7. TRIGONOMETRIC EQUATIONS-II ① 2025|26

Class: IX, MATHEMATICS

FOUNDATION+, SOLUTIONS

TEACHING TASK
JEE MAINS LEVEL

01. Given $\tan x + \tan(120^\circ + x) + \tan(x - 120^\circ) = 0$

put $x = 1$ in all the options, we get

A) $\frac{\pi}{3}$ B) $\frac{\pi}{6}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{2}$

Now, option verification.

Option: A; $x = \frac{\pi}{3} = 60^\circ$

$$LHS = \tan 60^\circ + \tan(120^\circ + 60^\circ) + \tan(60^\circ - 120^\circ)$$
$$= \tan 60^\circ + \tan 180^\circ + \tan(-60^\circ)$$
$$= \tan 60^\circ - 0 - \tan 60^\circ = 0 = RHS$$

Ans. A

02. We know,

$$\tan x \cdot \tan(120^\circ + x) \cdot \tan(120^\circ - x) = \tan 3x$$

$\therefore \tan 3x = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan 3x = \tan \frac{\pi}{6}$

$\Rightarrow 3x = n\pi + \frac{\pi}{6}$

$x = \frac{n\pi}{3} + \frac{\pi}{18}, n \in Z$

$= (6n+1)\frac{\pi}{18}$

Ans.. A

03 We know, $\cos^3\alpha + \cos^3(120^\circ+\alpha) + \cos^3(120^\circ-\alpha)$ (2)

$$= \frac{3}{4}\cos 3\alpha$$

$\therefore \frac{3}{4}\cos 3\alpha = \frac{3\sqrt{3}}{4}$

$\Rightarrow \cos 3\alpha = \sqrt{3} = 1.732.$

Since $-1 \le \cos 3\alpha \le 1$

This eq<u>n</u> does not have any real solutions

Ans: A

04 We know, $\cos\theta \cdot \cos(120^\circ+\theta) \cdot \cos(120^\circ-\theta) = \frac{1}{4}\cos 3\theta$

$\Rightarrow 4\cos\theta \cdot \cos(120^\circ+\theta) \cdot \cos(120^\circ-\theta) = \cos 3\theta$

$\therefore \cos 3\theta = \frac{1}{2}$

$\Rightarrow \cos 3\theta = \cos\frac{\pi}{3}$

$\Rightarrow 3\theta = 2n\pi \pm \frac{\pi}{3}$

$\Rightarrow \theta = \frac{2n\pi}{3} \pm \frac{\pi}{9}$ Ans: C

05 Given $x^2 + 4 + 3\sin(ax+b) - 2x = 0$

$\Rightarrow x^2 + 4 - 2x = -3\sin(ax+b)$

$\Rightarrow (x-1)^2 + 3 = -3\sin(ax+b)$

Clearly $(x-1)^2 + 3 \ge 3$

$\therefore -3\sin(ax+b) \ge 3$

$\Rightarrow \sin(ax+b) \le -1$

$\Rightarrow \sin(ax+b) = -1$

$\Rightarrow ax+b = \frac{3\pi}{2}$ or $\frac{7\pi}{2}$

But given $(a, b) \in [0, 2\pi]$

let $x = 1$, $a+b = \frac{3\pi}{2}$ Ans. B

06 Given $\cos x + \cos y = 1 \longrightarrow (1)$

$\cos x \cdot \cos y = \frac{1}{4}$

$\Rightarrow \cos x(1 - \cos x) = \frac{1}{4}$

$\Rightarrow 4\cos^2 x - 4\cos x + 1 = 0$

$\Rightarrow (2\cos x - 1)^2 = 0$

$\Rightarrow 2\cos x - 1 = 0$

$\Rightarrow \cos x = \frac{1}{2}$

Now, from (1), $\cos x + \cos y = 1$

$\Rightarrow \frac{1}{2} + \cos y = 1$

$\Rightarrow \cos y = \frac{1}{2}$

Consider, $\cos x = \frac{1}{2}$

$\Rightarrow \cos x = \cos \frac{\pi}{3}$

$\Rightarrow x = 2n\pi \pm \frac{\pi}{3}, n \in Z$

Similarly $y = 2m\pi \pm \frac{\pi}{3}, m \in Z$ Ans. B

07. Given $\tan A$, $\tan B$ are the roots of

$x^2 - bx + c = 0$

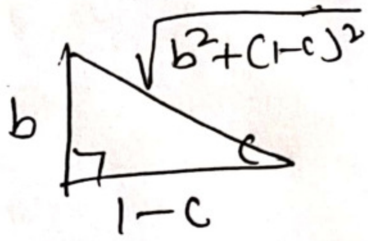
4

We have, $\tan A + \tan B = b$

$\tan A \cdot \tan B = c$

Now, $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$= \dfrac{b}{1-c}$



$\sin(A+B) = \dfrac{b}{\sqrt{b^2+(1-c)^2}}$

$\therefore \sin^2(A+B) = \dfrac{b^2}{b^2+(1-c)^2}$

Ans: D

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08 $\cos 6\theta + \cos 4\theta + \cos 2\theta + 1 = 0.$

option verification.

opt: A: let $\theta = \pi$

$\cos 6\pi + \cos 4\pi + \cos 2\pi + 1$

LHS $= \cos 6\pi + \cos 4\pi + \cos 2\pi + 1$

$= 1 + 1 + 1 + 1 = 4 \neq 0.$

opt. B: let $\theta = \frac{\pi}{2}$

$\cos 3\pi + \cos 2\pi + \cos \pi + 1$

LHS $= \cos 3\pi + \cos 2\pi + \cos \pi + 1$

$= -1 + 1 - 1 + 1 = 0$ ✓

Since $\frac{\pi}{2}$ is not in option C & option: D

option: B is Correct Answer

Ans: B

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09. $y = \dfrac{\cos^2\theta - 1}{\cos^2\theta + \cos\theta} = \dfrac{(\cancel{\cos\theta+1})(\cos\theta - 1)}{\cos\theta(\cancel{\cos\theta+1})}$

Since $\cos\theta + 1 \neq 0$

$$y = \frac{\cos\theta - 1}{\cos\theta}$$

$$y = 1 - \sec\theta$$

$\Rightarrow \sin\theta = 1 - y$

We know $\sec\theta \le -1$ and $\sec\theta \ge 1$

$1 - y \le -1$ and $1 - y \ge 1$

$\Rightarrow y \ge 2$ and $y \le 0$

Ans: B

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10 Given $e^{-\frac{1}{\sqrt{2}}}\left(e^{\sin x} + e^{\cos x}\right) = 2$

