HERON'S FORMULA 1

class IX . Mathematics

SOLUTIONS

TEACHING TASK

01. $a:b:c = 25:17:12$

$\Rightarrow a+b+c = 25x+17x+12x = 540$

$\Rightarrow x = 10$

Sides are 250, 170, 120

$\therefore s = \dfrac{250+170+120}{2} = 270$

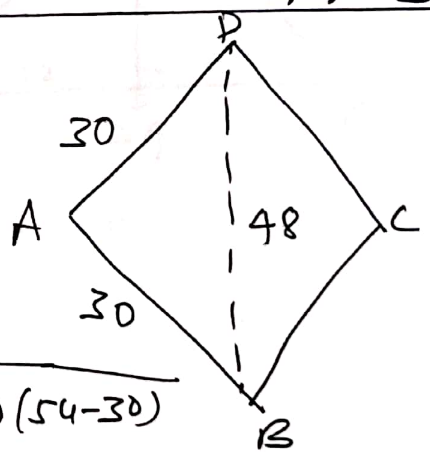
Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{270(270-250)(270-170)(270-120)}$

$= 9000$

Ans: B

02. $a=30$, $b=30$, $c=48$



$s = \dfrac{30+30+48}{2}$

$s = 54$

Area of $\triangle ABD = \sqrt{54(54-30)(54-30)(54-48)}$

$= 9 \times 48$

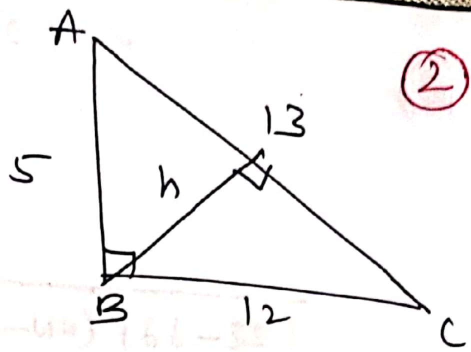
Total Area $\square ABCD = 2 \times 9 \times 48$

Area of grass for each cow $= \dfrac{2 \times 9 \times 48}{12}$

$= 72$

Ans: A

03 Area of $\triangle ABC$



2

$\frac{1}{2} \times 12 \times 5 = \frac{1}{2} \times 13 \times h$

$\Rightarrow h = \frac{60}{13}$

Ans: B

04. $S = \frac{7.5 + 6.5 + 7}{2} = \frac{21}{2} = 10.5$

Area of $\triangle ABC = \sqrt{10.5(10.5-7.5)(10.5-6.5)(10.5-7)}$

$= 21.$

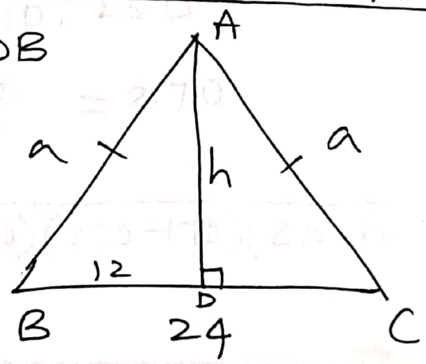
Area of $\|^{gm}$ DCED $= b \times h = 21$

$7 \times h = 21$

$\Rightarrow h = 3$

Ans: A

05 Ar $\triangle ABC = 2 \times$ Ar $\triangle ADB$



Ar $\triangle ABC = \frac{1}{2} \times 24 \times h = 192$

$\Rightarrow h = 16$

Area of $\triangle ABD = \frac{1}{2} \times 1$

We have $a^2 = h^2 + 12^2$

$= 16^2 + 12^2$

$= 400$

$a = 20$

perimeter $= a + a + 24$

$= 20 + 20 + 24$

$= 64$ cm

Ans: D

06 $a = 33$, $b = 44$, $c = 55$

$$s = \frac{33+44+55}{2}$$
$$= 66$$

$$\text{Area} = \sqrt{66(66-33)(66-44)(66-55)}$$
$$= \sqrt{66 \times 33 \times 22 \times 11}$$
$$= 726.$$

Total cost $= \text{Rs } 726 \times 1.20$

$$= \text{Rs } 871.20$$

Ans: A

---

07 $a = 12x$, $b = 17x$, $c = 25x$

$$a+b+c = 12x + 17x + 25x = 540$$
$$\Rightarrow \quad x = 10$$

Sides are ⇒ 120, 170, 250

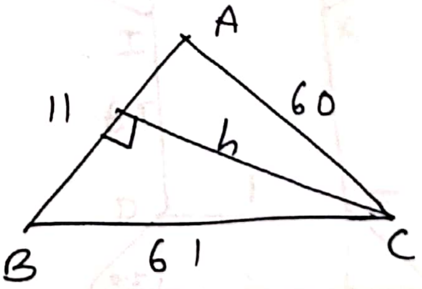
$$s = \frac{120+170+250}{2} = 270$$

$$\text{Area} = \sqrt{270(270-120)(270-170)(270-250)}$$
$$= 9000$$

Ans. D

---

08 $$s = \frac{11+61+60}{2} = 66$$



Area of $\triangle ABC$ =

$$\sqrt{66(66-11)(66-61)(61-60)}$$

$$= 330$$

$$\therefore \frac{1}{2} \times 11 \times h = 330$$

$$\Rightarrow h = 60$$

Ans: D

09. original side | new sides

a, b, c | $2a, 2b, 2c$

$S = \frac{a+b+c}{2}$ | $S_1 = \frac{2a+2b+2c}{2}$

$\Rightarrow a+b+c = 2S$ | $S_1 = a+b+c = 2S$

(4)

