HYPERBOLIC FUNCTIONS

Class: IX, Mathematics

IIT Advanced

SOLUTIONS

TEACHING TASK

01 $\sinh^3 x - \cosh^3 x =$

$= (\sinh x - \cosh x)(\sinh^2 x + \sinh x \cdot \cosh x + \cosh^2 x)$

$= (\sinh x - \cosh x)((\sinh x + \cosh x)^2 - \sinh x \cosh x)$

$= (-e^{-x})\left((e^x)^2 - \frac{e^{2x} - e^{-2x}}{4}\right)$

$= (-e^{-x})\left(\frac{3e^{2x} + e^{-2x}}{4}\right)$

$= \left(\frac{-3e^x - e^{-3x}}{4}\right) = \frac{(-3)e^x - e^{(-3)x}}{1 - (-3)}$

$\therefore \quad K = -3$

Ans : C

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02 $\sinh^{-1} x + \cosh^{-1} x = y$

Let $\sinh^{-1} x = A \mid \cosh^{-1} x = B$

$\Rightarrow \sinh A = x \mid \Rightarrow \cosh B = x$

$\boxed{\cosh^2 x - \sinh^2 x = 1}$

$A + B = y$

$\sinh(A+B) = \sinh A \cosh B + \cosh A \sinh B$

$\sinh(y) = x \cdot x + \sqrt{1+x^2}\sqrt{x^2-1}$

$= x^2 + \sqrt{x^4 - 1}$

Ans : A

Q3 $\sinh^{-1}(2\alpha) = 2\cosh^{-1}(\beta)$

let $\sinh^{-1}(2\alpha) = x$ and $2\cosh^{-1}(\beta) = x$

$\sinh x = 2\alpha$ and $\cosh\left(\frac{x}{2}\right) = \beta$

$\Rightarrow 2\sinh\left(\frac{x}{2}\right)\cosh\left(\frac{x}{2}\right) = 2\alpha$

$\Rightarrow 2\sinh\left(\frac{x}{2}\right).\beta = \alpha \Rightarrow \sin\left(\frac{x}{2}\right) = \frac{\alpha}{\beta}$

We know $\boxed{\cosh^2 x - \sinh^2 x = 1}$

$\beta^2 - \left(\frac{\alpha}{\beta}\right)^2 = 1$

$\Rightarrow \beta^4 - \alpha^2 = \beta^2$

$\Rightarrow \beta^4 = \alpha^2 + \beta^2$

Ans:

Q4 $\frac{\cosh 2\theta - 1}{\sinh 2\theta} = \frac{\cosh^2\theta + \sinh^2\theta - 1}{2\sinh\theta\cdot\cosh\theta} = \frac{2\sinh^2\theta}{2\sinh\theta\cdot\cosh\theta}$

$= \tanh\theta$

Ans: B

Q5 $\tanh^2 x = \cos\theta$

$\cosh 2x = \frac{1+\tanh^2 x}{1-\tanh^2 x}$

$= \frac{1+\cos\theta}{1-\cos\theta}$

$= \frac{2\cos^2\theta/2}{2\sin^2\theta/2}$

$= \cot^2\left(\frac{\theta}{2}\right)$

Ans: B)

06 ~~Hence~~ This is shortcut method

We can solve this problem in this method also.

Given x is an acute angle. let $x = 30^\circ$

$$y = \log_e\left(\tan\left(\frac{\pi}{4} + \frac{30^\circ}{2}\right)\right)$$

$$\Rightarrow y = \log_e(\tan 60^\circ)$$

$$\Rightarrow e^y = \sqrt{3} \Rightarrow e^{-y} = \frac{1}{\sqrt{3}}$$

$$\text{Now } \cos x \cdot \cosh y = \cos x \cdot \left(\frac{e^y + e^{-y}}{2}\right)$$

$$= \cos 30^\circ \left(\frac{\sqrt{3} + \frac{1}{\sqrt{3}}}{2}\right)$$

$$= \frac{\sqrt{3}}{2} \times \frac{4}{2\sqrt{3}} = 1$$

Ans : B

07. $\text{Sech}^{-1}(x) = \log\left(\frac{1 + \sqrt{1 - x^2}}{x}\right)$

$$\text{Sech}^{-1}(\sin\theta) = \log\left(\frac{1 + \sqrt{1 - \sin^2\theta}}{\sin\theta}\right)$$