Let $x = \frac{\pi}{4}$

$LHS = e^{-\frac{1}{\sqrt{2}}}\left(e^{\frac{1}{\sqrt{2}}} + e^{\frac{1}{\sqrt{2}}}\right)$

$= e^{-\frac{1}{\sqrt{2}}} \cdot 2 \cdot e^{\frac{1}{\sqrt{2}}} = 2 = RHS$

Ans: B

JEE ADVANCED LEVEL

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01. $\sqrt{2}\tan^2 x - \sqrt{10}\tan x + \sqrt{2} = 0$, $\left(0 < x < \frac{\pi}{2}\right)$

$$\Rightarrow \tan x = \frac{\sqrt{10} \pm \sqrt{10 - 8}}{2\sqrt{2}}$$

$$= \frac{\sqrt{10} \pm \sqrt{2}}{2\sqrt{2}}$$

$$= \frac{\sqrt{5} \pm 1}{2}$$

A) No. of real roots of the eqn 2 ✓

B) Sum $= \left(\frac{\sqrt{5}}{2} + \frac{1}{2}\right) + \left(\frac{\sqrt{5}}{2} - \frac{1}{2}\right) = \sqrt{5}$ ✓

Ans. A, B

02 Given $\sin\theta = \cos\phi$

$\Rightarrow \sin\theta = \sin\left(\frac{\pi}{2} - \phi\right)$

$\theta = n\pi + (-1)^n\left(\frac{\pi}{2} - \phi\right)$

Let $n = 0 \Rightarrow \theta = \frac{\pi}{2} - \phi$

$\Rightarrow \theta + \phi = \frac{\pi}{2} \longrightarrow (1)$

Let $n = 1 \Rightarrow \theta = \pi - \left(\frac{\pi}{2} - \phi\right)$

$\Rightarrow \theta = \frac{\pi}{2} + \phi$

$\therefore \Rightarrow \theta - \phi = \frac{\pi}{2} \rightarrow (2)$

From (1), (2) $\theta \pm \phi = \frac{\pi}{2}$

Let $n = 2 \Rightarrow \theta = 2\pi + \frac{\pi}{2} - \phi = \frac{5\pi}{2} - \phi$

$\therefore \theta + \phi = \frac{5\pi}{2}$

Given $\frac{1}{\pi}\left(\theta \pm \phi - \frac{\pi}{2}\right)$

$= \frac{1}{\pi}\left(\frac{\pi}{2} - \frac{\pi}{2}\right) = 0$

Also, $\frac{1}{\pi}\left(\frac{5\pi}{2} - \frac{\pi}{2}\right) = \frac{1}{\pi} \times 2\pi = 2$

$\therefore$ The possible values are 0 or 2

Ans: A/C

03 Given $\sin x + \cos x = \sqrt{y + \frac{1}{y}}$ $a = y,\ b = \frac{1}{y}$

We know, $A.M \geq G.M$ $Am = \frac{a+b}{2}$

$\Rightarrow \frac{y + \frac{1}{y}}{2} \geq \sqrt{y \times \frac{1}{y}}$ $G.m = \sqrt{ab}$

$\Rightarrow y + \frac{1}{y} \geq 2$

$\therefore$ The $\Rightarrow \sqrt{y + \frac{1}{y}} \geq \sqrt{2}$

$\therefore$ The minimum value of $\sqrt{y + \frac{1}{y}}$ is $\sqrt{2}$

When $x = \frac{\pi}{4}$, $\sin\frac{\pi}{4} + \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$

Also, when $y = 1$, $\sqrt{y + \frac{1}{y}} = \sqrt{2}$

Ans. A, C

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04 Statement I: $9^{\cos x} - 2.3^{\cos x} + 1 = 0$

$= (3^{\cos x})^2 - 2.3^{\cos x} + 1 = 0$

let $3^{\cos x} = t$

$t^2 - 2t + 1 = 0$

$\therefore$

$\Rightarrow (t-1)^2 = 0$

$\Rightarrow t = 1$

$\Rightarrow 3^{\cos x} = 3^0$

$\Rightarrow \cos x = 0$

$\Rightarrow x = (2n+1)\frac{\pi}{2}$, (True)

Statement II: $\sqrt{3}\sin x + \cos x = 4$

$$\Rightarrow \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = 2$$

$$\Rightarrow \sin x \cdot \cos\frac{\pi}{6} + \cos x \cdot \sin\frac{\pi}{6} = 2$$

$$\Rightarrow \sin\left(x + \frac{\pi}{6}\right) = 2$$

which is not possible, since max. value of Sine is 1. (False)

Ans: C

05 Statement I: $\sin\theta + \sin 3\theta + \sin 5\theta = 0$.

When $\theta = 0 \Rightarrow \sin 0 + \sin 3\times 0 + \sin 5\times 0 = 0$ ✓

When $\theta = \frac{\pi}{3} \Rightarrow \sin\frac{\pi}{3} + \sin\pi + \sin\frac{5\pi}{3}$

$$= \sin\frac{\pi}{3} + 0 + \sin\left(2\pi - \frac{\pi}{3}\right)$$

$$= \sin\frac{\pi}{3} + 0 - \sin\frac{\pi}{3} = 0$$ ✓ (True)

Statement II:

$$\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta \cdot \tan 2\theta = \sqrt{3}$$

$$\tan\theta + \tan 2\theta = \sqrt{3} - \sqrt{3}\cdot\tan\theta\cdot\tan 2\theta$$

$$\Rightarrow \tan\theta + \tan 2\theta = \sqrt{3}\left(1 - \tan\theta\cdot\tan 2\theta\right)$$

$$\Rightarrow \frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\cdot\tan 2\theta} = \sqrt{3}$$

$$\Rightarrow \tan(\theta + 2\theta) = \sqrt{3}$$

$$\Rightarrow \tan 3\theta = \tan\frac{\pi}{3}$$

$$\Rightarrow 3\theta = n\pi + \frac{\pi}{3}$$

$$\theta = \frac{n\pi}{3} + \frac{\pi}{9}$$

(True)

Ans: A

06. **Assertion:** $x+y+z=180^\circ$

$\Rightarrow x+y=180^\circ-z$

$\Rightarrow \tan(x+y)=\tan(180^\circ-z)$

$\Rightarrow \dfrac{\tan x+\tan y}{1-\tan x\cdot\tan y}=-\tan z$

$\Rightarrow \tan x+\tan y+\tan z=\tan x\cdot\tan y\cdot\tan z$ (True)

Reason; True

Ans- A

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07 **Assertion;** ~~$20^\circ+40^\circ=60^\circ$~~

~~$\Rightarrow \tan(20^\circ+40^\circ)=\tan 60^\circ$~~

~~$\Rightarrow \dfrac{\tan 20^\circ+\tan 40^\circ}{1-\tan 20^\circ\cdot\tan 40}=\tan 60^\circ$~~

$\Rightarrow \tan 20^\circ+\tan 40^\circ=\tan 60^\circ-\tan 20^\circ\cdot\tan 40^\circ\tan 60^\circ$

Please, discard this problems as this question has insufficient data.

