9th foundation + ws-11. Dot product

Task

(1)

(1)

Given $\vec{P} = 2i+3j-4k$ and $Q = 5\hat{i}+2\hat{j}+4\hat{k}$

$\vec{P}\cdot\vec{Q} = (2\hat{i}+3\hat{j}-4\hat{k})\cdot(5\hat{i}+2\hat{j}+4\hat{k})$

$= 2\cdot5 + 3\cdot2 - 4\cdot4 = 10+6-16 = 0$

$\cos\theta = \dfrac{\vec{P}\cdot\vec{Q}}{|\vec{P}|\,|\vec{Q}|} = \dfrac{0}{|\vec{P}||\vec{Q}|} = 0 \Rightarrow \theta = 90^\circ \rightarrow D$

(2)

Given $\vec{F} = 5\hat{i}-3\hat{j}+2\hat{k}$; $r_1 = 2\hat{i}+7\hat{j}+4\hat{k}$; $r_2 = 2\hat{j}+5\hat{i}+8\hat{k}$

Displacement $\vec{S} = r_2 - r_1 \Rightarrow 5\hat{i}+2\hat{j}+8\hat{k}-2\hat{i}-7\hat{j}-4\hat{k}$

$\Rightarrow 3\hat{i}-5\hat{j}+4\hat{k}$

$\therefore$ work done $\omega = \vec{F}\cdot\vec{S} = (5\hat{i}-3\hat{j}+2\hat{k})\cdot(3\hat{i}-5\hat{j}+4\hat{k})$

$= 5\cdot3 + (-3)(-5) + 2(4)$

$\Rightarrow 15+15+8 = 38$ units $\rightarrow A$

(3)

Given $\vec{A} = \hat{i}+2\hat{j}$; $|\vec{B}| = 3\sqrt{5}$

$\vec{C}$ is a vector which is $\perp^{ar}$ to $\vec{A}$ and $|\vec{C}| = |\vec{B}|$

let $\vec{C} = x\hat{i}+y\hat{j} \Rightarrow \vec{C}\cdot\vec{A} = 0 \Rightarrow (x\hat{i}+y\hat{j})(\hat{i}+2\hat{j}) = 0$

$\Rightarrow x+2y = 0 \rightarrow (1) \Rightarrow x = -2y$

also : $\sqrt{x^2+y^2} = 3\sqrt{5} \Rightarrow x^2+y^2 = 45 \rightarrow (2)$

$\Rightarrow (-2y)^2+y^2 = 45 \Rightarrow \cancel{5}y^2 = \cancel{45}\,9$

$y = \sqrt{9} = 3$

$\therefore x = -6$

$\therefore \vec{C} = -6\hat{i}+3\hat{j} \rightarrow B$

(4)

(2)

Given $\vec{P} = a\hat{i} + a\hat{j} + 3\hat{k}$ and $\vec{Q} = a\hat{i} - 2\hat{j} - \hat{k}$

$\therefore \quad \vec{P}\cdot\vec{Q} = 0$

$\Rightarrow (a\hat{i} + a\hat{j} + 3\hat{k})\cdot(a\hat{i} - 2\hat{j} - \hat{k}) = 0$

$\Rightarrow \quad a^2 - 2a - 3 = 0 \Rightarrow a^2 - 3a + a - 3 = 0$

$\Rightarrow a(a-3) + 1(a-3) = 0$

$\Rightarrow (a-3) = 0$ (or) $(a+1) = 0$

$\Rightarrow a = 3$ (or) $-1 \rightarrow D$

(5)

Given

$F = 2\hat{i} + 3\hat{j} + 2\hat{k}$ N ; $\vec{S} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ m ; $t = 4$ sec

work $= \vec{F}\cdot\vec{S} = (2\hat{i} + 3\hat{j} + 2\hat{k})\cdot(3\hat{i} + 4\hat{j} + 5\hat{k})$

$= 2\times3 + 3\times4 + 2\cdot5$

$\Rightarrow 6 + 12 + 10 = 28.$

Power $= \frac{w}{t} = \frac{28}{4} = 7\,W \rightarrow C.$

(6)

Given $\vec{A} = 2\hat{i} + 3\hat{j}$ & $\vec{B} = 2\hat{i} + 3\hat{k}.$

component of $\vec{B}$ along $\vec{A} = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}|}$

$= \frac{(2\hat{i} + 3\hat{j})\cdot(2\hat{i} + 3\hat{k})}{\sqrt{2^2 + 3^2}} = \frac{2\times2 + 3\cdot(0) + 3(0)}{\sqrt{4+9}}$