New Area $= \sqrt{\cancel{(a+b+c)} S_1 (S_1-2a)(S_1-2b)(S_1-2c)}$

$\therefore A' = \sqrt{2S(2S-2a)(2S-2b)(2S-2c)}$

$= 4\sqrt{s(s-a)(s-b)(s-c)}$

$A' = 4A$

% Increase $= \frac{\text{New Area} - \text{Original area}}{\text{Original Area}} \times 100$

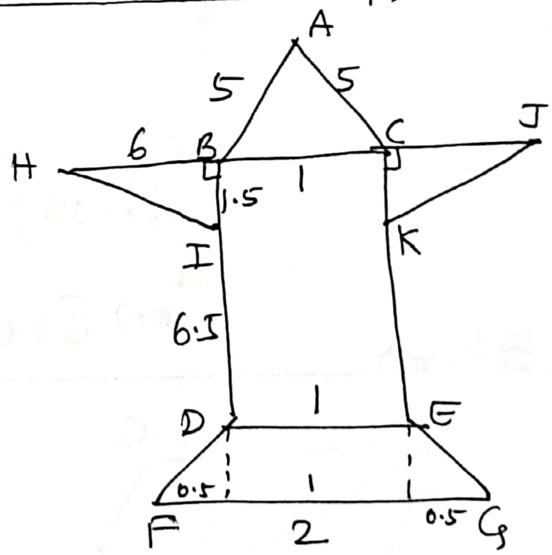
$= \frac{4A - \cancel{3}A}{A} \times 100$

$= 300\%$

Ans: D

---

10. Ar△ ABC = 2.48.



Ar▭BCDE = $l \times b$

$= 6.5$

Area DEFG = 0.5

Area of △HBI $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 1.5 \times 6$

$= 4.5$

Similarly △CJK = 4.5

Total Area $= 2.48 + 6.5 + 0.5 + 4.5 + 4.5$

$= 19.3$

11. $a = 10$ cm, $b = 34$ cm, $c = 36$ cm.

Area of each trianglular floor = Rs 169.68

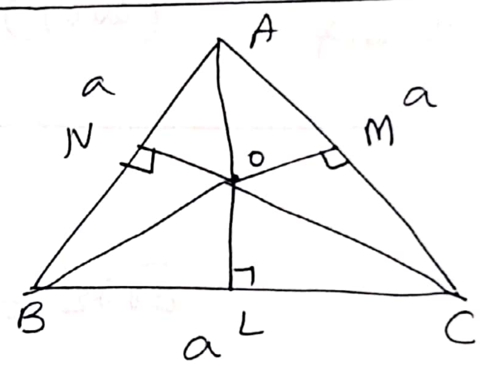
Area of 16 triangular tiles = 16×169.68
$= 2714.8\ cm^2$

Cost of polishing the tiles at the rate of Rs. 2.50 $= 2714.8 \times 2.50$
$=$ Rs. 6787

Ans: A, B, C

---

12 Ar $\triangle ABC$ = Ar $\triangle OAB$ + Ar $\triangle OBC$ + Ar $\triangle OCA$



$$\frac{\sqrt{3}}{4} a^2 = \frac{1}{2} \times a \times 14 + \frac{1}{2} \times a \times 10 + \frac{1}{2} \times a \times 6$$

$\Rightarrow \quad a = 20\sqrt{3}$

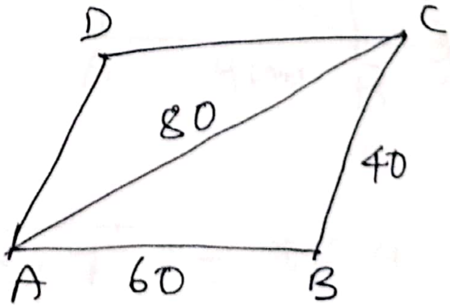
$$\text{Area} = \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} (20\sqrt{3})^2$$
$$= 300\sqrt{3}\ cm^2$$

Ans: A, B

---

13 $a = 40$, $b = 80$, $c = 60$

$$s = \frac{40 + 80 + 60}{2} = 90$$



Ar $\triangle ABC = \sqrt{90 \times 50 \times 10 \times 30}$
$= 30\sqrt{15}$
$= 30 \times 3.87 = 1161$

Total Area $= 2 \times 1161 = 2322$

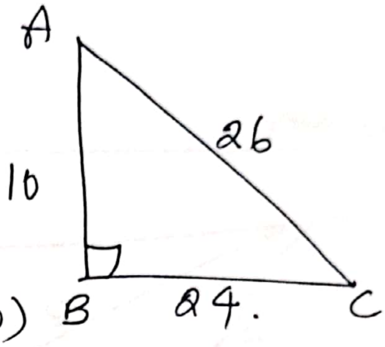
Statement II: Conceptual (True).