08 Given $7\cos^2\theta + 4\cos\theta - 1 = 0$

$$\Rightarrow \cos\theta = \frac{-4 \pm \sqrt{16+28}}{14}$$

$$= \frac{-4 \pm \sqrt{44}}{14}$$

$$= \frac{-4 \pm 2\sqrt{11}}{14}$$

$$= \frac{-2 \pm \sqrt{11}}{7}$$

Now, $\cos 2\theta = 2\cos^2\theta - 1$

$$= 2\left(\frac{-2 \pm \sqrt{11}}{7}\right)^2 - 1$$

$$= \frac{2}{49}\left(4 + 11 \pm 4\sqrt{11}\right) - 1$$

$$= \frac{30 \pm 8\sqrt{11} - 49}{49}$$

$$= \frac{-19 \pm 8\sqrt{11}}{49} \longrightarrow (1)$$

Given $p\cos^2 2\theta + q\cos 2\theta + r = 0$

$$\Rightarrow \cos 2\theta = \frac{-q \pm \sqrt{q^2 - 4pr}}{2p} \longrightarrow (2)$$

Comparing (1) & (2)

We get $2p = 49 \Rightarrow p = \frac{49}{2} \longrightarrow (3)$

$-q = -19 \Rightarrow q = 19 \longrightarrow (4)$

$$\sqrt{q^2 - 4pr} = 8\sqrt{11}$$

$\Rightarrow q^2 - 4pr = 704$

$\Rightarrow 19^2 - 4 \cdot \frac{49}{2} \cdot r = 704$

$\Rightarrow r = -\frac{7}{2}$

Given $p\cos^2 2\theta + q\cos 2\theta + r = 0$

$\Rightarrow \frac{49}{2}\cos^2 2\theta + 19\cos 2\theta - \frac{7}{2} = 0$

$\Rightarrow 49\cos^2 2\theta + 38\cos 2\theta - 7 = 0$

$\therefore p = 49, \quad q = 38, \quad r = -7$

Ans. B

08) $p = 49$

Ans: C

09) $q = 38$

Ans: A

10) $r = -7$

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11. $f(\theta) = 3\cos\theta + 4\sin\theta$

max. value $= \sqrt{3^2+4^2} = \sqrt{9+16} = 5$

min. value $= -\sqrt{3^2+4^2} = -5$

$\therefore -5 \le f(\theta) \le 5$

$\therefore \lambda \in [-5, 5]$

Ans: B

$\boxed{12}$ $\sqrt{3}\cos x + \sin x = 2$

$\Rightarrow \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = 1$

$\Rightarrow \cos x \cdot \cos\frac{\pi}{6} + \sin x \cdot \sin\frac{\pi}{6} = 1$ (12)

$\Rightarrow \cos\left(x - \frac{\pi}{6}\right) = \cos 0^\circ$

$\Rightarrow x - \frac{\pi}{6} = 2n\pi \pm 0 = 2n\pi$

$\Rightarrow x = 2n\pi + \frac{\pi}{6}$

~~Ans. B~~

$\boxed{\text{Ans.. A}}$

13 Given $x\cos^3 y + 3x\cos y \cdot \sin^2 y = 14 \longrightarrow (1)$

$x\sin^3 y + 3x \cdot \cos^2 y \cdot \sin y = 13 \longrightarrow (2)$

Now (1) + (2)

$x(\cos^3 y + 3\cos y \cdot \sin^2 y + 3\cos^2 y \sin y + \sin^3 y) = 27$

$\Rightarrow x(\cos y + \sin y)^3 = 27 \rightarrow (3)$

(1) − (2)

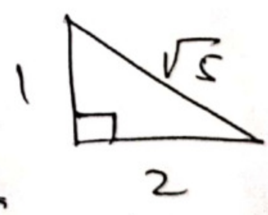
$\Rightarrow x(\cos y - \sin y)^3 = 1 \longrightarrow (4)$

Now, $\frac{(4)}{(3)} \Rightarrow \left(\frac{\cos y + \sin y}{\cos y - \sin y}\right)^3 = 27$

$\Rightarrow \frac{\cos y + \sin y}{\cos y - \sin y} = 3$

$\Rightarrow 4\sin y = 2\cos y$

$\Rightarrow \tan y = \frac{1}{2}$



$\Rightarrow \sin y = \pm\frac{1}{\sqrt{5}}$; $\cos y = \pm\frac{2}{\sqrt{5}}$

Now (1) $\Rightarrow x\cos^3 y + 3x \cdot \cos y \cdot \sin^2 y = 14$

$\Rightarrow x\left(\frac{2}{\sqrt{5}}\right)^3 + 3 \cdot x\left(\frac{2}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{5}}\right)^2 = 14$

$\Rightarrow x = 5\sqrt{5}$

Ans. A

(4) $\Rightarrow x(\cos y - \sin y)^3 = 1$

$\Rightarrow x\left(\pm \frac{2}{\sqrt{5}} \mp \frac{1}{\sqrt{5}}\right)^3 = 1$

$\Rightarrow x^{\frac{1}{3}}\left(\pm \frac{1}{\sqrt{5}}\right) = 1$

$\Rightarrow x^{1/3} = \pm\sqrt{5}$

$\Rightarrow x = \pm 5\sqrt{5}$

Ans: A

~~14~~ Case (i) $\sin y = \frac{1}{\sqrt{5}}$, $\cos y = \frac{2}{\sqrt{5}} \Rightarrow y \in Q_1$,

$\therefore y = 2n\pi + \alpha$ where $0 < \alpha < \frac{\pi}{2}$

Case (ii) $\sin y = -\frac{1}{\sqrt{5}}$ and $\cos y = -\frac{2}{\sqrt{5}}$

Here $y \in Q_3$

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14 We have $\tan y = \frac{1}{2}$

We know, $\tan y$ function has a period of π and has a one solution per period for a given value.

i.e one solution in $[0, \pi]$

Hence for $[0, 6\pi] = [0, \pi) \times 6$, there are six solutions

Ans. D

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15 $\sin^2 y + 2\cos^2 y = \left(\pm\frac{1}{\sqrt{5}}\right)^2 + 2\left(\pm\frac{2}{\sqrt{5}}\right)^2$