$\Rightarrow \frac{4}{\sqrt{13}} \rightarrow C.$

(7)

Given that component of $\vec{A}$ along $\vec{B}$ = $\sqrt{3}$ component of $\vec{B}$ along $\vec{A}$

$\Rightarrow A\cos\theta = \sqrt{3}\, B\cos\theta$

$\Rightarrow A = \sqrt{3}\, B \Rightarrow \frac{A}{B} = \frac{\sqrt{3}}{1}$ — A

(8)

Given $\vec{F} = 8\hat{i} + 4\hat{j}$ N ; $\vec{S} = 3\hat{i} - 3\hat{j}$ m ; Power = 0.6 W

work = $\vec{F}\cdot\vec{S} = (8\hat{i} + 4\hat{j})\cdot(3\hat{i} - 3\hat{j})$

$= 8\times3 + 4(-3) = 24 - 12 = 12$

Power $= \frac{W}{t} \Rightarrow t = \frac{W}{P} = \frac{12}{0.6} = \frac{120}{6} = 20 \rightarrow A$

(9)

Given $\vec{A} + \vec{B} = \vec{R} \rightarrow (1)$ $2\vec{A} + \vec{B}$ is perpendicular to $\vec{B}$

$\Rightarrow (2\vec{A} + \vec{B})\cdot\vec{B} = 0 \Rightarrow 2\,\vec{A}\cdot\vec{B} + \vec{B}\cdot\vec{B} = 0$

$\Rightarrow 2AB = -\vec{B}\cdot\vec{B}$

$\Rightarrow 2A\cancel{B} = -B^{\cancel{2}} \rightarrow (2)$

$\Rightarrow 2A = -B$

From $\vec{A} + \vec{B} = \vec{R}$

$\Rightarrow (\vec{A} + \vec{B})\cdot\vec{B} = \vec{R}\cdot\vec{B}$

$\Rightarrow \vec{A}\cdot\vec{B} + \vec{B}\cdot\vec{B} = \vec{R}\cdot\vec{B}$

$\Rightarrow \vec{A}\cdot\vec{B} - 2AB = \vec{R}\cdot\vec{B}$

$\Rightarrow -AB = \vec{R}\cdot\vec{B}$

$\Rightarrow -A\times(-2A) = \vec{R}(2A)$

$\Rightarrow 2A^2 = 2AR$

$\Rightarrow R = A$

(or)

From (1) $(\vec{A} + \vec{B})^2 = R^2$

$\Rightarrow A^2 + B^2 + 2AB = R^2$

$\Rightarrow A^2 - 2AB + 2AB = R^2$

$= R^2 = A^2$ [From (2)]

$\Rightarrow R = A$

$\rightarrow A$

(10)

(4)

Given $\vec{v} = 2\hat{i} + c\hat{j}$ and $\vec{a} = 3\hat{i} + 4\hat{j}$.

Given velocity and acceleration are perpendicular

$\Rightarrow \vec{v} \cdot \vec{a} = 0$

$\Rightarrow (2\hat{i} + c\hat{j}) \cdot (3\hat{i} + 4\hat{j}) = 0$

$\Rightarrow 2 \times 3 + c \times 4 = 0$

$\Rightarrow 6 + 4c = 0 \Rightarrow 4c = -6 \Rightarrow c = -\frac{3}{2} = -1.5$

$\rightarrow C$

Advanced level

(1)

Given $\vec{a} = m\vec{b} + \vec{c}$.

on doing dot product with $\vec{b}$ on both sides

$\vec{a} \cdot \vec{b} = (m\vec{b} + \vec{c}) \cdot \vec{b}$

$\Rightarrow \vec{a} \cdot \vec{b} = m\vec{b} \cdot \vec{b} + \vec{c} \cdot \vec{b}$

$\Rightarrow \vec{a} \cdot \vec{b} - \vec{c} \cdot \vec{b} = m b^2$

$\Rightarrow m = \dfrac{\vec{a} \cdot \vec{b} - \vec{c} \cdot \vec{b}}{b^2} \rightarrow A$

(3)

(c) $\sqrt{\vec{A} \cdot \vec{A}} = \sqrt{AA\cos\theta} = \sqrt{A^2 \cos\theta} = \sqrt{A^2} = A$ [Both $\vec{A}$ and $\vec{A}$ are parallel]

