FOUNDATION (V2)

Class: 8th : MATHEMATICS

1. FACTORISATION OF POLYNOMIALS

TEACHING TASK
JEE MAINS LEVEL

01. Given $f(x) = (k^2+4)x^2 + 13x + 4k$.

∵ one zero is reciprocal of the other

$\Rightarrow k^2 + 4 = 4k$

$\Rightarrow (k-2)^2 = 0$

$\Rightarrow k = 2$

Ans: C

02. $14pqr + 35pqr$

$= 7pq(2+5r)$

Ans: D

03 $f(x) = x^3 + ax^2 + bx - 8$

$f(1) = 1 + a + b - 8 = 0$

$\Rightarrow a + b = 7 \rightarrow (1)$

$f(-2) = -8 + 4a - 2b - 8 = 0$

$\Rightarrow 4a - 2b = 16$

$\Rightarrow 2a - b = 8 \rightarrow (2)$

Solving (1), (2) $\Rightarrow a = 5, b = 2$

Ans: D

04. $2y(y+z)-(x+y)(x+z)$

$= (2y^2+2yz)-(x^2+zx+xy+zy)$

$= 2y^2+2yz-x^2-zx-xy-zy$

$= 2y^2-x^2+yz-zx-xy$

option: C: $(y-x)(x+2y+z)$

$= (xy+2y^2+zy)-(x^2+2xy+zx)$

$= xy+2y^2+zy-x^2-2xy-zx$

$= 2y^2-x^2+yz-zx-xy$ Ans: C

05 $\alpha+\beta = 3, \quad \alpha.\beta = -2$

$\therefore x^2-(\alpha+\beta)x+\alpha\beta$

$= x^2-3x-2$ Ans: A

06 x^6+y^6

$= (x^2)^3+(y^2)^3$

$= (x^2+y^2)(x^4-x^2y^2+y^4)$ Ans: B

07 Horner's method.

$P(y) = y^3-7y+6$

$y=1$	1	0	−7	6
	0	1	1	−6
	1	1	−6	0

$\therefore y^2+y-6 = (y+3)(y-2)$ Ans: B

08 Given $\alpha+\beta+\gamma = 2 \rightarrow S_1$

$\alpha\beta+\beta\gamma+\gamma\alpha = -7 \rightarrow S_2$

$\alpha\beta\gamma = -14 \rightarrow S_3$

$\therefore x^3 - S_1x^2 + S_2x - S_3$

$= x^3 - 2x^2 - 7x + 14$

Ans: C

09. $p(x) = ax^3 + bx^2 + cx + d$.

product of the zeroes $= 0$

i.e $-\frac{d}{a} = 0$

$\Rightarrow d = 0$.

$\therefore p(x) = ax^3 + bx^2 + cx$

Since, there is no constant term,

∴ The curve passes through the origin

Also, one of the zero is zero

Ans: B,C

10. $V(x) = x^3 - 12x^2 + 47x - 60$.

$\alpha+\beta+\gamma = 12$

$\alpha+\beta = 7 \Rightarrow \gamma = 5$

∴ height = 5mt

Ans: B

JEE ADVANCED

01. conceptual :

Ans: A, B, C, D

02 Clearly

$(2x+\sqrt{6})(2x-\sqrt{6})(x+2)(x+1)$

$= 4x^4 + 12x^3 + 8x^2 - 6x - 12$

02 Given $f(x) = 4x^4 + 12x^3 + 8x^2 - 6x - 12$

$f(-1) = 4 - 12 + 8 + 6 - 12 \neq 0$

$\therefore (x+1)$ is not a factor

$f(-2) = 4(16) + 12(-8) + 8(4) - 6(-2) - 12$

$= 64 - 96 + 32 + 12 - 12$

$= 0$

$\therefore (x+2)$ is a factor.