But statement II does not resemblence statement I.

Ans: B

---

14. Statement I:



perimeter $= 10+24+26$
$= 60$ (True)

Also $26^2 = 10^2 + 24^2$ (True)

Statement II: Conceptual (True)

Ans: A

---

15. $3a = 36$

$\Rightarrow a = 12$

Area $= \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times 12^2 = 36\sqrt{3}$

Ans: D

---

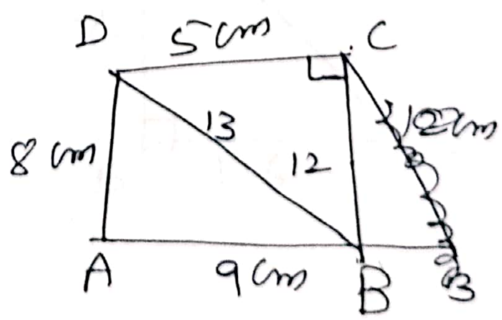
16 $a = 16$ cm

Area $= \frac{\sqrt{3}}{4} (16)^2 = 110.848$

Ans: D

---

17. $BD = 13$ cm



Ar$\triangle BCD = \frac{1}{2} \times 12 \times 5$
$= 30$

$a=9,\ b=13,\ c=8$

$s = \frac{9+13+8}{2} = 15$

Area $ABD = \sqrt{15 \times 6 \times 2 \times 7} = 35.4$

Total Area $= 30 + 35.4$
$= 65.4$

Ans: C

19. $\frac{\sqrt{3}}{4}a^2 = 400\sqrt{3}$

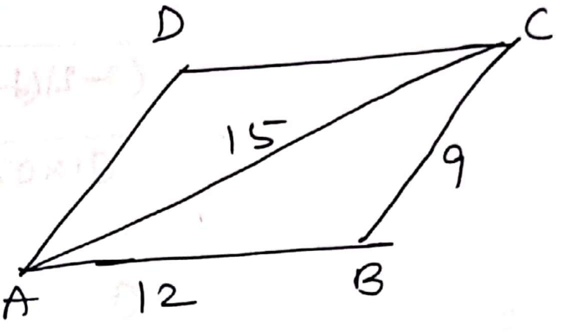
$\Rightarrow a = 40$

perimeter $= 3a = 3\times 40 = 120$

Now $\frac{m}{20} = \frac{120}{20} = 6$

Ans. 6

20. $a = 12,\ b = 15,\ c = 9$



$S = \frac{12+15+9}{2}$

$= 18$

Area $\triangle ABC$

$= \sqrt{18\times 6\times 3\times 9}$

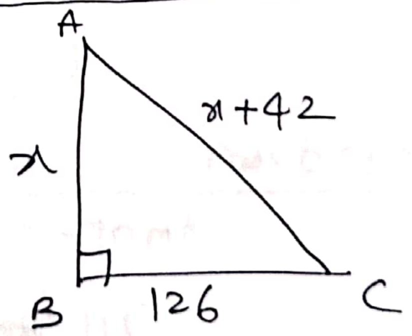
$= 54$

Area of $\parallel^{gm}$ $ABCD = 2\times 54 = 108 = K$

Given $\frac{K-100}{4} = \frac{108-100}{4} = 2$

Ans: 2

21. i) $(x+42)^2 = x^2 + 126^2$



$\Rightarrow x = 168$

$\therefore$ Area $= \frac{1}{2}\times 168\times 126$

$= 10584$

Ans: t

ii) $\frac{\sqrt{3}}{2}a = 8 \Rightarrow a = \frac{16}{\sqrt{3}}$

Area $= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}\left(\frac{16}{\sqrt{3}}\right)^2 = \frac{64\sqrt{3}}{3}$

⑧

iii) $a = 3x,\ b = 5x,\ c = 7x$

$s = 3x + 5x + 7x = 300$

$x = 20$

$a = 60,\ b = 100,\ c = 140$

~~Area~~ $s = \frac{60 + 100 + 140}{2} = 150$

$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{150 \times 90 \times 50 \times 10}$

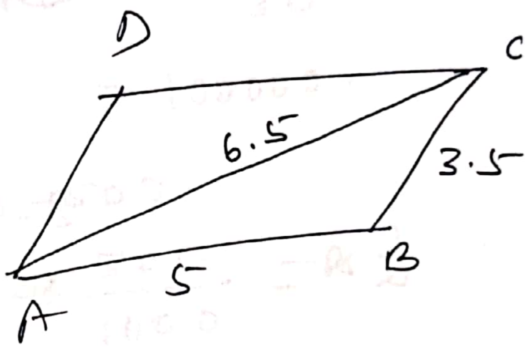
$= 1500\sqrt{3}$

iv)

