WS-2 8th advanced

Thank

(1) Given $y = x^{\frac{7}{2}}$

we know that $\frac{d}{dx} x^n = n x^{n-1}$

$\therefore \frac{dy}{dx} = \frac{d}{dx} [x^{\frac{7}{2}}]$

Here $n = \frac{7}{2}$

$= \frac{7}{2} (x)^{\frac{7}{2} - 1} = \frac{7}{2} x^{\frac{5}{2}}$

(2)

Given $a = x^{-3}$

we know that $\frac{d}{dx} x^n = n x^{n-1}$

$\therefore \frac{da}{dx} = \frac{d}{dx} [x^{-3}]$

Here $n = -3$

$= (-3\, x^{-3-1}) = -3 x^{-4}$

(3)

Given $a = t$

$\therefore \frac{da}{dt} = \frac{dt}{dt} = 1$

(4) Given $y = x^5 + x^3$

$\therefore \frac{dy}{dx} = \frac{d}{dx} [x^5 + x^3]$

$= \frac{d}{dx} x^5 + \frac{d}{dx} x^3$

we know $\frac{d}{dx} x^n = n x^{n-1}$

$\therefore$

$\Rightarrow 5 x^{5-1} + 3 x^{3-1}$

$= 5x^4 + 3x^2$

(5) Given $P = 5t^4 + 6t^3 + 9t$

$\therefore \frac{dP}{dt} = \frac{d}{dt} [5t^4 + 6t^3 + 9t]$

$= 5 \frac{d}{dt} t^4 + 6 \frac{d}{dt} t^3 + 9 \frac{dt}{dt}$

$\Rightarrow 5(4t^{4-1}) + 6(3t^{3-1}) + 9$

$= 20t^3 + 18t^2 + 9$

$[\because \frac{d}{dx} x^n = n x^{n-1}]$

(6) Given $s = t^2 + 5t + 3$.

$\therefore \frac{ds}{dt} = \frac{d}{dt}[t^2+5t+3]$

$= \frac{d}{dt}t^2 + 5\frac{dt}{dt} + \frac{d}{dt}(3)$

$= 2t^{2-1} + 5 + 0$

$= 2t + 5$

$[\frac{d}{dx}x^n = nx^{n-1} ; \frac{d}{dx}(\text{constant}) = 0]$

(7) Given $A = 3t^2 + 7 \ cm^2$

The rate of increase of area

$\frac{dA}{dt} = \frac{d}{dt}[3t^2+7]$

$= 3\frac{d}{dt}t^2 + \frac{d}{dt}(7)$

$= 3[2t^{2-1}] + 0$

$= 6t$; At $t = 5$

$\frac{dA}{dt} = 6(5) = 30 \ cm^2/sec$

(14)

(8)

From $\frac{d}{dx}x^n = nx^{n-1}$

(a) $\frac{d}{dx}x^4 = 4x^{4-1}$

$= 4x^3$

(b) $\frac{d}{dx}x^3 = 3x^{3-1}$

$= 3x^2$

(c) $\frac{d}{dy}y^{-5} = -5y^{-5-1}$

$= -5y^{-6}$

(10) A: $\frac{d}{dt}t^5 = 5t^{5-1} = 5t^4$.

(12)

Given $y = \sqrt{x} = x^{\frac{1}{2}}$

$\therefore \frac{dy}{dx} = \frac{d}{dx}x^{\frac{1}{2}} = \frac{1}{2}x^{\frac{1}{2}-1}$

$= \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{1}{2}\frac{1}{x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$

From $\frac{d}{dx}x^n = nx^{n-1}$

(13)

Given $a = t^{-\frac{3}{2}}$

$\frac{da}{dt} = \frac{d}{dt}(t^{-\frac{3}{2}}) = \frac{-3}{2}t^{-\frac{3}{2}-1}$

$\frac{da}{dt} = \frac{-3}{2}t^{-\frac{5}{2}}$

At $t = 4 \ sec$

$\frac{da}{dt} = \frac{-3}{2}[4]^{-\frac{5}{2}}$

$= \frac{-3}{2}[2^{\cancel{2}}]^{-\frac{5}{\cancel{2}}} = \frac{-3}{2}2^{-5}$

$= -\frac{3}{2} \times \frac{1}{2^5} = \frac{-3}{64}$

$\frac{1}{7} cm^2$

of area

(14)