Also, $f\left(-\frac{\sqrt{6}}{2}\right) \neq 0$, $f\left(\frac{\sqrt{6}}{2}\right) \neq 0$

$\therefore x+1$ is a factor of $f(x)$

Ans: D

03 Statement I: Conceptual (True)

Statement II: Conceptual (True)

Ans: A

04 Statement I: $f(x) = x^3 - x^2 - 8x + 12$

$f(2) = 8 - 4 - 16 + 12$

$= 0$ (True)

Statement II: Conceptual (True)

Ans: A

05 Assertion: $8a^3 - 27b^6$

$= (2a)^3 - (3b^2)^3$

$= (2a - 3b^2)(4a^2 + 6ab^2 + 9b^4)$ (True)

Reason: Conceptual (True)

Ans: A

06. **Assertion:**

Volume = lbh

$= (x-2)(x+1)(x-4)$

$= (x^2-x-2)(x-4)$

$= x^3-4x^2-x^2+4x-2x+8$

$= x^3-5x^2+2x+8$ (True)

Reason: Conceptual (True)

Ans: A

07 $(x+y+2z)(x+y)$

$= [(x+y+z)+z][(x+y+z)-z]$

$= (x+y+z)^2-z^2$

Ans: A

08 $(x+2a)(x+4a)(x+6a)(x+8a)+7a^4$

$= [(x+2a)(x+8a)][(x+4a)(x+6a)]+7a^4$

$= (x^2+10ax+16a^2)(x^2+10ax+24a^2)+7a^4$

Let $x^2+10ax+16a^2 = y$

$= (y)(y+8a^2)+7a^4$

$= y^2+8ya^2+7a^4$

$= y^2+2\cdot y\cdot 4a^2+16a^4-9a^4$

$= (y+4a^2)^2-(3a^2)^2$

$= (x^2+10ax+20a^2)^2-(3a^2)^2$

Ans: A

09. $A(x) = 2x^2 + 7x + 6$

$= (2x+3)(x+2)$

$\therefore l = 2x+3, \quad b = x+2$

Ans: A

10. $A(x) = 2x^2 + 7x + 6 = 10$

$\Rightarrow 2x^2 + 7x - 4 = 0$

$\Rightarrow 2x^2 + 8x - x - 4 = 0$

$\Rightarrow 2x(x+4) - 1(x+4) = 0$

$\Rightarrow (x+4)(2x-1) = 0$

$\Rightarrow x = -4$ or $\frac{1}{2} = 0.5$

Ans: B

11. $p(x) = x^2 - 5x + K$

$\alpha + \beta = 5$ $\quad \alpha = 3$

$\alpha - \beta = 1$ $\quad \beta = 2$

product of the zeroes $= K = 6$

Ans: 6

12. $f(x) = x^6 - ax^5 + x^4 - ax^3 + 3x - a - 2$

$f(a) = a^6 - a \cdot a^5 + a^4 - a \cdot a^3 + 3a - a - 2 = 0$

$\Rightarrow a^6 - a^6 + a^4 - a^4 + 2a - 2 = 0$

$\Rightarrow a = 1$

Ans: 1

13

(7)

13 a) Standard form of homogeneous expression of 3rd degree in x, y

$= ax^3 + bx^2y + cxy^2 + dy^3$ (s)

b) $ax^2 + by^2 + cz^2 + dxy + eyz + fzx$ (q)

c) $a(x^2+y^2+z^2) + b(xy+yz+zx) + c(x+y+z)$ (p)

d) $a(x^2+y^2+z^2) + b(xy+yz+zx)$ (r)

Ans: s, q, p, r

---

14 a) $6x^2 + x - 2 = (3x+2)(2x-1)$ (q)

b)

$x=3$	1	2	−11	−12
	0	3	15	12
	1	5	4	0

$\therefore x^2 + 5x + 4$ (r)

c)

$x=1$	1	−6	11	−6
	0	1	−5	6
	1	−5	6	0

$\therefore x^2 - 5x + 6$ (s)

d)

$x=-2$	2	9	7	−6
	0	−4	−10	6
	2	5	−3	0

$\therefore 2x^2 + 5x - 3$

Ans: q, r, s, —

LEARNERS TASK

CUQ 15

01. $f(x) = (x-y)^3 + (y-z)^3 + (z-x)^3$

$f(y) = (y-y)^3 + (y-z)^3 + (z-y)^3$

$= 0 + (y-z)^3 - (y-z)^3 = 0$

$\therefore (x-y)$ is a factor.

Since, the given expression is cyclic.