$a = 5$

$b = 3.5$

$c = 6.5$



$s = \frac{15}{2} = 7.5$

$\text{Ar}\triangle ABC = \sqrt{7.5 \times 2.5 \times 4 \times 1}$

$= 5\sqrt{3}$

Total Area $= 10\sqrt{3}$

Ans: 4 r, p, q

---

22 (i) $a = 90\text{mt},\ b = 70\text{mt},\ c = 70\text{mt}$

$s = \frac{90 + 70 + 70}{2} = 115$

$\text{Area} = \sqrt{115 \times 25 \times 45 \times 45}$

$= 2412.86.$

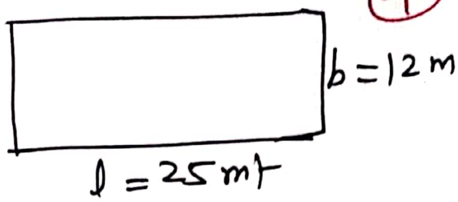
Cost $= 2412.86 \times 5$

$=$ Rs 12,060 (Approx)

9

(ii) $l = 25\,mt$ | $b = 12\,mt$

$l = 2500\,cm$ | $b = 1200\,cm$



$\text{Area} = l \times b$

$= 2500 \times 1200$

$= 3000000.$

Area of each brick $= 10 \times 3$

$= 30.$

Total bricks required $= \dfrac{3000000}{30}$

$= 100000.$

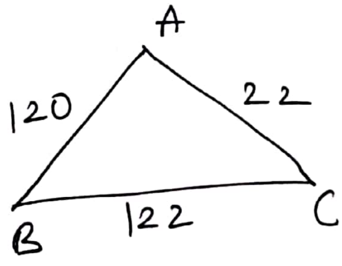
Given 1000 bircks = Rs 500.

1 brick = Rs $\dfrac{500}{1000} = \text{Rs } \dfrac{1}{2}.$

$\therefore$ Total cost $= \dfrac{100000}{2} = \text{Rs } 50000$

(iii) Area of $\triangle ABC$

$= 1320$



Rent of $1\,mt^2$ / 1 year = Rs 5000

Rent of $1\,mt^2$ / 1 month $= \dfrac{5000}{12}$

$\therefore$ Rent of $1320\,mt^2$ / 1 mont $= \dfrac{5000}{12} \times 1320$

for 3 months $= \dfrac{5000}{12} \times 1320 \times 3$

$=$ Rs 16,50,000

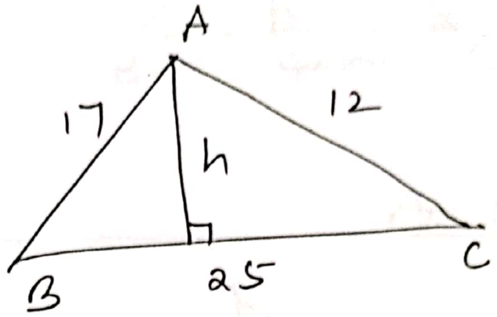
Ans: t, v, r, p

LEARNERS TASK

10

CUQ's

01. $a = 25,\ b = 12,\ c = 17$



$s = \dfrac{25+12+17}{2}$

$s = 27\,cm$

Area $\triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{27(27-25)(27-12)(27-17)}$

$= 90$

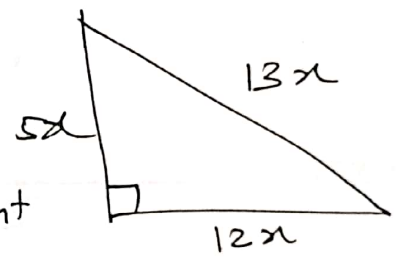
Area $\triangle ABC = \frac{1}{2} \times 25 \times h = 90$

$\Rightarrow h = 7.2\,cm$

Ans: B

---

02 Area $= \frac{1}{2} \times 12x \times 5x = ?$



perimeter =

$5x + 12x + 13x = 450\,mt$

$\Rightarrow x = 15$

Area $= \frac{1}{2} \times 12x \times 5x$

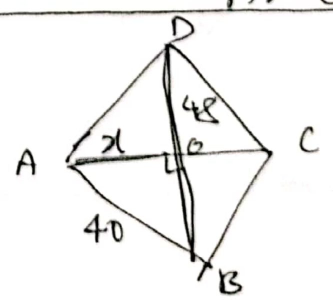
$= \frac{1}{2} \times 12 \times 5 \times (15)^2$