$= \frac{1}{5} + \frac{8}{5} = \frac{9}{5}$

Ans. B

16. $a\sin x + \cos 2x = 2a-7$

$\Rightarrow a\sin x + 1 - 2\sin^2 x = 2a-7$

$\Rightarrow 2\sin^2 x - a\sin x + 2a - 8 = 0$

$$\Rightarrow \sin x = \frac{a \pm \sqrt{a^2 - 4\cdot 2(2a-8)}}{4}$$

$$= \frac{a \pm \sqrt{a^2 - 16a + 64}}{4}$$

$$= \frac{a \pm \sqrt{(a-8)^2}}{4}$$

$$= \frac{a \pm (a-8)}{4}$$

$\therefore \sin x = \frac{a+(a-8)}{4} = \frac{a-4}{2}$ | $\sin x = \frac{a-(a-8)}{4} = 2$ which is not valid

$\therefore$ we know $-1 \le \sin x \le 1$

$\Rightarrow -1 \le \frac{a-4}{2} \le 1$

$\Rightarrow -2 \le a-4 \le 2$

$\Rightarrow 2 \le a \le 6$

$\therefore$ The maximum integral value of a is 6

Ans: 6

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17 $\sqrt{3}\sin\theta + \cos\theta = 5$

$\Rightarrow \frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta = \frac{5}{2}$

$\Rightarrow \sin\theta\cos\frac{\pi}{6} + \cos\theta\cdot\sin\frac{\pi}{6} = \frac{5}{2}$

$\Rightarrow \sin\left(\theta + \frac{\pi}{6}\right) = \frac{5}{2} = 2.5$

which is not possible, since max. value of $\sin\theta$ is $\underline{1}$.

Hence No. of solutions $= 0$ Ans. 0

18 a) $4\sin^2 x + \sin^2 2x = 3$

$\Rightarrow 4\sin^2 x + 4\sin^2 x \cdot \cos^2 x = 3$

$\Rightarrow 4\sin^2 x + 4\sin^2 x(1 - \sin^2 x) = 3$

$\Rightarrow 4\sin^4 x - 8\sin^2 x + 3 = 0$

$\Rightarrow (2\sin^2 x - 1)(2\sin^2 x - 3) = 0$

$\Rightarrow \sin x = \pm\frac{1}{\sqrt{2}}$ | $\sin x = \pm\sqrt{\frac{3}{2}}$

$\Rightarrow x = \pm\frac{\pi}{4}$

$\therefore \tan\frac{\pi}{4} = 1$

$\Rightarrow \tan x = 1$

$\Rightarrow x = \tan^{-1}(1)$. (P)

b) Given $4\cos^2 2x + 8\cos^2 x = 7$

let $x = \frac{\pi}{6}$

Now, LHS $= 4\cos^2 2\left(\frac{\pi}{6}\right) + 8\cos^2\left(\frac{\pi}{6}\right)$

$= 4\cos^2\frac{\pi}{3} + 8\cos^2\frac{\pi}{6}$

$= 4\left(\frac{1}{2}\right)^2 + 8\left(\frac{\sqrt{3}}{2}\right)^2$

$= 1 + 6$

$= 7 =$ RHS

$\therefore \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan x = \frac{1}{\sqrt{3}} \Rightarrow x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (r)

c) $3(1+\sin x) = 1+\cos 2x$

$\Rightarrow 3(1+\sin x) = 2\cos^2 x$

$\Rightarrow 3(1+\sin x) = 2(1-\sin^2 x)$

$\Rightarrow 2\sin^2 x + 3\sin x + 1 = 0$

$\Rightarrow (2\sin x+1)(\sin x+1) = 0$

$\Rightarrow \sin x = \frac{-1}{2}$ or $\sin x = -1$

$\Rightarrow x = \sin^{-1}(-1)$ (q)

d) $4x^4 + x^6 + \sin^2 5x = 0$

$x=0$, satisfies the above eqn. (s)

Ans. p, r, q, s

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19 a) $7\cos^2 x + \sin x \cdot \cos x - 3 = 0$

$\div \cos^2 x \Rightarrow 7 + \tan x - 3\sec^2 x = 0$

$\Rightarrow 7 + \tan x - 3(1+\tan^2 x) = 0$

$\Rightarrow 3\tan^2 x - \tan x - 4 = 0$

$\Rightarrow (3\tan x - 4)(\tan x + 1) = 0$

$\Rightarrow \tan x = \frac{4}{3}$ or $\tan x = -1$ (P, q)

b) $2\sin^2\left(\frac{\pi}{2}\cos^2 x\right) = 1 - \cos(\pi \sin 2x)$

$\Rightarrow 2\sin^2\left(\frac{\pi}{2}\cos^2 x\right) = 2\sin^2\left(\frac{\pi}{2}\sin 2x\right)$

$\Rightarrow \frac{\pi}{2}\cos^2 x = \frac{\pi}{2}\sin 2x$

$\Rightarrow \cos^2 x = 2\sin x\cos x$

$\Rightarrow \cos x(\cos x - 2\sin x) = 0$

$\Rightarrow \cos x = 0$ or $\tan x = \frac{1}{2}$ (r)

c) $6\sec^2 x - 11\tan x - 2 = 0$

$\Rightarrow 6(1+\tan^2 x) - 11\tan x - 2 = 0$

$\Rightarrow 6\tan^2 x - 11\tan x + 4 = 0$

$\Rightarrow (2\tan x - 1)(3\tan x - 4) = 0$

$\Rightarrow \tan x = \frac{1}{2}$ or $\frac{4}{3}$ (p, r)

d) $2\cos x - \sin x = \cos^2 x - 4\sin x$

$\Rightarrow 2\cos x - \cos^2 x = -3\sin x$

$\div \cos x \Rightarrow 2 - \cos x = -3\tan x$

$\Rightarrow \frac{1}{2+3\tan x} = \sec x$

$\Rightarrow \left(\frac{1}{2+3\tan x}\right)^2 = \sec^2 x = 1+\tan^2 x$

$\Rightarrow (1+\tan^2 x)(2+3\tan x)^2 = 1$

In column-II, No option satisfies this equation

Ans: (p, q), r, (p, r), –

LEARNER's TASK (CW/HS)

01. $4\cos^2\theta = 3 \Rightarrow \cos^2\theta = \frac{3}{4}$

$\Rightarrow \cos^2\theta = \left(\frac{\sqrt{3}}{2}\right)^2$

$\Rightarrow \cos^2\theta = \cos^2\frac{\pi}{6}$

$\theta = n\pi \pm \frac{\pi}{6}$

$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

Ans: A

02 $3\tan\theta = \cot\theta$

$\Rightarrow \tan^2\theta = \frac{1}{3}$

$\Rightarrow \tan\theta = \pm\frac{1}{\sqrt{3}} \Rightarrow \theta = \pm 30^\circ$