$\therefore (y-z), (z-x)$ are also factors

$\therefore (x-y)^3 + (y-z)^3 + (z-x)^3 = k(x-y)(y-z)(z-x)$

put $x=0, y=1, z=2$

$\Rightarrow (-1)^3 + (-1)^3 + (2)^3 = k(0-1)(1-2)(2-0)$

$\Rightarrow -1-1+8 = 1k(-1)(-1)(2)$

$\Rightarrow 6 = 2k$

$\Rightarrow k = 3$

$\Rightarrow (x-y)^3 + (y-z)^3 + (z-x)^3 = 3(x-y)(y-z)(z-x)$

Ans: A

02 option: B

$\frac{1}{2}[(x-y)^2 + (y-z)^2 + (z-x)^2]$

$= \frac{1}{2}[2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx]$

$= x^2 + y^2 + z^2 - xy - yz - zx$ Ans: B

03. $(x-y)^3 + 64$

$= (x-y)^3 + 4^3$

$= (x-y+4)\left((x-y)^2 - (x-y)\cdot 4 + 4^2\right)$

$= (x-y+4)\left((x-y)^2 - 4(x-y) + 16\right)$

$\therefore$ One of the factors $= x-y+4$ Ans: A

04 $\left(x-\frac{1}{x}\right)^2 = \left(x+\frac{1}{x}\right)^2 - 4x\cdot\frac{1}{x}$

$= (2)^2 - 4\cdot$

$= 0$

$\therefore x - \frac{1}{x} = 0$ Ans: A

05 $ax+by$ Ans: C

06. $2x^2+3xy+2y^2$ Ans: A

07 $\sum a^2 = a^2+b^2+c^2$ Ans: D

08 $f(x) = ax^2 + bxy \rightarrow$ homogeneous Ans: B

09. $\sum_{i=1}^{n} i = 1+2+3+\cdots+n = \frac{n(n+1)}{2}$ Ans: B

10. $f(x) = ax^4+bx^3+cx^2+dx+e$

$f(1) = a+b+c+d+e = 0 \rightarrow (1)$

$f(-1) = a-b+c-d+e = 0 \Rightarrow a+c+e = b+d \rightarrow (2)$

From (1), (2) $\Rightarrow a+c+e=0,\ b+d=0$ Ans: D

JEE MAINS LEVEL

01. $a^3+b^3 = (a+b)^3 - 3ab(a+b)$

$= (7)^3 - 3\cdot 5(7)$

$= 238$

Ans : D

02. $a^4 - 2a^3 + 2a^2 - 2a + 1$

$= (a^4 + 2a^2 + 1) - 2a^3 - 2a$

$= (a^2+1)^2 - 2a(a^2+1)$

$= (a^2+1)[a^2+1-2a]$

$= (a^2+1)(a-1)^2$

Ans: E

03. $(p^2 - 2pq + q^2) - r^2$

$= (p^2 - 2pq + q^2 + r)($

$= (p-q)^2 - r^2$

$= (p-q+r)(p-q-r)$

Ans: C

04. $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma}$

$= \frac{\left(\frac{c}{a}\right)}{\left(\frac{-d}{a}\right)} = \frac{-c}{d}$

Ans : C

05 $f(x) = x^3 + ax^2 + bx + c$

We know, $\alpha\beta\gamma = -c$

$\Rightarrow (-1)\beta\gamma = -c$

$\Rightarrow \beta\gamma = c$

We have, $f(-1) = -1 + a - b + c = 0$

$\Rightarrow c = 1 - a + b$

Ans.

