WS-3 Types of vector

Task

① Given vector is $\vec{A} = 6\hat{i} + 8\hat{j} + 10\hat{k}$

unit vector Parallel to $\vec{A} = \dfrac{\vec{A}}{|\vec{A}|}$

$$\Rightarrow \hat{A} = \frac{6\hat{i} + 8\hat{j} + 10\hat{k}}{\sqrt{6^2 + 8^2 + 10^2}}$$

$$\Rightarrow \hat{A} = \frac{6\hat{i} + 8\hat{j} + 10\hat{k}}{\sqrt{36 + 64 + 100}}$$

$$\Rightarrow \hat{A} = \frac{2(3\hat{i} + 4\hat{j} + 5\hat{k})}{\sqrt{200}}$$

$$\Rightarrow \hat{A} = \frac{2(3\hat{i} + 4\hat{j} + 5\hat{k})}{10\sqrt{2}}$$

$$\therefore \hat{A} = \frac{3\hat{i} + 4\hat{j} + 5\hat{k}}{5\sqrt{2}}$$

② let the Given position vector $\vec{R} = -6\hat{i} - 4\hat{j} - 12\hat{k}$

$\therefore$ unit vector Parallel to $\vec{R} = \dfrac{\vec{R}}{|\vec{R}|}$

$$\Rightarrow \vec{R} = \frac{-6\hat{i} - 4\hat{j} - 12\hat{k}}{\sqrt{(-6)^2 + (-4)^2 + (-12)^2}}$$

$$\Rightarrow \vec{R} = \frac{-2(3\hat{i} + 2\hat{j} + 6\hat{k})}{\sqrt{36 + 16 + 144}} = \frac{-2(3\hat{i} + 2\hat{j} + 6\hat{k})}{\sqrt{196}}$$

$$\vec{R} = \pm\frac{2(3\hat{i} + 2\hat{j} + 6\hat{k})}{14} \qquad (\sqrt{196} = \pm 14)$$

$$\vec{R} = \pm\frac{1}{7}(3\hat{i} + 2\hat{j} + 6\hat{k})$$

(3) If $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = 7\hat{i} + 24\hat{j}$

let C be the vector whose magnitude is same as that of $\vec{B}$ $\quad C = |\vec{B}|$

and als $\vec{C}$ is Parallel to $\vec{A}$ (i.e) $\vec{C} = \hat{A}$ $[|\hat{A}| = 1]$

we can write it as $\vec{C} = |\vec{B}| \times \hat{A}$

$$\Rightarrow \vec{C} = |\vec{B}| \times \frac{\vec{A}}{|\vec{A}|} \rightarrow (1)$$

$|\vec{B}| = \sqrt{7^2 + 24^2}$

$= \sqrt{49 + 576}$

$= \sqrt{625}$

$= 25$

$\therefore \vec{C} = \frac{25}{5}(3\hat{i} + 4\hat{j})$

$\vec{C} = 5(3\hat{i} + 4\hat{j})$

$\vec{C} = 15\hat{i} + 20\hat{j}$

$|\vec{A}| = \sqrt{3^2 + 4^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$

$= 5$

(4) Given force $\vec{F} = 6\hat{i} - 8\hat{j} + 10\hat{k}$; $a = 1\ m/s^2$

$$|\vec{F}| = \sqrt{6^2 + (-8)^2 + 10^2} = \sqrt{36 + 64 + 100}$$

$$= \sqrt{200} = 10\sqrt{2}$$

$\therefore$ The Mass of the body $m = \frac{|\vec{F}|}{a} = 10\sqrt{2}$ kg

(5) The velocity of the body $\vec{u} = 2\hat{i} - 3\hat{j} + 11\hat{k}$ m/s

$\vec{a} = 10\hat{i} + 10\hat{j} + 10\hat{k}$ m/s^2

After $t = 0.25$ sec the velocity of the body is

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\Rightarrow \vec{v} = 2\hat{i} - 3\hat{j} + 11\hat{k} + (10\hat{i} + 10\hat{j} + 10\hat{k})0.25$$