$= 6750$

Ans: C

---

03 $AB = 40,\ BO = 24$



$AB^2 = AO^2 + BO^2$

$40^2 = x^2 + 24^2$

$\Rightarrow x^2 = 1024$

$\Rightarrow x = 32$

Ar $\triangle AOB = \frac{1}{2} \times 32 \times 24 = 384$

Ar $\triangle ABCD$

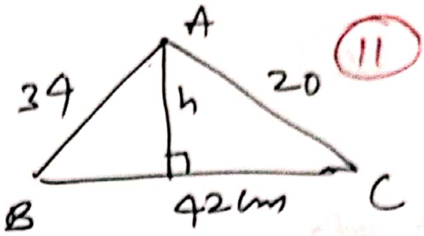
$= 4 \times 384$

$= 1536$ Ans: A

04 find area of $\triangle ABC$ and equate it with

$= \frac{1}{2} \times 42 \times h.$

we get h value



Ans: C

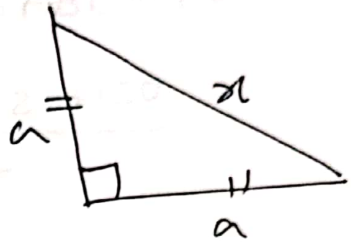
05 $\frac{1}{2} \times a \times a = 200$

$\Rightarrow a = 20$

$a^2 + a^2 = x^2$

$\Rightarrow 2a^2 = x^2$

$\Rightarrow 2 \times (20)^2 = x^2 \Rightarrow x = 20\sqrt{2}$



Ans: C

06 . $a = 9cm$, $b = 12cm$, $c = 15cm$

Use Heron's formula

$\Delta = 54$

Ans: D

07 $a = 13$, $b = 14$; $c = 15$

use Heron's formula

$\Delta = 84$

Ans: A

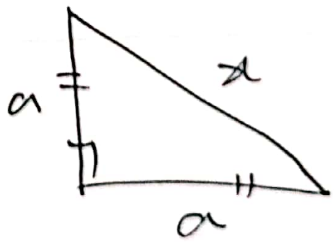
08 $\frac{1}{2} \times a \times a = 8$

$\Rightarrow a = 4$

$a^2 + a^2 = x^2$

$\Rightarrow 16 + 16 = x^2$

$\Rightarrow x = 4\sqrt{2}$



Ans: A

09 $3a = 90$

$\Rightarrow a = 30$

Area $= \frac{\sqrt{3}}{4} a^2 = 225\sqrt{3}$

Ans: C

10 $\frac{\sqrt{3}}{4}a^2 = 9\sqrt{3}$

(12)

$\Rightarrow a = 6$

perimeter $= 3a = 3 \times 6 = 18$

Ans: A

JEE MAINS LEVEL

01. Area of shaded region = Ar $\triangle ABC$ − Ar $\triangle BOC$

Now Ar $\triangle ABC$, $S = \frac{22+122+120}{2}$

$= 132$

$\triangle ABC = \sqrt{132(132-22)(132-122)(132-120)}$

$= 1320.$

Now, Ar $\triangle BOC$, $S = \frac{22+24+26}{2}$

$= 36$

Area $= \sqrt{36(36-22)(36-24)(26-26)}$

$= 245.93$

Required area $= 1320 - 245.93$

$= 1074$ (approx)

Ans: D

02 perimeter $= 2s = \alpha$

$\Rightarrow s = \frac{\alpha}{2}$

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{\frac{\alpha}{2}\left(\frac{\alpha}{2}-p\right)\left(\frac{\alpha}{2}-q\right)\left(\frac{\alpha}{2}-r\right)}$.

Ans: D

03 $(s-a)+(s-b)+(s-c) = 5+10+1$

$\Rightarrow 3s-(a+b+c) = 16$

$\Rightarrow 3s - 2s = 16$

$s = 16$

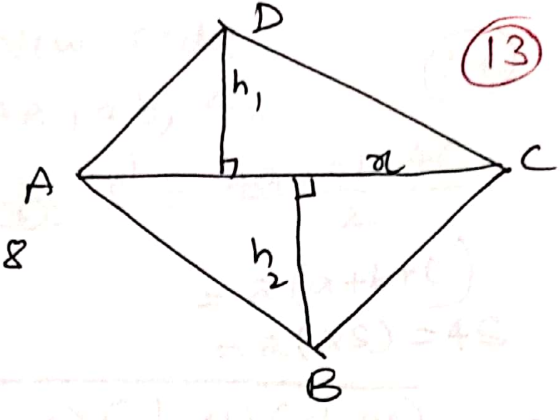
$\Delta = \sqrt{16 \times 5 \times 10 \times 1}$

$= 20\sqrt{2}$

Ans: C

04) Ar ABCD = 360

$h_1 = 10$, $h_2 = 8$



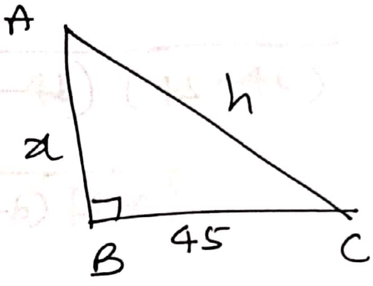
$360 = \frac{1}{2} \times x \times 10 + \frac{1}{2} \times x \times 8$