$$= \log\left(\frac{1 + \cos\theta}{\sin\theta}\right)$$

$$= \log\left(\frac{2\cos^2\theta/2}{2\sin\frac{\theta}{2}\cos\theta/2}\right)$$

$$= \log\left(\cot\frac{\theta}{2}\right)$$

Ans: D

08 $\sinh^{-1}(2) + \sinh^{-1}(3) = x$

$\Rightarrow \log(2+\sqrt{5}) + \log(3+\sqrt{10}) = x$

$\Rightarrow \log(6 + 2\sqrt{10} + 3\sqrt{5} + \sqrt{50}) = x$

08. $\sinh^{-1}(2) + \sinh^{-1}(3) = x$ (4)

$\Rightarrow \log(2+\sqrt{5}) + \log(3+\sqrt{10}) = x$

$\Rightarrow \log\left[(2+\sqrt{5})(3+\sqrt{10})\right] = x$

Now $\cosh(x) = \dfrac{e^x + e^{-x}}{2} = \dfrac{e^{\log((2+\sqrt{5})(3+\sqrt{10}))} + e^{-\log((2+\sqrt{5})(3+\sqrt{10}))}}{2}$

$= \left((2+\sqrt{5})(3+\sqrt{10}) - \dfrac{1}{(2+\sqrt{5})(3+\sqrt{10})}\right) \times \dfrac{1}{2}$

= on further simplification

we get $\frac{1}{2}(12 + 2\sqrt{50}) =$ Ans: C

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09 $\cosh 2x = 99$

$\Rightarrow \dfrac{1+\tanh^2\theta}{1-\tanh^2\theta} = 99 \Rightarrow \tanh\theta = \dfrac{7}{5\sqrt{2}}$ Ans: B

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10 $\cosh 3x = 4\cosh^3 x - 3\cosh x$

$= 4\left(\frac{5}{4}\right)^3 - 3\left(\frac{5}{4}\right)$

$= \dfrac{65}{16}$ Ans: D

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11 A) $\cosh x = \frac{5}{4} \Rightarrow \cosh(3x) = 4\cosh^3 x - 3\cosh x$

$= 4\left(\frac{5}{4}\right)^3 - 3\left(\frac{5}{4}\right) = \frac{65}{16}$ (True)

B) $\sinh(x) = \frac{3}{4} \Rightarrow \sinh(3x) = 3\sinh x + 4\sinh^3 x$

$= 3\left(\frac{3}{4}\right) + 4\left(\frac{3}{4}\right)^3 = \frac{63}{16}$ (True)

c) $\tanh(x) = \frac{3}{5} \Rightarrow \cosh(2x) = \dfrac{1+\tanh^2 x}{1-\tanh^2 x} = \dfrac{1+9/25}{1-9/25} \neq \frac{17}{8}$ (False)

Ans: A, B

12. $\cosh(2) + \sinh(2)$

$= \frac{e^2 + e^{-2}}{2} + \frac{e^2 - e^{-2}}{2} = e^2$ or $\frac{1}{e^{-2}}$ (5)

Ans: A, C

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13 Statement I: $f(x) \cdot f(y) = f(K)$

$(\cosh x + \sinh x)(\cosh y + \sinh y) = f(K)$

$\cosh x \cdot \sinh y + \sinh x \cdot \cosh y + \sinh x \sinh y + \cosh x \cdot \sinh y = f(K)$

$\Rightarrow \cosh(x+y) + \sinh(x+y) = f(K)$

$\therefore K = x + y$ (True)

Statement II:

$x = \log\left(\frac{1}{y} + \sqrt{\frac{1}{y^2} - 1}\right)$ put $y = \operatorname{sech} x$

$\Rightarrow x = \log\left(\cosh x + \sqrt{\cosh^2 x - 1}\right)$

$\Rightarrow x = \log(\cosh x + \sinh x) = \log(e^x) = x$ (which is true)

Ans: A

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14 Statement I:

$\sinh^{-1}(2) + \sinh^{-1}(3) = \alpha \Rightarrow \log(2+\sqrt{5}) + \log(3+\sqrt{10}) = \alpha$