Given $y = \sqrt{x}$; $a = t^{-\frac{3}{2}}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$: $\frac{da}{dt} = -\frac{3}{2} \frac{1}{t^{\frac{5}{2}}}$

$\therefore \frac{dy}{dx} \times \frac{da}{dt} = \frac{1}{2\sqrt{x}} \left[-\frac{3}{2}\right] \frac{1}{t^{\frac{5}{2}}}$

$= \frac{-3}{4\sqrt{x t^5}}$

iTask

SAQ's

(1) $y = x^{15}$

$\therefore \frac{dy}{dx} = \frac{d}{dx} x^{15}$

$= 15 x^{15-1}$

$= 15x^{14}$

(2) $a = t^{\frac{5}{2}}$

we know $\frac{d}{dx} x^n = nx^{n-1}$

$\therefore \frac{da}{dt}$, Differentiation w.r.t 'x' is not possible because a is a function of 't'

(3) Given $y = 5t^2 + 3t$

$\frac{dy}{dt} = \frac{d}{dt}[5t^2 + 3t]$

$= 5\frac{d}{dt} t^2 + 3\frac{dt}{dt}$

$= 5[2t^{2-1}] + 3$

$= 10t + 3$

(4) Given $y = -3t^{-3}$

$\therefore \frac{dy}{dt} = \frac{d}{dt}[-3t^{-3}]$

$= -3\left(\frac{d}{dt} t^{-3}\right) \quad \left[\because \frac{d}{dx} x^n = nx^{n-1}\right]$

$= -3(-3 t^{-3-1})$

$= 9t^{-4}$

(5)

Given $a = bx^3 + cx^4$

$\therefore \frac{da}{dx} = \frac{d}{dx}[bx^3 + cx^4]$

we know $\frac{d}{dx} x^n = nx^{n-1}$

$\Rightarrow \frac{da}{dx} = b\frac{d}{dx}x^3 + c\frac{d}{dx}x^4$

$= b[3x^{3-1}] + c[4x^{4-1}]$

$= 3bx^2 + 4cx^3$

(6) Given $v = 9t^2 + 3$.

then $\frac{dv}{dt} = \frac{d}{dt}[9t^2+3]$

$= 9\frac{d}{dt}t^2 + \frac{d}{dt}(3)$

$= 9[2t^{2-1}] + 0$

$= 18t$

(7)

Given $s = 4t^2 + 2t$

Rate of increase in displacement

$\frac{ds}{dt} = \frac{d}{dt}[4t^2 + 2t]$

$= 4\frac{d}{dt}t^2 + 2\frac{dt}{dt}$

$= 4[2t^{2-1}] + 2$

$= 8t + 2$

At $t = 2$ sec

$\frac{ds}{dt} = 8(2) + 2$

$= 16 + 2$

$= 18$

(10) Given $y = 4x^3 + 2x^2$

$\therefore \frac{dy}{dx} = \frac{d}{dx}[4x^3 + 2x^2]$

$= 4\frac{d}{dx}x^3 + 2\frac{d}{dx}x^2$

$= 4[3x^{3-1}] + 2[2x^{2-1}]$

$= 12x^2 + 4x$

at $x = 1$

$\frac{dy}{dx} = 12(1)^2 + 4(1)$

$= 12 + 4$

$= 16$

(11) ~~Given~~

(12) Given $y = 4x^3 + 2x^2$

$\therefore \frac{dy}{dx} = \cancel{12x^2 + 4x}.\frac{d}{dx}(4x^3 + 2x^2)$

$= 4\frac{d}{dx}x^3 + 2\frac{d}{dx}x^2$

$= 12x^2 + 4x$

~~At~~

At $x = \frac{1}{2}$ $\therefore \frac{dy}{dx} = 12(\frac{1}{2})^2 + 4(\frac{1}{2})$

$= 3 + 2 = 5$

(11) Given $y = 4x^3 + 2x^2$

$\frac{dy}{dx} = \frac{d}{dx}[4x^3 + 2x^2]$

$= 4\frac{d}{dx}x^3 + 2\frac{d}{dx}x^2$

$= 12x^2 + 4x$

At $x = 3 \Rightarrow \frac{dy}{dx} = 12(3)^2 + 4(3)$

$= 108 + 12 = 120$