06. $\alpha + \beta = 2, \quad \alpha\beta = -\frac{3}{5}$

$f(y) = y^3 - (\alpha+\beta)y + \alpha\beta$

$= y^3 - 2y - \frac{3}{5}$

$= 5y^3 - 10y - 3$

Ans: A

07 $4(a+b)^2 - 9(a-b)^2$

$= [2(a+b)]^2 - [3(a-b)]^2$

$= (2a+2b)^2 - (3a-3b)^2$

$= (2a+2b+3a-3b)(2a+2b-3a+3b)$

$= (5a-b)(5b-a)$

Ans: D

08 $f(x) = 2x^4 - 7x^3 + ax + b$

$f(3) = 2(3)^4 - 7(3)^3 + 3a + b = 0$

$\Rightarrow b + 3a = 27$

Ans: A

9. $f(x) = x^2 - 5x + 6$

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4(\alpha\beta)$

$= (5)^2 - 4 \cdot 6$

$= 1$

Ans: A

10 $f(x) = x^2 + x + c$

$(1+\alpha)(1+\beta) = 1 + (\alpha + \beta) + \alpha\beta$

$= 1 - 1 + c$

$= c$

Ans: D

11. A) $f(x) = x^n - 1$

$f(1) = 1^n - 1 = 0$ ✓

B) $f(x) = x^3 - 3ax^2 + 3ax - 1$

$f(1) = 1 - 3a + 3a - 1 = 0$ ✓

C) $f(x) = x^2 + 2x + 1$

$f(1) = 1 + 2 + 1 \neq 0$ ✗

D) $f(x) = x^2 - 2x + 1$

$f(1) = 1^2 - 2 \cdot 1 + 1 = 0$ ✓

∴ Ans: A, B, D

12 $f(x) = 2x^4 - 3x^3 - 3x^2 + 6x - 2$

let $\alpha = \sqrt{2}$, $\beta = -\sqrt{2} \Rightarrow x^2 - 0x - 2 = 0 \Rightarrow x^2 - 2 = 0$

Divide $f(x)$ with $x^2 - 2$, we get quotient

as $2x^2 - 3x + 1 = (x-1)(2x-1)$

$\therefore \quad x = 1, 1/2$

Ans: A, D

13. Statement I:

$(\sqrt{3}x+2)(4x-\sqrt{3})$

$= 4\sqrt{3}x^2 - 3x + 8x - 2\sqrt{3}$

$= 4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ (True)

Statement II Conceptual (True)

Ans: A

14 Statement I: Conceptual (True)

Statement II: Conceptual (True)

Ans: A

15 Assertion: $64x^3 + 125$

$= (4x)^3 + (5)^3$

$= (4x+5)((4x)^2 - 4x \cdot 5 + 5^2)$

$= (4x+5)(16x^2 - 20x + 25)$ (True)

Ans: A

Reason: Conceptual (True)

16 Assertion: $x^4 - 5x^2 + 4$.

Let $y = x^2$

$\therefore y^2 - 5y + 4 = (y-1)(y-4) = 0$

$y = 1, \; y = 4$

$x^2 = 1, \; x^2 = 4$

$x = \pm 1$

$x = \pm 2$ (True)

Ans: A

Reason: Conceptual (True)

17. $\sum (x^2y + xy^2)$

$= \sum xy(x+y)$

$= xy(x+y) + yz(y+z) + zx(z+x)$

Ans: C

18 $\sum_{i=1}^{n} 2i = 210$

$\Rightarrow 2\sum_{i=1}^{n} i = 210$

$\Rightarrow 2 \times \frac{n(n+1)}{2} = 210$

$n(n+1) = 210$

$\Rightarrow n = 14$ or [redacted]

Ans: 14

19 $27p^3 - 8q^3$

$= (3p)^3 - (2q)^3$

$= (3p-2q)((3p)^2 + 3p \cdot 2q + (2q)^2)$

$= (3p-2q)(9p^2 + 6pq + 4q^2)$

Ans: A

20 $1000 + y^3 = (10)^3 + y^3 = (10+y)(100 + 10y + y^2)$

Ans: C

21. $\alpha^2 + \beta^2 = 40$

$\Rightarrow (\alpha+\beta)^2 - 2\alpha\beta = 40$

$\Rightarrow (8)^2 - 2 \cdot K = 40$

$\Rightarrow K = 12$

Ans: 12

22 $(2x^3+5x-1)(4x^4-3x^2+x)$ (15)

$= 8x^7 + \cdots$

$\therefore$ Degree $= 7$

Ans. 7

23 a) $a^3(b-c)+b^3(c-a)+c^3(a-b)$

factors are $(a-b)(b-c)(c-a) \rightarrow q$

b) $a^4(b-c)+b^4(c-a)+c^4(a-b)$

factors are $(a-b)(b-c)(c-a) \rightarrow q$

c) $x^3(y-z)+y^3(z-x)+z^3(x-y)$

factors are $(x-y)(y-z)(z-x) \rightarrow r$

d) $x^4(y-z)+y^4(z-x)+z^4(x-y)$

factors are $(x-y)(y-z)(z-x) \rightarrow r$

Ans: q, q, r, r

24 a) $x^3-64 = x^3-4^3 = (x-4)(x^2+4x+16)$ (P)

b) $x^3+27 = x^3+3^3 = (x+3)(x^2-3x+9)$ (S)

c) $8x^3-1 = (2x)^3-1^3 = (2x-1)(4x^2+2x+1)$ (r)

d) $x^3-8 = x^3-2^3 = (x-2)(x^2+2x+4)$ ( )

Ans: P, S, r, —

$\Rightarrow$ THE END $\Leftarrow$