Ans: A

03 $\cos m\theta = \sin n\theta$

$\Rightarrow \cos m\theta = \cos\left(\frac{\pi}{2} - n\theta\right)$

$\Rightarrow m\theta = 2K\pi \pm \left(\frac{\pi}{2} - n\theta\right), \quad K \in \mathbb{Z}$

$\Rightarrow (m \pm n)\theta = 2K\pi \pm \frac{\pi}{2}$

$\Rightarrow \theta = \frac{2K\pi \pm \frac{\pi}{2}}{m \pm n}$

Ans. C

04 $\tan m\theta = \cot n\theta$

$\Rightarrow \tan m\theta = \tan\left(\frac{\pi}{2} - \theta\right)$

$\Rightarrow m\theta = K\pi + \frac{\pi}{2} - \theta \quad , K \in \mathbb{Z}$

$\Rightarrow (m+1)\theta = K\pi + \frac{\pi}{2}$

$\Rightarrow \theta = \frac{(2K+1)\pi}{2(m+n)}, \forall K \in \mathbb{Z}$

Ans: B

05 $\sqrt{3}\sin\theta = \cos\theta$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \tan\frac{\pi}{6}$

$\Rightarrow \theta = n\pi + \frac{\pi}{6}$

Ans: D

06 $\sin 2x = 4\cos x$ (19)

<u>Short cut method</u>

Let $x = \frac{\pi}{2}$

$LHS = \sin 2x = \sin 2\times\frac{\pi}{2} = \sin\pi = 0$

$RHS = 4\cos x = 4\cos\frac{\pi}{2} = 4\times 0 = 0$

$\therefore$ General solution $(2n+1)\frac{\pi}{2}$ Ans. A

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07 $\tan\left(\frac{\pi}{2}\sin\theta\right) = \cot\left(\frac{\pi}{2}\cos\theta\right)$

$\Rightarrow \tan\left(\frac{\pi}{2}\sin\theta\right) = \tan\left(\pm\frac{\pi}{2} - \frac{\pi}{2}\cos\theta\right)$

$\Rightarrow \frac{\pi}{2}\sin\theta = \pm\frac{\pi}{2} - \frac{\pi}{2}\cos\theta$

$\Rightarrow \sin\theta = \pm 1 - \cos\theta$

$\Rightarrow \sin\theta + \cos\theta = \pm 1$

$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) = \pm\frac{1}{\sqrt{2}}$ Ans. B

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08 $\tan(\pi\cos x) = \cot(\pi\sin x)$

$\Rightarrow \tan(\pi\cos x) = \tan\left(\pm\frac{\pi}{2} - \pi\sin x\right)$

$\Rightarrow \pi\cos x = \pm\frac{\pi}{2} - \pi\sin x$

$\Rightarrow \cos x = \pm\frac{1}{2} - \sin x$

$\Rightarrow \cos x + \sin x = \pm\frac{1}{2}$

$\Rightarrow \frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x = \pm\frac{1}{2\sqrt{2}}$

$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \pm \frac{1}{2\sqrt{2}}$ Ans. C

09 $\tan(\cot x) = \cot(\tan x)$

$\Rightarrow \tan(\cot x) = \tan\left(\frac{\pi}{2} - \tan x\right)$

$\Rightarrow \cot x = n\pi + \frac{\pi}{2} - \tan x \quad , n \in \mathbb{Z}$

$\Rightarrow \tan x + \cot x = n\pi + \frac{\pi}{2}$

$\Rightarrow \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = n\pi + \frac{\pi}{2}$

$\Rightarrow \frac{1}{\cos x \sin x} = \frac{2n\pi + \pi}{2} = \frac{(2n+1)\pi}{2}$

$\Rightarrow \frac{1}{2\sin x \cos x} = \frac{(2n+1)\pi}{4}$ Ans. B

$\Rightarrow \sin 2x = \frac{4}{(2n+1)\pi}$

10 $\sin\left(\frac{\pi}{4}\cot\theta\right) = \cos\left(\frac{\pi}{4}\tan\theta\right)$

Let $\theta = \frac{\pi}{4}$

$LHS = \sin\left(\frac{\pi}{4} \cdot \cot\frac{\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$RHS = \cos\left(\frac{\pi}{4} \cdot \tan\theta\right) = \cos\left(\frac{\pi}{4} \cdot \tan\frac{\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

Hence, $\theta = n\pi + \frac{\pi}{4}$ Ans. B

JEE MAINS LEVEL

$\boxed{01}$ $\frac{\tan 2x - \tan x}{1 + \tan 2x \cdot \tan x} = 1 \Rightarrow \tan(2x - x) = \tan\frac{\pi}{4}$

$\Rightarrow x = n\pi + \frac{\pi}{4}$ $\boxed{\text{Ans: C}}$

02 $\sin x \cdot \sin(60^\circ + x) \cdot \sin(60^\circ - x) = \frac{1}{8}$

We know, $\sin x \cdot \sin(60^\circ + x) \cdot \sin(60^\circ - x) = \frac{1}{4} \sin 3x$

$\therefore \frac{1}{4} \sin 3x = \frac{1}{8} \Rightarrow \sin 3x = \frac{1}{2}$

$\Rightarrow \sin 3x = \sin\frac{\pi}{6}$

$\Rightarrow 3x = n\pi + (-1)^n \frac{\pi}{6}$

$\Rightarrow x = \frac{n\pi}{3} + (-1)^n \frac{\pi}{18}, n \in Z$

Ans: B

03 Given $\cot^2\theta - \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)\cot\theta + 1 = 0.$

let $\theta = \frac{\pi}{6}$

$LHS = \cot^2\frac{\pi}{6} - \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)\cot\frac{\pi}{6} + 1$

$= (\sqrt{3})^2 - \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)(\sqrt{3}) + 1$