$\Rightarrow x = 40$

Ans: D

---

05 Area $= \frac{1}{2} \times 45 \times x$



cost $\Rightarrow$ 2 $Km^2 \rightarrow 2700$

$\Rightarrow$ 1 $Km^2 \rightarrow 1350$

$\therefore$ Cost of $\frac{1}{2} \times 45 \times x$ sq = Rs 1350

$\Rightarrow$ $x = 60$

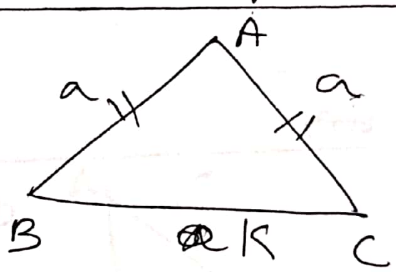
$\Rightarrow$ $h = 75$ Km

Ans: D

---

06. Given $a : k = 3 : 2$

$\Rightarrow a = 3x, k = 2x$



perimeter = 32

$3x + 3x + 2x = 32$

$\Rightarrow$ $8x = 32$

$\Rightarrow$ $x = 4$

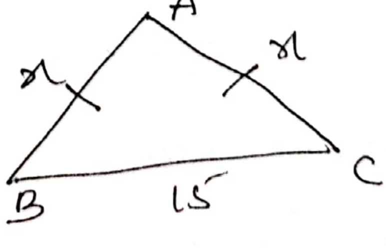
Sides are 12, 12, 8

$\Rightarrow$ Area = 160.75 (use Heron's formula

Ans: A

---

07 $x + x + 15 = 60$



$\Rightarrow x = \frac{45}{2} = 22.5$

$\therefore$ The sides are 22.5, 22.5, 15.

Area = 159.07 (use Heron's formula)

Ans: B

08 original sides a, b, c

$s = a+b+c/2$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

New sides $4a, 4b, 4C$

$\Delta' = s' = \dfrac{4a+4b+4C}{2}$

$= 2(a+b+c)$

$= 2(2s) = 4s$

New Area $= \sqrt{s'(s'-4a)(s'-4b)(s'-4c)}$

$= \sqrt{4s(4s-4a)(4s-4b)(4s-4c)}$

$= \sqrt{4^4\, s(s-a)(s-b)(s-c)}$

$\therefore \Delta' = 16 \cdot \Delta$

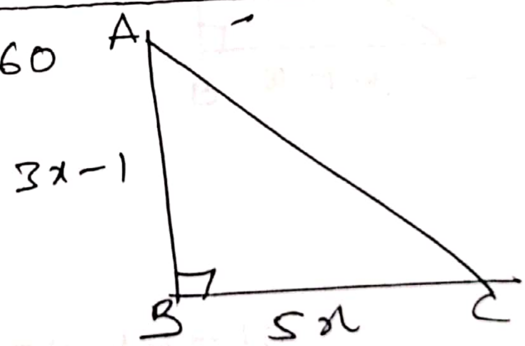
percentage change in Area $= \dfrac{\text{New} - \text{original}}{\text{original}} \times 100$