(d) $\vec{a} = m\vec{b}$ on doing dot product with $\vec{b}$ on both side

$\Rightarrow \vec{a} \cdot \vec{b} = m\,\vec{b} \cdot \vec{b}$

$\Rightarrow \vec{a} \cdot \vec{b} = m b^2 \Rightarrow m = \dfrac{\vec{a} \cdot \vec{b}}{b^2}$ (A, B, C, D)

(4)

Given $\vec{A} = 5\hat{i}+6\hat{j}+3\hat{k}$ an $\vec{B} = 6\hat{i}-2\hat{j}-6\hat{k}$

$\vec{A}\cdot\vec{B} = [5\hat{i}+6\hat{j}+3\hat{k}]\cdot[6\hat{i}-2\hat{j}-6\hat{k}]$

$= 5\times 6 + 6(-2) + 3(-6) = 30-12-18 = 0.$

$\vec{B}\cdot\vec{A} = 0 \rightarrow A, B, D.$

(5)

Given $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{C}$

$\Rightarrow AB\cos\theta_1 = BC\cos\theta_2$ if $\theta_1 = \theta_2$ then $A = C$

Other wise $\vec{A}$ is not equal to c.

$\rightarrow D$

(6)

Given $\theta = 60^\circ$

$\vec{a}\cdot\vec{b} = ab\cos\theta = ab\cos 60^\circ = \frac{ab}{2} \rightarrow C$

(7)

Component of $\vec{b}$ along $\vec{a} = b\cos\theta = \frac{\vec{b}\cdot\vec{a}}{|\vec{a}|}$

$= b\frac{\vec{a}}{|\vec{a}|} = b\hat{a} \rightarrow B$

(9)

Given
let $\vec{A} = \hat{i}+y\hat{j}+2\hat{k}$ & $\vec{B} = 2\hat{i}+4\hat{j}-3\hat{k}$

$\vec{A}\perp^{r}$ to $\vec{B}$ $(\vec{A}\cdot\vec{B}) = 0 \Rightarrow (\hat{i}+y\hat{j}+2\hat{k})\cdot(2\hat{i}+4\hat{j}-3\hat{k}) = 0$

$\Rightarrow 2+4y-6 = 0$

$\Rightarrow 4y-4 = 0 \Rightarrow 4y = 4 \Rightarrow y = 1$

(8)

Given $\vec{P} = \hat{i} + 2\hat{j} - 4\hat{k}$ $\quad \vec{Q} = \hat{i} + 2\hat{j} - \hat{k}$

$\vec{P} + \vec{Q} = \hat{i} + 2\hat{j} - 4\hat{k} + \hat{i} + 2\hat{j} - \hat{k} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

$\vec{P} - \vec{Q} = \cancel{\hat{i}} + \cancel{2\hat{j}} - 4\hat{k} - \cancel{\hat{i}} - \cancel{2\hat{j}} + \hat{k} = -3\hat{k}$

$(\vec{P} + \vec{Q}) \cdot (\vec{P} - \vec{Q}) = (2\hat{i} + 4\hat{j} - 5\hat{k}) \cdot (-3\hat{k})$

$= (-5)(-3) = 15$

(20)

(a)

$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$

$\Rightarrow A^2 \cancel{+B^2} + 2AB\cos\theta = A^2 \cancel{+B^2} - 2AB\cos\theta$

$\Rightarrow 2AB\cos\theta = -2AB\cos\theta \Rightarrow 4AB\cos\theta = 0$

$\Rightarrow \cos\theta = 0$

$\theta = 90^\circ$

(b)

$|\vec{A} \times \vec{B}| = \vec{A} \cdot \vec{B}$

$\Rightarrow \cancel{AB}\sin\theta = \cancel{AB}\cos\theta \Rightarrow \sin\theta = \cos\theta \Rightarrow \theta = 45^\circ$

(c)

$\vec{A} \cdot \vec{B} = \frac{AB}{2}$

$\Rightarrow \cancel{AB}\cos\theta = \frac{\cancel{AB}}{2} \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$

(d)

$|\vec{A} \times \vec{B}| = \frac{AB}{2}$

$\Rightarrow \cancel{AB}\sin\theta = \frac{\cancel{AB}}{2} \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30^\circ.$

L Task

CVQ's

(2)