$\Rightarrow \log[(2+\sqrt{5})(3+\sqrt{10})] = \alpha \Rightarrow e^{\alpha} = (2+\sqrt{5})(3+\sqrt{10})$

Now $\sinh\alpha = \frac{e^{\alpha} - e^{-\alpha}}{2} = \frac{e^{\log[(2+\sqrt{5})(3+\sqrt{10})]} - e^{-\log(2+\sqrt{5})(3+\sqrt{10})}}{2}$

$= \left[(2+\sqrt{5})(3+\sqrt{10}) - \frac{1}{(2+\sqrt{5})(3+\sqrt{10})}\right]\frac{1}{2} \neq 2\sqrt{10} + 3\sqrt{5}$ (False)

Statement II: Conceptual (True)

Ans: D

15 $\sinh^{-1}x = \log(x+\sqrt{x^2+1})$

$= \log(5+\sqrt{5^2+1}) \Rightarrow x = 5$ Ans: —

16 $\sinh^{-1}(2^{3/2}) = \log(2^{3/2}+\sqrt{(2^{3/2})^2+1})$

$= \log(\sqrt{8}+\sqrt{9}) = \log(3+\sqrt{8})$ Ans: B

17. $x = \tanh^{-1}(y)$

$x = \frac{1}{2}\log\left(\frac{1+y}{1-y}\right) \Rightarrow 2x = \log_e\left(\frac{1+y}{1-y}\right)$ Ans: C

18 $\tanh^{-1}x = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right) \Rightarrow a = \frac{1}{2}$ Ans: C

19. $\cosh^{-1}(k) = \log_e(3+\sqrt{3^2-1})$

$= \log_e(3+\sqrt{3^2-1})$

$= \log_e(3+2\sqrt{2}) \Rightarrow k = 3$ Ans: 3

20 $\tanh^{-1}x = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$

$2\tanh^{-1}(\frac{1}{2}) = \log\left(\frac{1+x}{1-x}\right) \Rightarrow \frac{1+x}{1-x} =$

$= \log\left(\frac{1+1/2}{1-1/2}\right) = \log(3) = \log k \Rightarrow k = 3$

Ans: 3

2) a) $\tan\frac{x}{2}.\coth\left(\frac{x}{2}\right)=1$

$\Rightarrow \tan\frac{x}{2}=\tanh\left(\frac{x}{2}\right)$

Now $\cos x.\cosh x$

$=\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)\cosh x \Rightarrow \left(\frac{1-\tanh^2\frac{x}{2}}{1+\tanh^2\frac{x}{2}}\right)\cosh x$

$= \operatorname{sech} x.\cosh x = 1$

b) $\operatorname{sech}^2\left(\tanh^{-1}\frac{1}{2}\right)+\operatorname{cosech}^2\left(\coth^{-1}3\right) \rightarrow (1)$

Let $\tanh^{-1}\frac{1}{2}=x$ | $\coth^{-1}3=y$

$\Rightarrow \tanh x=\frac{1}{2}$ | $\Rightarrow \coth y=3$

$(1)\Rightarrow 1-\tanh^2(x)+\coth^2(y)-1$

$= 1-\left(\frac{1}{2}\right)^2+(3)^2-1 = 9-\frac{1}{4}=\frac{35}{4}$

c) $\sinh(0)=\log\left(0+\sqrt{0^2+1}\right)=0$

d) $\cosh x=\frac{5}{3} \Rightarrow \sinh x=\sqrt{\cosh^2 x-1} = \sqrt{\left(\frac{5}{3}\right)^2-1}$

$=\sqrt{\frac{16}{9}}=\frac{4}{3}$

Ans: q, s, p, r

22 a) $\cosh x = \frac{5}{2}$ (8)

$\sinh x = \sqrt{\cosh^2 x - 1} = \sqrt{\frac{25}{4} - 1} = \frac{\sqrt{21}}{2}$

Now, $\cosh(2x) = \cosh^2 x + \sinh^2 x$

$= \frac{25}{4} + \frac{21}{4} = \frac{46}{4} = \frac{23}{2}$

b) $\operatorname{sech} x = \frac{3}{5} \Rightarrow \cosh x = \frac{5}{3}$

$\Rightarrow \sinh x =$

$\operatorname{sech}^2 x + \tanh^2 x = 1 \Rightarrow \tanh x = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