$= \dfrac{16\Delta - \Delta}{\Delta} \times 100$

$= 1500\ \%$

Ans: 8

09. Area $= \frac{1}{2} \times 5x \times (3x-1) = 60$



$\Rightarrow 15x^2 - 5x = 120$

$\Rightarrow 15x^2 - 5x - 120 = 0$

$\Rightarrow 3x^2 - x - 24 = 0$

$\Rightarrow (x-3)(3x+8) = 0$

$\Rightarrow x = 3$

Sides are $= 8, 15$

Now $h^2 = 8^2 + 15^2$

$= 64 + 225$

$= 289$

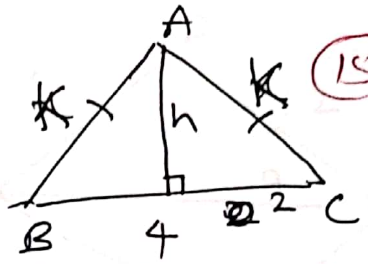
$\therefore h = 17\,\text{cm}$

Ans: A

15

10 $\text{Area} = \cancel{2\times}\frac{1}{2}\times 2\times h = 16$

$\Rightarrow h = 8$ cm



$\text{Area} = \frac{1}{2}\times 4\times h = 16$

$\Rightarrow h = 8$ cm

from diagram $h^2 + 2^2 = K^2$

$\Rightarrow 64 + 4 = K^2$

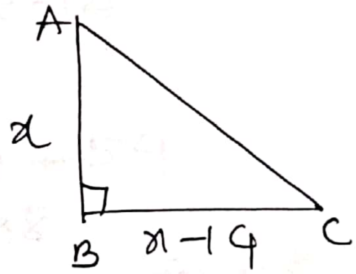
$\Rightarrow K = \sqrt{68}$

$\Rightarrow K = 2\sqrt{17}$

$= 8.24.$

$\therefore K - 1.24 = 7$

11 Difference of sides $= 14$



$\text{Area} = \frac{1}{2}\cdot x\cdot(x-14) = 120$

$\Rightarrow x(x-14) = 240$

$\Rightarrow \boxed{x = 24}, -10$

a) one side $= 24$

b) other side $= x - 14 = 24 - 14 = 10$

c) Hypotenus $h^2 = 24^2 + 10^2$

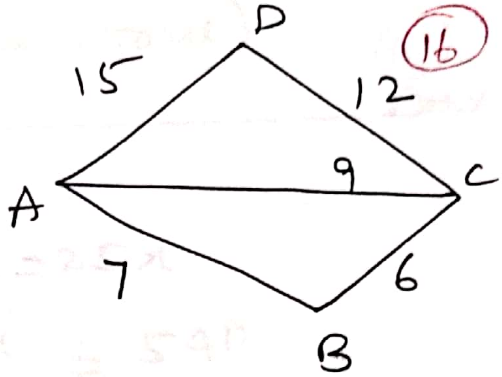
$\Rightarrow h = 26$

d) perimeter $= 24 + 10 + 26$

$= 60$ cm

Ans: A, B, C, D

12 For $\triangle ABC$



$a = 6, b = 9, c = 7.$

$s = \frac{6+9+7}{2}$

$= \frac{22}{2}$

$= 11$

$\therefore$ Area $= \sqrt{11(11-6)(11-9)(11-7)}$

$= \sqrt{11 \times 5 \times 2 \times 4}$

$= 20.98$

For $\triangle ACD$

$a = 9, \quad b = 15, \ c = 12$

$S = \frac{9+15+12}{2} = \frac{36}{2} = 18$

$\therefore$ Area $= \sqrt{18(18-9)(18-15)(18-12)}$

$= 54.$

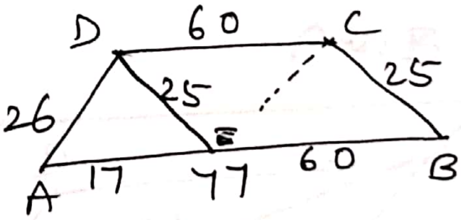
Total Area $\square ABCD = 20.98 + 54$

$= 74.98$

Ans: A, B, C

---

13 Statement: A



Ar $\triangle AED = 204$ (By Heron's formula)

Ar $\triangle$ ~~BEC~~ =

$\therefore \frac{1}{2} \times b \times h = 204$

$\Rightarrow \frac{1}{2} \times 17 \times h = 204$

$\Rightarrow h = 24$ cm

Area $\|^{gm} = b \times h$

$= 60 \times 24 = 1440$

Total trapezium Area $= 1440 + 204$

$=$

$= 1644$

(True)

Statement II: Conceptual (True)

Ans: A

14. Statement I:

$a = 12x,\quad b = 17x,\quad c = 25x$

$S = \dfrac{12x + 17x + 25x}{2} = 540$

$\Rightarrow 34x = 540$

$\Rightarrow x = 10$

$\therefore$ Sides are ~~240~~ 120, ~~340~~ 170, ~~500~~ 250

By Heron's formula, Area = 9000 (False)

Statement II: Conceptual (True) Ans: D

15. $a = 34,\quad b = 20,\quad c = 42$

$S = \dfrac{34 + 20 + 42}{2}$

$= \dfrac{96}{2} = 48$

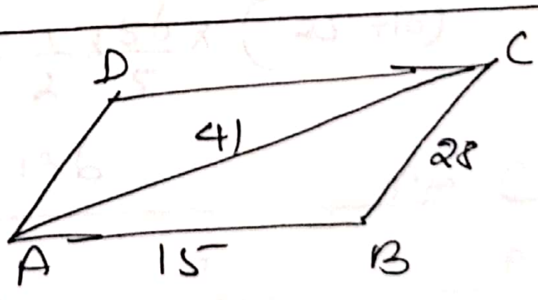
$\Delta = \sqrt{48(48-34)(48-20)(48-42)}$