Given $\vec{A}+\vec{B}=\vec{C}$

on doing dot Product with $\vec{B}$ on both sides

$$(\vec{A}+\vec{B})\cdot\vec{B}=\vec{C}\cdot\vec{B}$$

$$\Rightarrow \vec{A}\cdot\vec{B}+\vec{B}\cdot\vec{B}=\vec{C}\cdot\vec{B}$$

$$\Rightarrow \vec{B}^2 \Rightarrow \vec{C}\cdot\vec{B}-\vec{A}\cdot\vec{B}$$

$$\Rightarrow \vec{B}=\sqrt{\vec{C}\cdot\vec{B}-\vec{A}\cdot\vec{B}}$$

(5)

Given $\vec{A}\cdot\vec{B}=0$ ie $\vec{A}$ is Perpendicular to $\vec{B}$

$\vec{A}\cdot\vec{C}=0$ ie $\vec{A}$ is Perpendicular to $\vec{C}$

(ie) A must lie along the direction that is perpendicular to the plane formed by $\vec{B}$ and $\vec{C}$.

so $\vec{A}=\vec{B}\times\vec{C}$

(6)

we know Torque $\tau=\vec{r}\times\vec{F} \rightarrow (1)$

$\therefore r\cdot\tau = r\cdot(\vec{r}\times\vec{F}) \Rightarrow$ ~~r·τ...~~

The dot product of a vector $(\vec{r})$ with a cross Product of itself and another vector(F) always zero $r\cdot\tau=0$

Again from (1) $F\cdot\vec{\tau}=F\cdot(\vec{r}\times\vec{F}) \Rightarrow F\cdot\tau=0$ $\rightarrow (2)$

(By using same above rule for eqn (2)

(8) (1)

let the given vectors be $\vec{A} = \hat{i} + \hat{j}$ and $\vec{B} = \hat{j} + \hat{k}$

$\vec{A} \cdot \vec{B} = (\hat{i} + \hat{j}) \cdot (\hat{j} + \hat{k}) = \hat{j} \cdot \hat{j} = 1$

$|\vec{A}| = \sqrt{1^2 + 1^2} = \sqrt{2}$; $|\vec{B}| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|} = \dfrac{1}{\sqrt{2}\sqrt{2}} = \dfrac{1}{2} \Rightarrow \theta = 60^\circ \rightarrow C$

(9)

Given $\vec{A} = (4, -2, 5)$ $\vec{B} = (3, 1, -2)$

$\vec{A} \cdot \vec{B} = (4 \cdot 3) + (-2 \cdot 1) + (5 \times -2) = 12 - 2 - 10 = 0$

$\cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|} = \dfrac{0}{|\vec{A}||\vec{B}|} = 0 \Rightarrow \theta = 90^\circ - C$

(10)

Given $\vec{A} = 2\hat{i} + 3\hat{j}$

Angle made by the vector with y-axis

$\tan\theta = \left(\dfrac{A_x}{A_y}\right) = \dfrac{2}{3}$

$\theta = \tan^{-1}\left(\dfrac{2}{3}\right) \rightarrow B$

L Task

Jee main's level

① Given $\vec{A} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{B} = \hat{i} - \hat{k}$

$\vec{A} \cdot \vec{B} = (2\hat{i} + \hat{j} - \hat{k}) \cdot (\hat{i} - \hat{k}) = 2 \times 1 + (-1)(-1) = 2 + 1 = 3$

$|\vec{A}| = \sqrt{2^2 + 1^2 + (-1)^2}$; $|\vec{B}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$= \sqrt{6}$

$\cos\theta = \dfrac{\vec{A} \cdot \vec{B}}{|\vec{A}|\,|\vec{B}|} = \dfrac{3}{\sqrt{6}\sqrt{2}} = \dfrac{3}{2\sqrt{3}} = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \theta = 30^\circ \rightarrow A$

② Given $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = \hat{i} + \hat{j}$?

Component of $\vec{A}$ along $\vec{B} = \dfrac{\vec{A} \cdot \vec{B}}{|\vec{B}|} = \dfrac{(3\hat{i} + 4\hat{j}) \cdot (\hat{i} + \hat{j})}{\sqrt{1^2 + 1^2}}$

$= \dfrac{3+4}{\sqrt{2}} = \dfrac{7}{\sqrt{2}} \rightarrow B$

③ Given $|\vec{a}_1| = |\vec{a}_2| = 1.$

also $|\vec{a}_1 + \vec{a}_2| = \sqrt{3}.$

$\Rightarrow \sqrt{a_1^2 + a_2^2 + 2\vec{a}_1 \cdot \vec{a}_2} = \sqrt{3}$

$\Rightarrow 1^2 + 1^2 + 2\vec{a}_1 \cdot \vec{a}_2 = (\sqrt{3})^2$

$\Rightarrow 2 + 2\vec{a}_1 \cdot \vec{a}_2 = 3$

$\Rightarrow 2\vec{a}_1 \cdot \vec{a}_2 = 1$

$\therefore (\vec{a}_1 - \vec{a}_2) \cdot (2\vec{a}_1 + \vec{a}_2) = 2\vec{a}_1 \cdot \vec{a}_1 + \vec{a}_1 \cdot \vec{a}_2 - 2\vec{a}_1 \cdot \vec{a}_2 - \vec{a}_2 \cdot \vec{a}_2$