$\tanh 2x = \frac{2\left(\frac{4}{5}\right)}{1 + \frac{16}{25}} = \frac{8}{5} \times \frac{25}{41} = \frac{40}{41}$

c) $\cosh^{-1} x = \log\left(x + \sqrt{x^2 - 1}\right)$

$\cosh^{-1}(1) = \log\left(1 + \sqrt{1^2 - 1}\right) = 0$

d) $\cosh^{-1}(0) = \log\left(2 + \sqrt{2^2 - 1}\right) \Rightarrow x = 2$

Ans: t, p, q, r

22 a) $\cosh x = \frac{5}{2}$ (8)

$\sinh x = \sqrt{\cosh^2 x - 1} = \sqrt{\frac{25}{4} - 1} = \frac{\sqrt{21}}{2}$

Now, $\cosh(2x) = \cosh^2 x + \sinh^2 x$

$= \frac{25}{4} + \frac{21}{4} = \frac{46}{4} = \frac{23}{2}$

b) $\operatorname{sech} x = \frac{3}{5} \Rightarrow \cosh x = \frac{5}{3}$

$\Rightarrow \sinh x =$

$\operatorname{sech}^2 x + \tanh^2 x = 1 \Rightarrow \tanh x = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

$\tanh 2x = \frac{2\left(\frac{4}{5}\right)}{1 + \frac{16}{25}} = \frac{8}{5} \times \frac{25}{41} = \frac{40}{41}$

c) $\cosh^{-1} x = \log\left(x + \sqrt{x^2 - 1}\right)$

$\cosh^{-1}(1) = \log\left(1 + \sqrt{1^2 - 1}\right) = 0$

d) $\cosh^{-1}(2) = \log\left(2 + \sqrt{2^2 - 1}\right) \Rightarrow x = 2$

Ans: t, P, Q, R

LEARNER'S TASKS

CUQ's

01 Domain of $\operatorname{cosech} x = R - \{0\}$ (or) $(-\infty, 0) \cup (0, \infty)$ Ans:

Ans: —

02 The range of $\coth x = R - [-1, 1]$ (or) $(-\infty, -1) \cup (1, \infty)$

Ans: A

03 $\cosh^4 x - \sinh^4 x = (\cosh^2 x + \sinh^2 x)(\cosh^2 x - \sinh^2 x)$

$= \cosh 2x \times 1$

$= \cosh 2x$.

Ans: B

04. $\sinh x = \frac{3}{4}$

$\cosh^2 x - \sinh^2 x = 1$

$\Rightarrow \sinh^2 x = \cosh^2 x - 1$ (or) $\cosh^2 x = 1 + \sinh^2 x$

$= 1 + \left(\frac{3}{4}\right)^2$

$= \frac{25}{16}$

$\therefore \cosh x = \frac{5}{4}$

Now, $\sinh 2x = 2 \cdot \sinh x \cdot \cosh x$

$= 2 \cdot \frac{3}{4} \cdot \frac{5}{4} = \frac{30}{16} = \frac{15}{8}$ Ans: B

05 $\sinh 2x = \frac{2\tanh x}{1 - \tanh^2 x}$

$= \frac{2\left(\frac{3}{5}\right)}{1 - \left(\frac{3}{5}\right)^2} = \frac{6}{5} \times \frac{25}{16_8} = \frac{15}{8}$ Ans: A

06 $\sinh x$, $\operatorname{cosech} x$, $\tanh x$ are odd functions Ans: D

07 Conceptual. Ans: D

08 Conceptual. Ans: D

09 Conceptual Ans: D

10 Conceptual Ans: C

JEE MAINS LEVEL QUESTIONS

01. $\cosh 2x = \frac{1 + \tanh^2 x}{1 - \tanh^2 x}$

$= \frac{1 + \left(\frac{3}{5}\right)^2}{1 - \left(\frac{3}{5}\right)^2} = \frac{17}{8}$

Ans: D

02 $\tanh(3x) = \dfrac{3\tanh x + \tanh^3 x}{1 + 3\tanh^2 x}$ (10)

$= \dfrac{3\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^3}{1 + 3\left(\frac{1}{2}\right)^2} = \dfrac{13}{14}$

Ans: A

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03. $x = \log_e\left[\cot\left(\frac{\pi}{4} + \theta\right)\right]$

$\Rightarrow e^x = \cot\left(\frac{\pi}{4} + \theta\right)$

Let $\theta = 15^\circ$.