$\Delta = 336$

Ans: B

16. for ΔABC

$a = 28,\ b = 41,\ c = 15$



Ar ΔABC = ~~168~~ 126

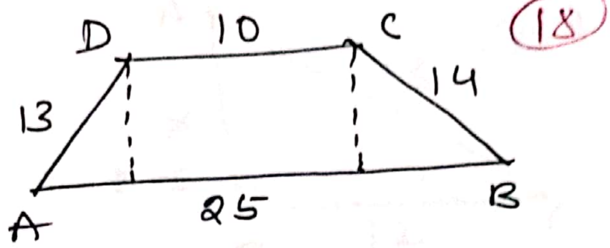
$\therefore$ Ar ▱ABCD = 2×126

= ~~336 cm~~

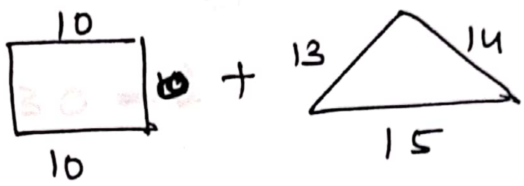
= 252 cm^2

Ans: D

17



∴ Area of trapezium =



<u>for △ triangle</u>

$a = 15,\ b = 14,\ c = 13$

$s = \frac{15+14+13}{2}$

$= 21$

Area of $\triangle = \sqrt{21(21-15)(21-14)(21-13)}$

$= 84.$

∴ ~~Total area of trapezium~~ = ~~10×10 + 84~~

=

∴ $\frac{1}{2} \times 15 \times h = 84$

$\Rightarrow h = \frac{56}{5}$

∴ Area of trapezium $= \frac{1}{2} h(a+b)$

$= \frac{1}{2} \times \frac{56}{5} \times (25+10)$

$= 196$

Ans: C

18 for triangle

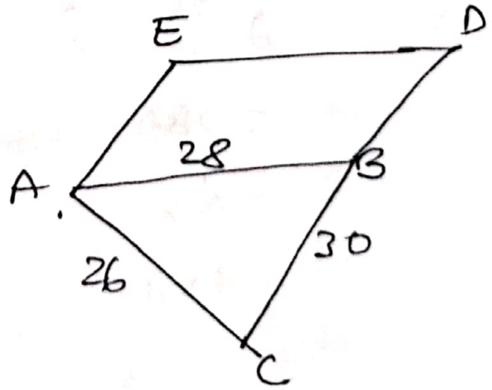
$a = 30,\ b = 26,\ c = 28$

Ar $\triangle ABC = 12 \times 28\ cm^2$

Area ∥gm $= b \times h$

$28 \times h = 12 \times 28$

$\Rightarrow h = 12\,cm$



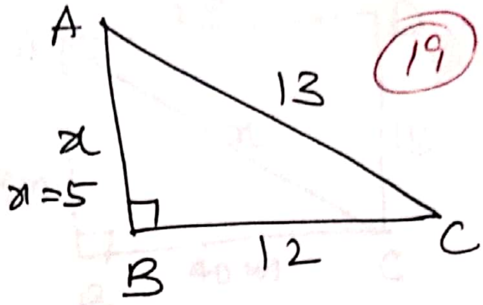
Ans: C

19. $x^2+12^2=13^2$

$\Rightarrow x^2+144=169$

$\Rightarrow x^2=25$

$\Rightarrow x=5$



$\therefore$ Area $=\frac{1}{2}\times 12\times 5=30=l$

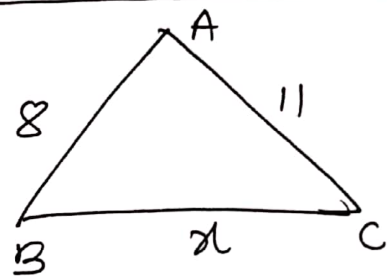
Now $=\frac{3l}{10}=\frac{3\times 30}{10}=9$

Ans: 9

---

20. $8+11+x=32$

$\Rightarrow x=13\text{ cm}$



$\therefore a=8,\ b=11,\ c=32$

Area $=8\sqrt{30}=k\sqrt{30}$

$\therefore k=8$

Ans: 8

---

21. 1) $a=26,\ b=28,\ c=30$

Area $=336$ By (Heron's formula)

$\therefore \frac{1}{2}\times 28\times h=336$

$\Rightarrow h=24\text{ cm}$

2) $\frac{\sqrt{3}}{4}a^2=4\sqrt{3}$

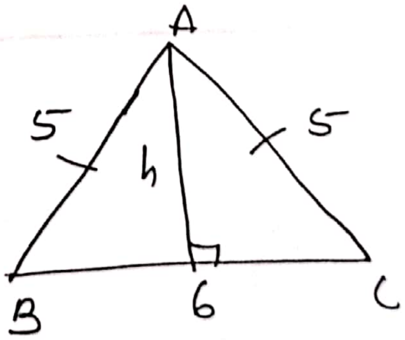
$\Rightarrow a=4$

perimeter $=3a=12$

3) $\frac{\sqrt{3}}{2}a=3\sqrt{3}$

$\Rightarrow a=6$

4)



Area $\triangle ABC=12$

$\therefore \frac{1}{2}\times 6\times h=12$

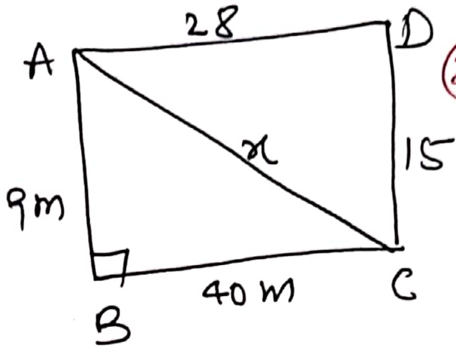
$\Rightarrow \frac{1}{2}\times 6\times h=12$

$\Rightarrow h=4$

Ans: r, s, p, q

22 1) $Ar \triangle ABC = \frac{1}{2} \times 40 \times 9$

$= 180 \text{ mt}^2$



(20)

2) $x^2 = 9^2 + 40^2$

$= 81 + 1600$

$= 1681$

$\therefore x = 41.$

Now for $\triangle ACD$

$a = 41, \; b = 15, \; c = 28$

$Ar \triangle ACD = 126 \text{ mt}^2$

3) Total Area ABCD $= 180 + 126$

$= 306$

Ans: s, q, r, p

D) $\therefore AC = 41$ mt

$\Longrightarrow$ THE END $\Longleftarrow$

Scanned with OKEN Scanner