$\Rightarrow 2 \times 1 \times 1 + \dfrac{1 \times 1}{2} - 1 - 1$

$\Rightarrow \dfrac{4+1}{2} - 2$

$\Rightarrow \dfrac{5}{2} - 2$

$\Rightarrow \dfrac{1}{2} \rightarrow C.$

7)

(4)

Given that $(\vec{a} + 2\vec{b}) \cdot (5\vec{a} - 4\vec{b}) = 0$

$$\Rightarrow 5|\vec{a}|^2 - 8|\vec{b}|^2 - 4\vec{a}\cdot\vec{b} + 10\,\vec{b}\cdot\vec{a} = 0$$

$$\Rightarrow -3|\vec{a}|^2 + 6\vec{a}\cdot\vec{b} = 0$$

$$\Rightarrow 6|\vec{a}||\vec{b}|\cos\theta = 3$$

$$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60^\circ \rightarrow D$$

(5)

Given $\vec{A} = 5\hat{i} + 7\hat{j} - 3\hat{k}$ and $\vec{B} = 2\hat{i} + 2\hat{j} - c\hat{k}$

$\vec{A}$ and $\vec{B}$ are Pendicular to each other.

$$\vec{A}\cdot\vec{B} = 0$$

$$\Rightarrow (5\hat{i} + 7\hat{j} - 3\hat{k})\cdot(2\hat{i} + 2\hat{j} - c\hat{k}) = 0$$

$$\Rightarrow 5\times 2 + 7\times 2 - 3(-c) = 0$$

$$\Rightarrow 10 + 14 + 3c = 0 \quad \Rightarrow 3c = -24$$

$$\Rightarrow c = -8 \rightarrow B$$

(6)

Given

Force $F = 3\hat{i} + 4\hat{k}$ N. : displacement $\vec{S} = 2\hat{i} + 4\hat{j}$ m

time = 1 sec.

$$\text{Power} = \frac{w}{t} = \frac{\vec{F}\cdot\vec{S}}{t} = \frac{(3\hat{i} + 4\hat{k})\cdot(2\hat{i} + 4\hat{j})}{1}$$

$$P = \frac{3\cdot 2 + 4\cdot 4}{1} = \frac{6 + 16}{1} = \frac{22}{1}$$

$$P = 22\,W$$

(7)

let $\vec{A} = 2\hat{i}+5\hat{k}$ & $\vec{B} = 3\hat{j}+4\hat{k}$

Given $\vec{A}\cdot\vec{B} = (2\hat{i}+5\hat{k})\cdot(3\hat{j}+4\hat{k})$

$\Rightarrow \cancel{2\times}\ 5\times4 = 20 \rightarrow A$

(8)

Given

$(3\hat{i}-2\hat{j}+2\hat{k})\cdot(2\hat{i}-x\hat{j}+3\hat{k}) = -12$

$\Rightarrow 3\times2+(-2)(-x)+2\times3 = -12$

$\Rightarrow 6+2x+6 = -12$

$\Rightarrow 2x = -12-12 \Rightarrow 2x = -24$

$\Rightarrow x = -12. \rightarrow D$

(9)

If $\vec{A} = 6\hat{i}+5\hat{j}$ and $\vec{B} = 4\hat{i}-3\hat{j}$

$\vec{A}+\vec{B} = 6\hat{i}+5\hat{j}+4\hat{i}-3\hat{j} \Rightarrow 10\hat{i}+2\hat{j}$

$\vec{A}-\vec{B} = 6\hat{i}+5\hat{j}-4\hat{i}+3\hat{j} \Rightarrow 2\hat{i}+8\hat{j}$

$(\vec{A}+\vec{B})\cdot(\vec{A}-\vec{B}) = (10\hat{i}+2\hat{j})\cdot(2\hat{i}+8\hat{j})$