$\therefore e^x = \cot(45^\circ + 15^\circ) = \cot 60^\circ = \frac{1}{\sqrt{3}}$

$\therefore e^{-x} = \sqrt{3}$

Now $\sinh x = \dfrac{\frac{1}{\sqrt{3}} - \sqrt{3}}{2} = \dfrac{-1}{\sqrt{3}} = -\tan 2\times 15^\circ$

$= -\tan 2\theta$.

Ans: C

NOTE: This is one ~~way~~ method of solving the problem. you can approach the solution by expanding $\cot\left(\frac{\pi}{4} + \theta\right)$.

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04 $\dfrac{\tanh x}{\operatorname{sech} x - 1} + \dfrac{\tanh x}{\operatorname{sech} x + 1}$

$= \dfrac{\operatorname{sech} x \tanh x + \tanh x + \operatorname{sech} x \tanh x - \tanh x}{\operatorname{sech}^2 x - 1}$

$= \dfrac{2\operatorname{sech} x \cdot \tanh x}{-\tanh^2 x} = -2\operatorname{cosech} x$

Ans: D

05 $\sinh^{-1}(K) = \log\left(K + \sqrt{K^2+1}\right)$ (11)

$= \log\left(2 + \sqrt{2^2+1}\right) \quad \because K = 2$

Ans: D

06. $\operatorname{sech}^{-1}(x) = \log\left(\frac{1+\sqrt{1-x^2}}{x}\right)$

$\operatorname{sech}^{-1}\left(\frac{1}{3}\right) = \log\left(\frac{1+\sqrt{1-\left(\frac{1}{3}\right)^2}}{1/3}\right)$

$= \log(3+\sqrt{8})$

Ans: A

~~07~~. $\tanh^{-1} x = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$

$\Rightarrow \log\left(\frac{1+x}{1-x}\right) = 2\tanh^{-1} x.$

Given $x > 0$, let $x = \frac{1}{2}$.

$\tanh^{-1}\left(\frac{x^2-1}{\quad}\right)$

07 $\tanh^{-1} x = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$

$\tanh^{-1}\left(\frac{x^2-1}{x^2+1}\right) = \frac{1}{2}\log\left(\frac{1+\left(\frac{x^2-1}{x^2+1}\right)}{1-\left(\frac{x^2-1}{x^2+1}\right)}\right)$

$= \frac{1}{2}\log\left(\frac{2x^2}{x}\right) = \log_e x$

Ans: B

08 $$\frac{\sinh(2x)}{1+\cosh(2x)} = \frac{2\sinh x \cdot \cosh x}{1+\cosh^2 x + \sinh^2 x}$$

$$= \frac{2\sinh x \cdot \cosh x}{2\cosh^2 x}$$

$$= \tanh x$$

Ans: A

09. $\tanh^2 x = \tan^2\theta$

We know $\cosh(2x) = \frac{1+\tanh^2 x}{1-\tanh^2 x}$

$$= \frac{1+\tan^2\theta}{1-\tan^2\theta} = \sec 2\theta$$

Ans: B

10 $$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$= \frac{\tan\alpha \cdot \tanh\beta + \cot\alpha \cdot \tanh\beta}{1 - (\tan\alpha \cdot \tanh\beta)(\cot\alpha \cdot \tanh\beta)}$$

$$= \frac{\tanh\beta\,(\tan\alpha + \cot\alpha)}{1 - \tanh^2\beta}$$

$$= \left(\frac{\tanh\beta}{1-\tanh^2\beta}\right) \times \left(\frac{1+\tan^2\alpha}{\tan\alpha}\right)$$

$$= \left(\frac{2\tanh\beta}{1-\tanh^2\beta}\right) \times \left(\frac{1+\tan^2\alpha}{2\tan\alpha}\right)$$

$$= \sinh 2\beta \times \operatorname{cosec} 2\alpha$$

Ans: B

11. $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \infty$ (13)

