Exercise - 4.4 :-

Question-1 Find the nature of the roots of the following quadratic equatics, if the real roots exist, find them:-

i, $2x^2 - 3x + 5 = 0$ compare with $ax^2+bx+c=0$

$a=2, b=-3, c=5$

$b^2 - 4ac = (-3)^2 - 4(2)(5)$

$= 9 - 40 = -31 < 0$

So, no real roots ~~for~~ this eqn.

ii, $3x^2 - 4\sqrt{3}x + 4 = 0$ compare with $ax^2+bx+c=0$

$a=3, b=-4\sqrt{3}, c=4$

$b^2 - 4ac \Rightarrow (-4\sqrt{3})^2 - 4(3)(4)$

$\Rightarrow 48 - 48 = 0$

When $b^2 - 4ac = 0$, equation will have equal roots.

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \Rightarrow \frac{-b \pm \sqrt{0}}{2a}$$

$$x = -b/2a, -b/2a$$

$$x = \frac{-(-4\sqrt{3})}{2(3)} \Rightarrow \frac{4\sqrt{3}}{6} \Rightarrow \frac{2\sqrt{3}}{3\sqrt{3}}$$

$x = 2/\sqrt{3}, 2/\sqrt{3}$ (Equal roots) iii, $2x^2 - 6x + 3 = 0$

Here, if you compare with $ax^2+bx+c=0$

$a=2, b=-6, c=3$.

$b^2-4ac = (-6)^2 - 4(2)(3)$

$\Rightarrow 36 - 24 \Rightarrow 12 > 0$ [So, real roots are possible]

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{12}}{2(2)} \Rightarrow \frac{6 \pm 2\sqrt{3}}{4}$$

$$\Rightarrow \frac{3 \pm \sqrt{3}}{2}$$

So, $x = \frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$ [two real solutions]

Question-2 Find the values of K for each of the following quadratic equations, so that they have two equal roots.

i, $2x^2+Kx+3=0$.

equal roots mean that $b^2-4ac=0$.

Here, $a=2$, $b=K, c=3$.

$b^2-4ac \Rightarrow K^2-4(2)(3) \Rightarrow K^2-24$

$K^2-24=0$

$K^2=24$

$K=\sqrt{24}$

$\boxed{K = \pm 2\sqrt{6}}$

ii) $kx(x-2)+6=0$

$kx^2-2kx+6=0$

Here $a=k$, $b=-2k$, $c=6$.

$b^2-4ac=0$.

$(-2k)^2-4(k)(6)=0$

$4k^2-24k=0 \Rightarrow 4k(k-6)=0$

$\cancel{4k^2=24k}$ Here $4k=0$ or $k-6=0$.

$\cancel{k^2=6k}$ $k=0$ or $k=6$

$\cancel{k=6}$ If $k=0$, equation will not have x^2 term. and x term.

So, $\boxed{k=6}$.

Question-3:- Is it possible to design a rectangular mango grove whose length is twice its breadth and the area is $800\ m^2$? If ~~it~~ so, find its length and breadth.

A) let breadth be x m, length $=2x$ m.

Area = length × breadth $\Rightarrow (2x)(x)=2x^2$

given as $800\ m^2$.

$2x^2=800$

$x^2=400$ [length/breadth cannot be negative]

$x=\pm 20$

So, $\boxed{x=20}$

$\therefore$ Breadth, $x = 20$ m.

length, $2x = 2(20) = 40$ m.

(Yes it is possible to design in that way).

Question-4:-

Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

~~A) Assume age of 1st friend as x and 2nd friend as y~~

~~4 yrs ago, they will be~~

A) Assume age of 1st friend as x years

age of 2nd friend $= 20 - x$ years.

4 years ago, they will be $x-4$ and $16-x$ resp.

given, $(x-4)(16-x) = 48$

$\Rightarrow 16x - x^2 - 64 + 4x = 48$

$\Rightarrow 20x - x^2 - 64 - 48 = 0$

$\Rightarrow 20x - x^2 - 112 = 0$

$\Rightarrow -(x^2 - 20x + 112) = 0$

$\Rightarrow x^2 - 20x + 112 = 0$. Here $a=1, b=-20$, $c=112$

Here calculate $b^2 - 4ac \Rightarrow$

$(-20)^2 - 4(1)(112)$

$\Rightarrow 400 - 448 \Rightarrow -48 < 0$.

So, no real solutions.

So, this situation is not possible.

Question-5

Is it possible to design a rectangular park of perimeter 80 m and area 400 m². Find its length and breadth.

A) Assume length as x and breadth as y.

perimeter $\Rightarrow 2(x+y) = 80$ m

$\cancel{x+y = 80\,m}$

$x+y = 40$

$y = (40-x)$ m

Area = length × breadth

$\Rightarrow (x) \times (40-x) = 400$

$\Rightarrow 40x - x^2 = 400$

$\Rightarrow -x^2 + 40x - 400 = 0$

$\Rightarrow x^2 - 40x + 400 = 0$

$\Rightarrow x^2 - 20x - 20x + 400 = 0$

$\Rightarrow x(x-20) - 20(x-20) = 0$

$\Rightarrow (x-20)(x-20) = 0$

$\Rightarrow x = 20, 20$

If $x = 20$, length = 20 m

Breadth = $40 - x = 40 - 20$

$= 20$ m.

So, length and breadth are 20 m and 20 m, respectively.