$\Rightarrow 10\times2+2\times8$

$\Rightarrow 20+16 = 36 \rightarrow B$

(10)

let $|\vec{A}| = 3$; $|\vec{B}| = 4$ similar $\vec{A}\cdot\vec{B} = 6$

$\Rightarrow \vec{A}\cdot\vec{B} = 6$

$\Rightarrow AB\cos\theta = 6$

$\Rightarrow \cancel{3}\times4\times\cos\theta = \cancel{6}\ 2$

$\Rightarrow 4\cos\theta = 2$

$\Rightarrow \cos\theta = \frac{2}{4} = \frac{1}{2}$

$\Rightarrow \theta = 60^\circ \rightarrow A$

(12)

let $\vec{A} = 6\hat{i}+3\hat{j}+2\hat{k}$ and $\vec{B} = 2\hat{i}-2\hat{j}-\hat{k}$

$\vec{A}\cdot\vec{B} = (6\hat{i}+3\hat{j}+2\hat{k})\cdot(2\hat{i}-2\hat{j}-\hat{k})$

$\Rightarrow 6\times2 + 3\times(-2) + 2(-1) \Rightarrow 12-6-2 = 4$

$|\vec{A}| = \sqrt{6^2+3^2+2^2} = \sqrt{49} = 7$

$|\vec{B}| = \sqrt{2^2+(-2)^2+(-1)^2} = \sqrt{9} = 3$

$\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \dfrac{4}{7\times3} = \dfrac{4}{21}.$

$\theta = \cos^{-1}\left(\frac{4}{21}\right) \rightarrow c$

(13)

$P\cdot Q = PQ \Rightarrow P\cdot Q = PQ$

$\Rightarrow \cancel{PQ}\cos\theta = \cancel{PQ} \Rightarrow \cos\theta = 1$

$\Rightarrow \theta = 0^\circ$

$\vec{P}\cdot\vec{Q} = \frac{\sqrt{3}}{2} PQ$

$\Rightarrow \cancel{PQ}\cos\theta = \frac{\sqrt{3}}{2}\cancel{PQ} \Rightarrow \cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30^\circ$

(14), (15)

Given $\vec{F} = 5\hat{i}-3\hat{j}+2\hat{k}$; $\vec{r}_1 = 2\hat{i}+7\hat{j}+4\hat{k}$; $\vec{r}_2 = 5\hat{i}+2\hat{j}+8\hat{k}$

Displacement $\vec{s} = \vec{r}_2 - \vec{r}_1 = 5\hat{i}+2\hat{j}+8\hat{k} - 2\hat{i}-7\hat{j}-4\hat{k}$

$= 3\hat{i}-5\hat{j}+4\hat{k} \rightarrow c$

work done $= \vec{F}\cdot\vec{S} \Rightarrow (5\hat{i}-3\hat{j}+2\hat{k})\cdot(3\hat{i}-5\hat{j}+4\hat{k})$

$\Rightarrow 5\cdot3 + (-3)(-5) + 2(4)$

$\Rightarrow 15+15+8$

$\Rightarrow 38 \rightarrow c.$

(16)

Given

$(3\hat{i}+4\hat{j}-7\hat{k})\cdot(2\hat{i}+4\hat{j}+p\hat{k})=8$

$\Rightarrow 3\cdot 2+4\cdot 4-7P=8$

$\Rightarrow 6+16-7P=8 \quad \Rightarrow 7P=22-8$

$\Rightarrow 7P=14 \Rightarrow P=2.$

(17)

Given vector is $\sqrt{3}\,\hat{i}+\hat{j}$; $\theta=\frac{\pi}{k}$

Angle made by the vector with x-axis

$\tan\theta=\frac{y\text{-comp}}{x\text{-comp}}=\frac{1}{\sqrt{3}}$

$\Rightarrow \theta=30^\circ=\frac{\pi}{6}$

$\Rightarrow \frac{\pi}{k}=\frac{\pi}{6} \Rightarrow k=6$

(18)

Angle between two vectors is 90° means the vectors are Perpendicular to each other. Their dot Product is 0.

$\Rightarrow (\hat{i}-\hat{j}-3\hat{k})\cdot(-2\hat{i}-c\hat{j}+\hat{k})=0$

$\Rightarrow 1\times(-2)+(-1)(-c)+(-3)(1)=0$

$\Rightarrow -2+c-3=0$

$\Rightarrow c=5$

(19)

If $\vec{A}\cdot\vec{B}=\frac{AB}{2}$

$\Rightarrow AB\cos\theta=\frac{AB}{2} \qquad \Rightarrow \theta=60^\circ$

$\Rightarrow \cos\theta=\frac{1}{2}$