$= \sum_{n=0}^{\infty} \frac{x^n}{n!}$

Ans: A, B

12. $\sinh x = \frac{e^x - e^{-x}}{2}$

$= \frac{x}{1!} + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots \infty$

Ans: A, B

13. Statement I:

$$\frac{\sinh(x+y) + \sinh(x-y)}{\cosh(x+y) - \cosh(x-y)}$$

$$= \frac{2\sinh x \cdot \cosh y}{2\sinh x \cdot \sinh x} = \coth y \text{ (True)}$$

Statement II: Conceptual (True)

Ans: A

14. Statement I:

$$\operatorname{sech}^{-1}\left(\frac{1}{\sqrt{2}}\right) + \operatorname{cosech}^{-1}(-1)$$

$$= \log\left(\frac{1 + \sqrt{1 - \frac{1}{2}}}{\frac{1}{\sqrt{2}}}\right) + \log\left(\frac{1 - \sqrt{1 - (-1)^2}}{-1}\right)$$

$$= \log 1$$

$$= 0$$

Statement II: Conceptual (True)

Ans: A

15. $\cosh^2 x - \sinh^2 x = 1$

$\Rightarrow \cosh^2 x - (2)^2 = 1$

$\Rightarrow \cosh x = \sqrt{10}$

Ans: D

16. $\cosh^2 x - \sinh^2 x = 1$

$\Rightarrow (2)^2 - \sinh^2 x = 1$

$\Rightarrow \sinh x = \sqrt{3}$

$\Rightarrow x = \sinh^{-1}(\sqrt{3})$

$\Rightarrow x = \log\left(\sqrt{3} + \sqrt{(\sqrt{3})^2 + 1}\right)$

$\Rightarrow x = \log(2 + \sqrt{3})$

Ans: A

17. $\cosh^2 x - \sinh^2 x = 1$

$\Rightarrow \cosh^2 x - (2)^2 = 1$

$\Rightarrow \cosh x = \sqrt{5}$

Ans: B

18. $\cosh^2 x - \sinh^2 x = 1$

$\Rightarrow (3)^2 - \sinh^2 x = 1$

$\Rightarrow \sinh x = \sqrt{8} = 2\sqrt{2}$

Ans: B

19. $\cosh^2 x - \sinh^2 x = 1$

$\cosh^2 x - (1)^2 = 1$

$\cosh x = \sqrt{2}$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$= 2 + 1$

$= 3$

Ans: 3

20 $\cosh x = 1$

$\Rightarrow x = \cosh^{-1}(1)$

$= \log(1+\sqrt{1^2-1})$

$= \log 1 = 0$

Ans: 0

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21. 1) $\operatorname{cosech} x - \operatorname{sech} x$

$= \frac{2}{e^x - e^{-x}} - \frac{2}{e^x + e^{-x}} = \frac{4e^x}{e^{4x}-1}$

$= 2e^{-x}\left(\frac{2\cdot e^{2x}}{e^{4x}-1}\right)$

$= 2e^{-x}\cdot \operatorname{cosech} 2x$

2) $\frac{1+\tanh x}{1-\tanh x} = \frac{1+\left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)}{1-\left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)} = e^{2x}$

$= \cosh 2x + \sinh 2x$

3) $\log_e\left(\cot\frac{\theta}{2}\right) =$

$\tanh^{-1}(\cos\theta) = \frac{1}{2}\log\left(\frac{1+\cos\theta}{1-\cos\theta}\right) = \frac{1}{2}\log\left(\frac{2\cos^2\theta/2}{2\sin^2\theta/2}\right)$

$= \log\left(\cot\frac{\theta}{2}\right)$

4) $\log\left[(x-1)+\sqrt{(x-1)^2-1}\right] = \cosh^{-1}(x-1)$

Ans: r, t, p, v

22 1) Domain of $\operatorname{sech} x = R$ (20)

2) Domain of $\coth^{-1} x = R - [-1, 1]$

3) Domain of $\operatorname{cosh}^{-1} x = [1, \infty)$

4) Domain of $\coth x = R - \{0\}$

Ans: C, A, D, B

$\Rightarrow$ THE END $\in$