$= 3 - 3 - 1 + 1$

$= 0 = RHS$

Hence, the general solution is $\theta = n\pi + \frac{\pi}{6}$

$n \in z$ $\boxed{\text{A: B}}$

04 We know, $\sin\theta \cdot \sin(60^\circ + \theta) \cdot \sin(60^\circ - \theta) = \frac{1}{4} \sin 3\theta$

$\therefore \frac{1}{4} \sin 3\theta = \frac{1}{4}$

$\sin 3\theta = 1$

$\Rightarrow \sin 3\theta = \sin\frac{\pi}{2}$

$\Rightarrow 3\theta = n\pi + (-1)^n \cdot \frac{\pi}{2}$

$\Rightarrow \theta = \frac{n\pi}{3} + (-1)^n \cdot \frac{\pi}{6}$

Ans: B

05 $\sin x + \cos x = \sqrt{2}\cos\alpha$

$\Rightarrow \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \cos\alpha$

$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \cos\alpha$

$\Rightarrow x - \frac{\pi}{4} = 2n\pi \pm \alpha$

$\Rightarrow x = 2n\pi + \frac{\pi}{4} \pm \alpha$,

Ans. C

06 $\sin\left(x + \frac{\pi}{4}\right) = \sin 2x$

$\Rightarrow x + \frac{\pi}{4} = n\pi + (-1)^n \cdot 2x$, $n \in \mathbb{Z}$

$\Rightarrow x - (-1)^n 2x = n\pi - \frac{\pi}{4}$

$\Rightarrow x\left[1 - (-1)^n \cdot 2\right] = n\pi - \frac{\pi}{4}$

$\Rightarrow x = \dfrac{n\pi - \frac{\pi}{4}}{1 - (-1)^n \cdot 2}$, $n \in \mathbb{Z}$

Ans: D

07 $81^{\sin^2 x} + 81^{\cos^2 x} = 30$.

option verification technique

let $x = \frac{\pi}{6}$

LHS $= 81^{\frac{1}{4}} + 81^{\frac{3}{4}}$

$= 3 + 27 = 30$

let $x = \frac{\pi}{3}$

LHS $= 81^{\frac{3}{4}} + 81^{\frac{1}{4}}$

$= 27 + 3$

$= 30$

Ans: A

08 $\sec^2(a+2)x + (a^2-1) = 0$

$\Rightarrow \sec^2(a+2)x - 1 + a^2 = 0$

$\Rightarrow \tan^2(a+2)x + a^2 = 0$

$\Rightarrow \tan^2(a+2)x = 0$ and $a^2 = 0$ (one possibility)

$\Rightarrow a = 0$

$\Rightarrow \tan^2 2x = 0$

$\Rightarrow 2x = 0, \pi, -\pi$ since $-\pi < x < \pi$

$\Rightarrow x = 0, \frac{\pi}{2}, \frac{-\pi}{2}$

So, the required ordered pairs are

$(0,0), (0, \frac{\pi}{2}), (0, \frac{-\pi}{2})$.

$\therefore$ 3 possible ordered pairs Ans. C

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09 $1 + \sin^2\theta = 3\sin\theta\cos\theta$

$\div \cos^2\theta \Rightarrow \sec^2\theta + \tan^2\theta = 3\tan\theta$

$\Rightarrow (1+\tan^2\theta) + \tan^2\theta = 3\tan\theta$

$\Rightarrow 2\tan^2\theta - 3\tan\theta + 1 = 0$

$\Rightarrow (2\tan\theta - 1)(\tan\theta - 1) = 0$

$\Rightarrow \tan\theta = \frac{1}{2}$ or $\tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$

$\Rightarrow \theta = \tan^{-1}(\frac{1}{2})$ Ans. B

---

10 Given $2\cos x + \cos 2\pi x = 3$

One of the possible cases is

$2\cos x = 2$ and $\cos 2\pi x = 1$

$\Rightarrow \cos x = 1$

$\Rightarrow \cos x = \cos 0$

$\Rightarrow x = 2n\pi, n \in \mathbb{Z}$

Let $\cos 2\lambda x = 1$

If $\lambda \in \mathbb{Q}$, there are infinitely many solutions and, if $\lambda \notin \mathbb{Q}$, then $x = 0$ is the only one solution satisfying both the equations.

Hence, λ is an Irrational number

Ans. C

JEE ADVANCED LEVEL

01 Given $\sin^4 x + \cos^4 x = 1$

option verification technique.

put $n = 1$ in all the options

A) π B) $\pi \pm \sin^{-1}\left(\sqrt{\frac{2}{5}}\right)$ C) $\frac{2\pi}{3}$ D) $\pi \pm \frac{\pi}{2}$

A) Let $x = \pi$ $\sin^4 \pi + \cos^4 \pi = 1$ ✓

D) $\pi \pm \frac{\pi}{2}$, let $x = \frac{\pi}{2}$ $\sin^4 \frac{\pi}{2} + \cos^4 \frac{\pi}{2} = 1$ ✓

Ans.. A, D

02 $a \cos 2\theta + b \sin 2\theta = c$

$\Rightarrow a\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + b\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = c$

$\Rightarrow a - a\tan^2\theta + 2b\tan\theta = c + c\tan^2\theta$

$\Rightarrow (a+c)\tan^2\theta - 2b\tan\theta + (c-a) = 0$

$\tan\theta_1$ & $\tan\theta_2$ are the roots.

$$\tan\theta_1 \cdot \tan\theta_2 = \frac{c-a}{c+a}$$ ~~Ans. (B)~~

~~c x x x x x x~~

(25)

Also
$$\tan(\theta_1+\theta_2) = \frac{\tan\theta_1 + \tan\theta_2}{1-\tan\theta_1 \cdot \tan\theta_2}$$

$$= \frac{\frac{2b}{c+a}}{1-\left(\frac{c-a}{c+a}\right)} = \frac{b}{a}$$

$\therefore$ Ans: A, B

03. Statement I: $\sqrt{3}\tan\theta - 1 = 0$

$$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan\theta = \tan\frac{\pi}{6}$$

$$\Rightarrow \theta = n\pi + \frac{\pi}{6} \quad \text{(True)}$$

Ans. A

Statement II: Conceptual (True)

04 Statement I:

$\sin\theta = \frac{1}{2}$ | $\tan\theta = \frac{1}{\sqrt{3}}$

$\sin\frac{\pi}{6} = \frac{1}{2}$ | $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$ (True)

Statement II:
$$\cos\frac{3\pi}{4} = \cos\left(\pi + \frac{\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{1}{\sqrt{2}}$$

$$\tan\frac{3\pi}{4} = \tan\left(\pi + \frac{\pi}{4}\right) = -\tan\frac{\pi}{4} = -1$$

(True)

Ans. A

05 **Assertion:** $2\cos\theta + 2\sin\theta$.

$$\text{max. value} = c + \sqrt{a^2+b^2}$$

$$= 0 + \sqrt{2^2+2^2} = \sqrt{8} \text{ (True)}$$

Reason: Conceptual (True)

Ans. - A

06 **Assertion:**

$$\tan 15^\circ + \tan 45^\circ + \tan 75^\circ$$

$$= \tan 45^\circ + \tan(45^\circ - 30^\circ) + \tan(45^\circ + 30^\circ)$$

here $x = 45^\circ$

Formula = $3\tan 3x = 3\tan 3\times 45^\circ$

$$= 3\tan 135^\circ$$

$$= 3\tan(180^\circ - 45^\circ)$$

$$= -3 \text{ (True)}$$

Reason: Conceptual (True)