$$= 2\hat{i} - 3\hat{j} + 11\hat{k} + 2.5\hat{i} + 2.5\hat{j} + 2.5\hat{k}$$

$$|\vec{v}| = 4.5\hat{i} - 0.5\hat{j} + 13.5\hat{k}$$

$$|\vec{V}| = \sqrt{(4.5)^2 + (-0.5)^2 + (13.5)^2}$$

$$= \sqrt{\left(\frac{9}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 + \left(\frac{27}{2}\right)^2}$$

$$= \sqrt{\left(\frac{1}{2}\right)^2 [9^2 + (-1)^2 + (27)^2]} = \frac{1}{2}\sqrt{81+1+729}$$

$$|\vec{V}| = \frac{1}{2}\sqrt{811} \text{ m/s}$$

(6) Given two vectors are

let $\vec{A} = \underset{l_1}{\hat{i}} + 2\hat{j} + 3\hat{k}$ & $\vec{B} = \underset{l_2}{3\hat{i}} + \underset{m_2}{m\hat{j}} - \underset{n_2}{9\hat{k}}$

If any two vectors are parallel then the condition is $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$

$$\Rightarrow \frac{1}{3} = \frac{2}{m} = \frac{-3}{-9}$$

$\therefore$ either from $\frac{1}{3} = \frac{2}{m}$ (or) $\frac{2}{m} = \frac{+3}{+9}$

$\Rightarrow m = 6$ $\Rightarrow \frac{2}{m} = \frac{1}{3} \Rightarrow m = 6$

(7) let the two vectors are given by

$\vec{a} = 2\hat{i} + \hat{j} - 3\hat{k}$ & $\vec{b} = 5\hat{i} + 3\hat{j} - 2\hat{k}$

Given condition is $3\vec{a} + 2\vec{b} - \vec{c} = 0$

$\therefore 3(2\hat{i} + \hat{j} - 3\hat{k}) + 2(5\hat{i} + 3\hat{j} - 2\hat{k}) - \vec{c} = 0$

$\Rightarrow 6\hat{i} + 3\hat{j} - 9\hat{k} + 10\hat{i} + 6\hat{j} - 4\hat{k} - \vec{c} = 0$

$\Rightarrow 16\hat{i} + 9\hat{j} - 13\hat{k} - \vec{c} = 0$

$\Rightarrow \vec{c} = 16\hat{i} + 9\hat{j} - 13\hat{k}$

(8) Given three forces are

$\vec{F_1} = 3\hat{i} - 4\hat{j} + 2\hat{k}$; $\vec{F_2} = 2\hat{i} + 3\hat{j} - \hat{k}$; $\vec{F_3} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

∴ The resultant force is $\vec{F} = \vec{F_1} + \vec{F_2} + \vec{F_3}$

$\therefore \vec{F} = 3\hat{i} - 4\hat{j} + 2\hat{k} + 2\hat{i} + 3\hat{j} - \hat{k} + 2\hat{i} + 4\hat{j} - 5\hat{k}$

$\Rightarrow \vec{F} = 7\hat{i} + 3\hat{j} - 4\hat{k}$

(9) Given $\vec{A} = 6\hat{i} - 2\hat{k}$ Given relation is $\vec{A} - 2\vec{B} = -3(\vec{A} + \vec{B})$

$\Rightarrow \vec{A} - 2\vec{B} = -3\vec{A} - 3\vec{B}$

$\Rightarrow 3\vec{B} - 2\vec{B} = -3\vec{A} - \vec{A}$

$\Rightarrow \vec{B} = -4\vec{A} = -4[6\hat{i} - 2\hat{k}]$

$\vec{B} = -24\hat{i} + 8\hat{k}$

(10) Given $\vec{A} = 3\hat{i} + 5\hat{j} - 2\hat{k}$ and $\vec{B} = -3\hat{j} + 6\hat{k}$

Given condition is $2\vec{A} + 7\vec{B} + 4\vec{C} = 0$

$\therefore 2(3\hat{i} + 5\hat{j} - 2\hat{k}) + 7(-3\hat{j} + 6\hat{k}) + 4\vec{C} = 0$

