x| = f(x)$

period $= 2\pi$ (t)

Ans : r, p, q, t

$\Rightarrow$ THE END $\Leftarrow$