Ans. A

07 The relation is valid only for the angles of a triangle

Ans. B

08 $\tan 60^\circ + \tan 60^\circ + \tan 60^\circ = 3\tan 60^\circ = 3\sqrt{3}$ Ans. D

09 $\tan 40^\circ \times \tan 20^\circ \times \tan 60^\circ$

$\tan 40^\circ \times \tan(40^\circ - 20^\circ) \times \tan(40^\circ + 20^\circ)$

$$= \tan 3\times 40^\circ$$

$$= \tan 120^\circ$$

$$= \tan(180^\circ - 60^\circ) = -\tan 60^\circ = -\sqrt{3}$$

Ans B

10 Triple Angle Ans. B

11. $2\sin^2\theta + 3\sin\theta + 1 = 0$

$\Rightarrow (2\sin\theta + 1)(\sin\theta + 1) = 0$

$\Rightarrow \sin\theta = -\frac{1}{2}$ and $\sin\theta = -1$

$\Rightarrow \sin\theta = \sin\left(-\frac{\pi}{6}\right)$

$\Rightarrow \theta = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$

$\Rightarrow \theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{5\pi}{6}$

$\sin\theta = \sin\left(-\frac{\pi}{2}\right)$

$\Rightarrow \theta = n\pi + (-1)^n\left(-\frac{\pi}{2}\right)$

$\Rightarrow \theta = \frac{3\pi}{2}$

$\therefore$ Total 4 solutions in $(0, 2\pi)$ Ans. 4

12 $3\sin x + 4\cos x = 5$

$3\left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right) + 4\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right) = 5$

For Convinient let $\tan\frac{x}{2} = a$

$\therefore \frac{6a}{1+a^2} + 4\left(\frac{1-a^2}{1+a^2}\right) = 5$

$\Rightarrow 6a + 4 - 4a^2 = 5 + 5a^2$

$\Rightarrow 9a^2 - 6a + 1 = 0$

$\Rightarrow (3a-1)^2 = 0$

$\Rightarrow a = \frac{1}{3}$

$\Rightarrow \tan\frac{x}{2} = \frac{1}{3} = \frac{a}{b} \Rightarrow a+b = 4$

Ans. 4

13 a) $\cos x = \frac{-1}{2}$

Now, $\cos\frac{8\pi}{3} = \cos\left(2\pi + \frac{2\pi}{3}\right) = \cos\frac{2\pi}{3} = \frac{-1}{2}$ (r)

b) $\sin x = \frac{\sqrt{3}}{2}$

$\sin\frac{8\pi}{3} = \sin\left(2\pi + \frac{2\pi}{3}\right) = \sin\frac{2\pi}{3} = \sin\left(\pi - \frac{\pi}{3}\right)$
$= \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ ( )

$\sin\frac{7\pi}{3} = \sin\left(2\pi + \frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ (p, r)

c) $\tan x = \frac{1}{\sqrt{3}}$

$\tan\frac{19\pi}{6} = \tan\left(2\pi + \frac{7\pi}{6}\right) = \tan\frac{7\pi}{6} = \tan\left(\pi + \frac{\pi}{6}\right)$
$= \tan\frac{\pi}{6}$
$= \frac{1}{\sqrt{3}}$ (q)

d) $\cot x = -1$

$\Rightarrow \tan x = -1$

$\tan\frac{11\pi}{4} = \tan\left(2\pi + \frac{3\pi}{4}\right) = \tan\frac{3\pi}{4} = \tan\left(\pi - \frac{\pi}{4}\right)$
$= -\tan\frac{\pi}{4}$
$= -1$ (s)

Ans: r, (p, r), q, s

14 a)

14 a) Given $\sqrt{p}\cos x - 2\sin x = \sqrt{2}+\sqrt{2-p}$

here $\sqrt{p}$ is defined only when $p \geq 0 \rightarrow (1)$

$\sqrt{2-p}$ is defined only when $2-p \geq 0$

$\Rightarrow p \leq 2 \rightarrow (2)$

from (1) & (2) $\quad 0 \leq p \leq 2$

from the options $p \in [\sqrt{5}-1, 2]$ is valid (s)

b) $\cos 2x + a\sin x = 2a-7$

$\Rightarrow 1-2\sin^2 x + a\sin x = 2a-7$

$\Rightarrow 2\sin^2 x - a\sin x + (2a-8) = 0$

$$\sin x = \frac{a \pm \sqrt{a^2-4.2.(2a-8)}}{4}$$

$$= \frac{a \pm \sqrt{a^2-16a+64}}{4}$$

$$= \frac{a \pm \sqrt{(a-8)^2}}{4}$$

$$= \frac{a \pm (a-8)}{4} = \frac{a-4}{2} \text{ or } 2$$

$\because$ $\sin x = 2$ not valid

$\therefore$ $\sin x = \frac{a-4}{2}$

$\Rightarrow -1 \leq \sin x \leq 1$

$-1 \leq \frac{a-4}{2} \leq 1$

$-2 \leq a-4 \leq 2$

$2 \leq a \leq 6$

$a \in [2,6]$ (r)

d) $\sin^4 x + \cos^4 x + \sin 2x + a = 0$

$= (\sin^2 x)^2 + (\cos^2 x)^2 + \sin 2x + a = 0$

$\Rightarrow 1 - 2\sin^2 x \cos^2 x + \sin 2x + a = 0$

$\Rightarrow 1 - \frac{1}{2}\sin^2 2x + \sin 2x + a = 0$

$\Rightarrow 2 - \sin^2 2x + 2\sin 2x + 2a = 0$

$\Rightarrow \sin^2 2x - 2\sin 2x - (2a+2) = 0$

$$\boxed{\cancel{\sin 2x = \frac{2 \pm \sqrt{4 + 4(2a+2)}}{2} = \frac{2 \pm \sqrt{8a+12}}{2}}}$$

Let $\sin 2x = y$

$y^2 - 2y - (2a+2) = 0$

$\Rightarrow (y-1)^2 - 1 - 2a - 2 = 0$

$\Rightarrow (y-1)^2 = 2a + 3$

$\Rightarrow 2a = (y-1)^2 - 3$

Now $-1 \le y \le 1$ since $\sin 2x = y$

$\Rightarrow -2 \le y - 1 \le 0$

$\Rightarrow 0 \le (y-1)^2 \le 4$

$\Rightarrow -3 \le (y-1)^2 - 3 \le 1$

$\Rightarrow -3 \le 2a \le 1$

$\Rightarrow -\frac{3}{2} \le a \le \frac{1}{2} \Rightarrow a \in \left[-\frac{3}{2}, \frac{1}{2}\right]$ (P)