$\Rightarrow 6\hat{i} + 10\hat{j} - 4\hat{k} - 21\hat{j} + 42\hat{k} + 4\vec{C} = 0$

$\Rightarrow 6\hat{i} - 11\hat{j} + 38\hat{k} + 4\vec{C} = 0$

$\Rightarrow 4\vec{C} = -6\hat{i} + 11\hat{j} - 38\hat{k}$

$\Rightarrow \vec{C} = -\frac{6}{4}\hat{i} + \frac{11}{4}\hat{j} - \frac{38}{4}\hat{k}$

$\vec{C} = -1.5\hat{i} + 2.75\hat{j} - 9.5\hat{k}$

LTask CUQ's

(9) Given $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ then the magnitude of $\vec{A} = |\vec{A}| = \sqrt{3^2+4^2+5^2} = \sqrt{9+16+25} = \sqrt{50}$

$$|\vec{A}| = 5\sqrt{2}$$

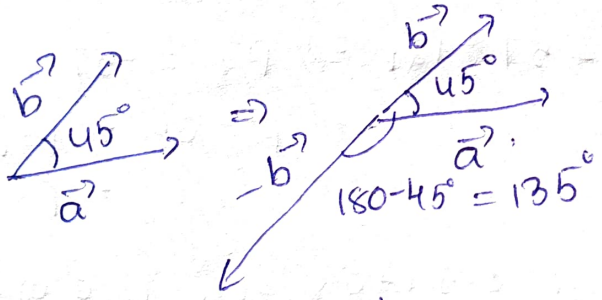
(12) Given $\vec{Q} = 10\hat{i} + 8\hat{j} + 6\hat{k}$

magnitude in yz-plane $= \sqrt{y^2+z^2}$

$$= \sqrt{8^2+6^2} = \sqrt{64+36}$$

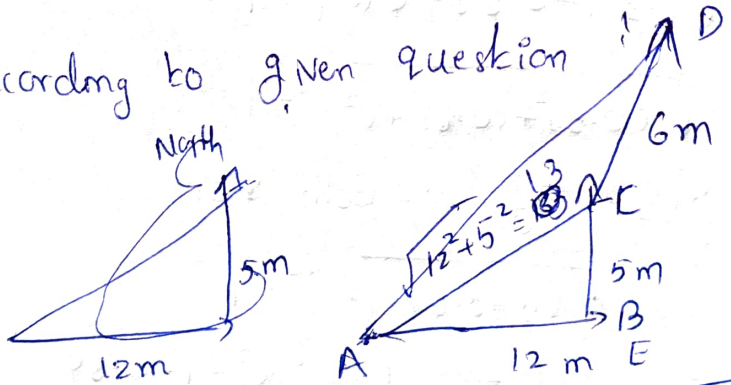
$$= \sqrt{100} = 10$$

(13) The angle between $\vec{a}$ and $\vec{b}$ is 45°



SAQ's

(1) According to given question



The displacement of a particle $= \sqrt{12^2+5^2+6^2}$

$$= \sqrt{144+25+36}$$

$$= \sqrt{205}$$

$$= 14.32 \text{ m}$$

(2) Given vector is $\frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$

∴ The magnitude is $= \sqrt{\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2}$

$= \sqrt{\frac{4}{9} + \frac{1}{9} + \frac{4}{9}} = \sqrt{\frac{9}{9}} = 1$

(3) Given Vector let it be $\vec{A} = 5\hat{i} + p\hat{j} + 4\sqrt{2}\hat{k}$

Given that $|\vec{A}| = 11$.