ADDITIONAL PRACTICE QUESTIONS ~~FOR~~

01 Given $K = \cos 20^\circ$

$\cos x = 2K^2 - 1$

$= 2\cos^2 20^\circ - 1$

$= \cos 2 \times 20^\circ$

$= \cos 40^\circ$

$\therefore \cos x = \cos 40^\circ$

$\Rightarrow x = 40^\circ$

$\cos x = \cos(360^\circ - 40^\circ)$

$= \cos 320^\circ$

$x = 320^\circ$

Ans: C

02 Given $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$

This is a Q.E in x

$\therefore$ Discriminant ≥ 0

$\Rightarrow \cos^2 p - 4(\cos p - 1)\sin p \geq 0 \rightarrow (1)$

clearly $\cos^2 p \geq 0 \quad \forall\ p \in R$

We have $-1 \leq \cos p \leq 1$

$\Rightarrow \cos p - 1 \leq 0.$

i.e. $\cos^2 p$ is positive, $\cos p - 1$ is negative

$(1) \Rightarrow \underbrace{\cos^2 p}_{\text{positive}} - 4\underbrace{(\cos p - 1)}_{\text{Negative}} \sin p \geq 0$

$\therefore \Rightarrow \sin p \geq 0$

$p \in [0, \pi]$

$\Rightarrow p \in (0, \pi)$

Ans. D

03 $\tan\left(\theta+\frac{\pi}{4}\right) = 3\cdot\tan 3\theta$

$\Rightarrow \dfrac{\tan\theta+1}{1-\tan\theta} = 3\left(\dfrac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right)$

$\Rightarrow 3\tan^4\theta - 6\tan^2\theta + 8\tan\theta - 1 = 0$

Sum of the roots $= \tan\alpha+\tan\beta+\tan\gamma+\tan\delta$

Ans.. B

$= \frac{0}{3} = 0$

---

04 Given $\sin^{50}x - \cos^{50}x = 1$

$\Rightarrow \sin^{50}x = 1+\cos^{50}x$

$\therefore$ The maximum value of $\sin^{50}x$ is $\underline{1}$.

also, $\cos^{50}x$ is always positive and must be equal to 0

$\cos^{50}x = 0$

$\therefore \cos x = 0$

$\Rightarrow x = (2n+1)\frac{\pi}{2}, \; n\in Z$

Ans: C

$\Rightarrow x = n\pi + \frac{\pi}{2}$

---

05 $\cos^4x - (a+2)\cos^2x - (a+3) = 0$

Let $\cos^2x = t$

$\therefore t^2 - (a+2)t - (a+3) = 0$

$\Rightarrow t^2 - [(a+3)-1]t - (a+3) = 0$

$\Rightarrow t^2 + t - (a+3)t - (a+3) = 0$

$\Rightarrow t(t+1) - (a+3)[(t+1)] = 0$

$\Rightarrow (t+1)(t-(a+3)) = 0$

$\Rightarrow t+1 = 0$ or $t-(a+3) = 0$

$\Rightarrow t = -1$ or $t = a+3$

$\Rightarrow \cos^2 x = -1$

not possible

$\cos^2 x = a+3$

We know

$0 \le \cos^2 x \le 1$

$0 \le a+3 \le 1$

$-3 \le a \le -2$

$\therefore a \in [-3, -2]$ Ans. A

---

06 Given $\sin x - \sqrt{3} \cos x = \dfrac{4m-6}{4-m}$

$\Rightarrow \dfrac{1}{2} \sin x - \dfrac{\sqrt{3}}{2} \cos x = \dfrac{2m-3}{4-m}$

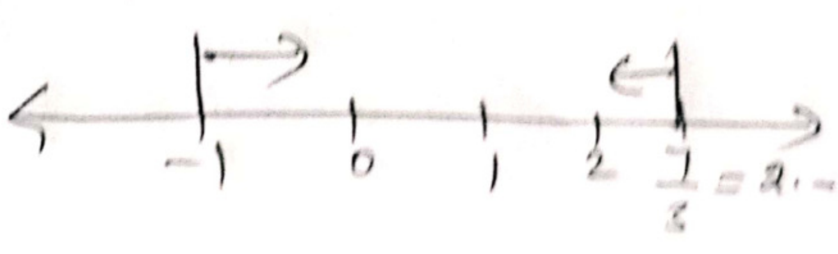
$\Rightarrow \sin\left(x - \dfrac{\pi}{3}\right) = \dfrac{2m-3}{4-m}$

$\therefore -1 \le \sin\left(x - \dfrac{\pi}{3}\right) \le 1$

$\Rightarrow -1 \le \dfrac{2m-3}{4-m} \le 1$

$\Rightarrow -1 \le \dfrac{2m-3}{4-m}$ and $\dfrac{2m-3}{4-m} \le 1$

$\Rightarrow m \ge -1$ and $m \le \dfrac{7}{3}$



$\therefore m \in \{-1, 0, 1, 2\}$

No. of integral solutions of m are 4

Ans. 4

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07 Given $3\sec\theta - 5 = 4\tan\theta$

~~let us draw the graph of $y = 3\sec\theta - 5$~~

~~and $y = 4\tan\theta$~~

$$\Rightarrow \frac{3}{\cos\theta} - 5 = \frac{4\sin\theta}{\cos\theta}$$

$\Rightarrow 3 - 5\cos\theta = 4\sin\theta$ since $\cos\theta \neq 0$

$\Rightarrow 3 - 4\sin\theta = 5\cos\theta$

$\Rightarrow 9 - 24\sin\theta + 16\sin^2\theta = 25\cos^2\theta$

$\Rightarrow 9 - 24\sin\theta + 16\sin^2\theta = 25(1 - \sin^2\theta)$

$\Rightarrow 41\sin^2\theta - 24\sin\theta - 16 = 0$

$$\Rightarrow \sin\theta = \frac{24 \pm \sqrt{3200}}{82} = \frac{24 \pm 56.57}{82}$$

$$\Rightarrow \sin\theta = \frac{80.57}{82} \approx 0.983$$

$$= \frac{-32.57}{82} \approx -0.397$$

For $\sin\theta = 0.983 \rightarrow$ there are two solutions one in Q_1 and another in Q_2

For $\sin\theta = -0.397 \rightarrow$ There are two solutions one in Q_3 and another in Q_4

$\therefore$ In $[0, 2\pi]$ there are possible 4 solutions

$\therefore$ In $[0, 4\pi]$ there are 8 possible solutions

Ans. 8

$\Rightarrow$ THE END $\Leftarrow$