$\Rightarrow \sqrt{5^2 + p^2 + (4\sqrt{2})^2} = 11$ By squaring on both sides

$\Rightarrow \left[\sqrt{25 + p^2 + 32}\right]^2 = 11^2$

$\Rightarrow p^2 + 57 = 121 \Rightarrow p^2 = 121 - 57$

$\Rightarrow p^2 = 64 \Rightarrow p = \sqrt{64}$

$\Rightarrow p = \pm 8$

(4) Given vector $0.5\hat{i} + 0.8\hat{j} + c\hat{k}$ is a unit vector

$\Rightarrow |0.5\hat{i} + 0.8\hat{j} + c\hat{k}| = 1$

$\Rightarrow \sqrt{(0.5)^2 + (0.8)^2 + c^2} = 1$

By squaring on both sides.

$\Rightarrow \left[\sqrt{0.25 + 0.64 + c^2}\right]^2 = 1$

$\Rightarrow 0.89 + c^2 = 1 \Rightarrow c^2 = 1 - 0.89$

$\Rightarrow c^2 = 0.11$

$\Rightarrow c = \sqrt{0.11}$

(5) Given vector is $3\hat{i} + \hat{j} + 2\hat{k}$

It's length in XY plane $= \sqrt{x^2 + y^2}$

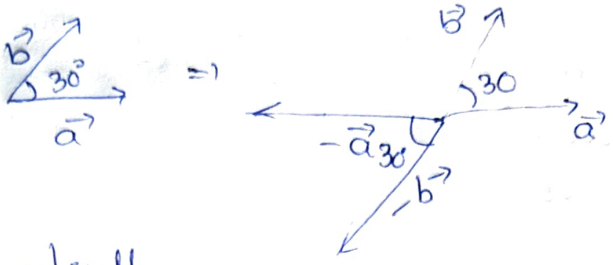
$= \sqrt{3^2 + 1}$

$= \sqrt{9 + 1}$

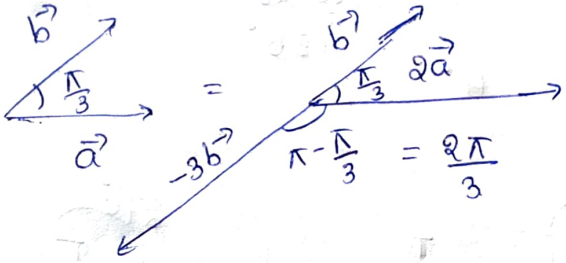
$= \sqrt{10}$

(6) Given angle between $\vec{a}$ and $\vec{b}$ is 30°

(i.e)



(7) Graphically



(8) Given $|x\hat{i} - 3\hat{j} + 5\hat{k}| = \sqrt{98}$

i.e $\sqrt{x^2 + (-3)^2 + 5^2} = \sqrt{98}$

By doing squaring on both sides we get

$\Rightarrow \left(\sqrt{x^2 + 9 + 25}\right)^2 = \left(\sqrt{98}\right)^2$

$\Rightarrow x^2 + 34 = 98$

$\Rightarrow x^2 = 64 \Rightarrow x = \pm 8$

(9) (a) $|3\hat{i} + 4\hat{j}| = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$

(b) $|2\hat{i} + 4\hat{j} + 6\hat{k}| = \sqrt{2^2 + 4^2 + 6^2} = \sqrt{4+16+36} = \sqrt{56}$

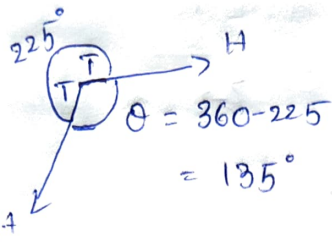
(c) $|2\hat{i} + 2\hat{j} + 2\hat{k}| = \sqrt{2^2 + 2^2 + 2^2} = \sqrt{4+4+4} = \sqrt{12}$

(10) The vector is $3\hat{i} + 4\hat{j} + 5\hat{k}$

The unit vector $= \dfrac{\text{Vector}}{\text{magnitude}} = \dfrac{3\hat{i} + 4\hat{j} + 5\hat{k}}{\sqrt{3^2 + 4^2 + 5^2}}$

$= \dfrac{3\hat{i} + 4\hat{j} + 5\hat{k}}{\sqrt{9+16+25}} = \dfrac{3\hat{i} + 4\hat{j} + 5\hat{k}}{5\sqrt{2}}$

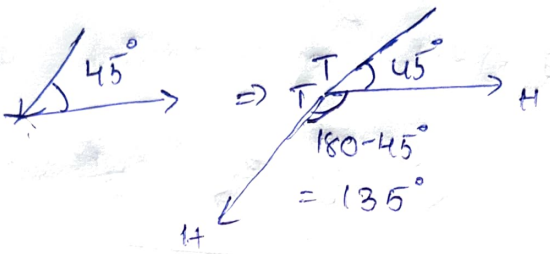
(13) Use Head-Head (or) Tail-Tail method



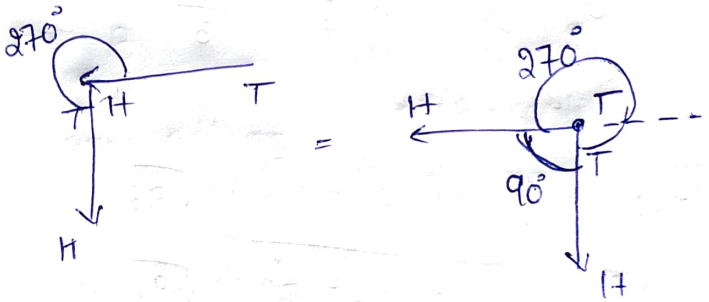
$\theta = 360 - 225$

$= 135°$

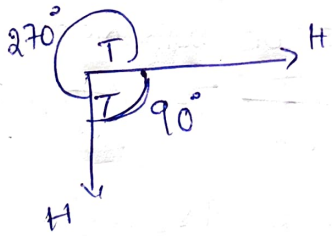
(14)



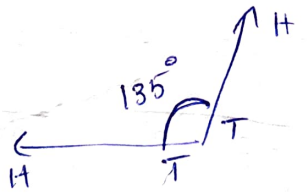
(15)



(16)

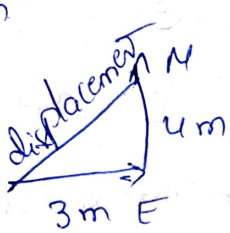


(17)



(18) According to given question

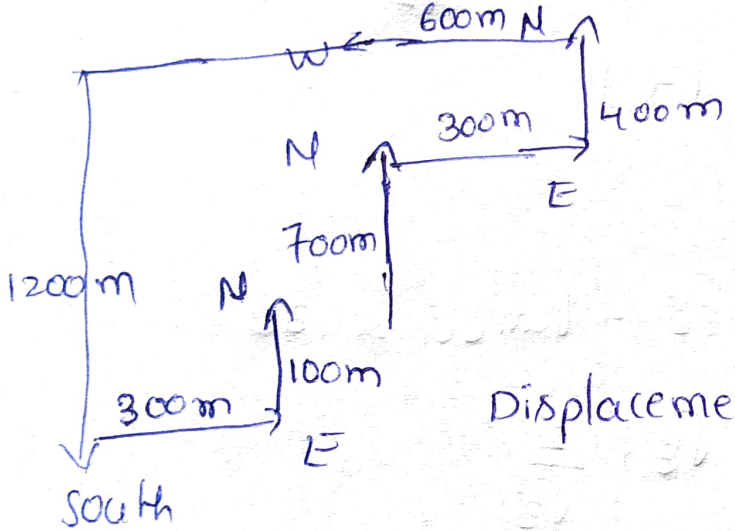
$\therefore$ displacement $= 3\hat{i} + 4\hat{j}$



$= \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5\,m$

East is represented by a unit vector $\hat{i}$ and North by $\hat{j}$

(19) Acc to given question



$\hat{i}$ → unit vector along East

$\hat{j}$ → " " " North

$-\hat{i}$ → unit vector along west

$-\hat{j}$ → unit vector along south

Displacement = $700\hat{j} + 300\hat{i} + 400\hat{j} - 600\hat{i}$

$- 1200\hat{j} + 300\hat{i} + 100\hat{j}$

$\vec{S} = -400\hat{j} + 400\hat{j} \Rightarrow \vec{S} = 0$

8th class