Class: IX

TRIGONOMETRIC EQUATIONS–II

WORKSHEET–6

TEACHING TASK

01. We know,

$\tan x + \tan(120^\circ + x) + \tan(x - 120^\circ) = 3\tan 3x$

$\therefore \quad 3\tan 3x = 0$

$\Rightarrow \tan 3x = 0$

$\Rightarrow 3x = n\pi, \quad n \in \mathbb{Z}$

$\Rightarrow x = \frac{n\pi}{3}, \quad \forall n \in \mathbb{Z}$

Ans: A

02 We know

$\tan x \cdot \tan(120^\circ + x) \cdot \tan(120^\circ - x) = \tan 3x$

$\therefore \quad \tan 3x = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan 3x = \tan\frac{\pi}{6}$

$\Rightarrow 3x = n\pi + \frac{\pi}{6}$

$\Rightarrow x = \frac{n\pi}{3} + \frac{\pi}{18}$

$\Rightarrow x = (6n+1)\frac{\pi}{18}, \quad n \in \mathbb{Z}$

Ans: A

03 We know, $\cos^3\alpha + \cos^3(120^\circ+\alpha) + \cos^3(120^\circ-\alpha)$

$$= \frac{3}{4}\cos 3\alpha$$ (2)

$$\therefore \frac{3}{4}\cos 3\alpha = \frac{3\sqrt{3}}{4}$$

$$\Rightarrow \cos 3\alpha = \sqrt{3}$$

Since $-1 \le \cos 3\alpha \le 1$

This equation does not ~~any~~ have any solution.

Ans: A

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04 We know $\cos\theta \cdot \cos(120^\circ+\theta) \cdot \cos(120^\circ-\theta) = \frac{1}{4}\cos 3\theta$

~~Given~~

$$\Rightarrow 4\cos\theta \cdot \cos(120^\circ+\theta) \cdot \cos(120^\circ-\theta) = \cos 3\theta$$

$$\therefore \cos 3\theta = \frac{1}{2}$$

$$\Rightarrow \cos 3\theta = \cos\frac{\pi}{3}$$

$$\Rightarrow 3\theta = 2n\pi \pm \frac{\pi}{3}$$

$$\Rightarrow \theta = \frac{2n\pi}{3} \pm \frac{\pi}{9}, \quad n \in \mathbb{Z}$$

Ans: C

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05 Given $x^2 + 4 + 3\sin(ax+b) - 2x = 0$

$$\Rightarrow x^2 + 4 - 2x = -3\sin(ax+b)$$

$$\Rightarrow (x-1)^2 + 3 = -3\sin(ax+b)$$

(3)

clearly $(x-1)^2+3 \geq 3$

$\therefore -3\sin(ax+b) \geq 3$

$\Rightarrow \sin(ax+b) \leq -1$

$\Rightarrow \sin(ax+b) = -1$

$\Rightarrow ax+b = \frac{3\pi}{2}$ or $\frac{7\pi}{2}$

But given $(a,b) \in [0, 2\pi]$

$\therefore$ let $x=1$ $a+b = \frac{3\pi}{2}$

Ans: B

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06 Given $\cos x + \cos y = 1 \rightarrow (1)$

$\cos x \cdot \cos y = \frac{1}{4}$

$\Rightarrow \cos x(1-\cos x) = \frac{1}{4}$

$\Rightarrow 4\cos x - 4\cos^2 x = 1$

$\Rightarrow 4\cos^2 x - 4\cos x + 1 = 0$

$\Rightarrow (2\cos x - 1)^2 = 0$

$\Rightarrow 2\cos x - 1 = 0$

$$\Rightarrow \cos x = \frac{1}{2}$$

Now From (1), $\cos x + \cos y = 1$

$$\Rightarrow \frac{1}{2} + \cos y = 1$$

$$\Rightarrow \cos y = \frac{1}{2}$$

Consider $\cos x = \frac{1}{2}$

$$\Rightarrow \cos x = \cos\frac{\pi}{3}$$

$$\Rightarrow x = 2n\pi \pm \frac{\pi}{3}, \quad n \in \mathbb{Z}$$

~~Cos~~ similarly $y = 2m\pi \pm \frac{\pi}{3}, \quad m \in \mathbb{Z}$

Ans: B

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07. Given $\tan A$, $\tan B$ are the roots of

$$x^2 - bx + c = 0.$$

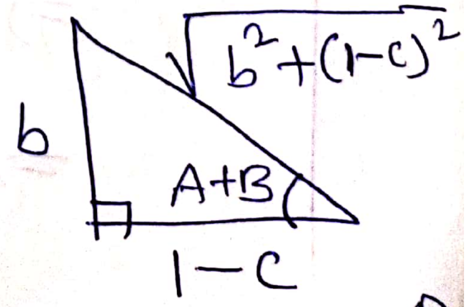
We have $\tan A + \tan B = b$

$$\tan A \cdot \tan B = c$$

Now $$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$= \frac{b}{1-c}$$

$$\therefore \sin(A+B) = \frac{b}{\sqrt{b^2 + (1-c)^2}}$$



$$\Rightarrow \sin^2(A+B) = \frac{b^2}{b^2 + (1-c)^2}$$

Ans: D

08 Given $\cos 6\theta + \cos 4\theta + \cos 2\theta + 1 = 0$

$$\Rightarrow 2\cos\left(\frac{6\theta+4\theta}{2}\right)\cdot\cos\left(\frac{6\theta-4\theta}{2}\right) + 2\cos^2\theta = 0$$

$$\Rightarrow 2\cos 5\theta \cdot \cos\theta + 2\cos^2\theta = 0$$

$$\Rightarrow 2\cos\theta\left[\cos 5\theta + \cos\theta\right] = 0$$

$$\Rightarrow 2\cos\theta \cdot \cos 3\theta \cdot \cos 2\theta = 0$$

$$\Rightarrow \cos\theta = 0 \text{ or } \cos 3\theta = 0 \text{ or } \cos 2\theta = 0.$$

Consider

$$\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$$

Consider, $\cos 3\theta = 0 \Rightarrow 3\theta = (2n+1)\frac{\pi}{2}$

$$\Rightarrow 3\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

$$\Rightarrow \theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6},$$

Consider $\cos 2\theta = 0 \Rightarrow 2\theta = (2n+1)\frac{\pi}{2}$

$$\Rightarrow 2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

$$\Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}$$

Since $0 \le \theta \le \pi$

$\theta \in \left\{ \frac{\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{\pi}{6}, \frac{5\pi}{6} \right\}$ Ans: B

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09. Given $y = \dfrac{\cos^2\theta - 1}{\cos^2\theta + \cos\theta}$

$\Rightarrow y = \dfrac{(\cos\theta + 1)(\cos\theta - 1)}{\cos\theta\,(\cos\theta + 1)}$

$\Rightarrow y = \dfrac{\cos\theta - 1}{\cos\theta}$ Since $\cos\theta + 1 \neq 0$

$\Rightarrow y = 1 - \sec\theta$

We know $\sec\theta \le -1$ and $\sec\theta \ge 1$

$\Rightarrow 1 - \sec\theta \ge 2$ and $1 - \sec\theta \le 0$

$\Rightarrow y \ge 2$ and $y \le 0$

Ans: B

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10. ~~Consider~~

Given $e^{\frac{-1}{\sqrt{2}}}\left(e^{\sin x} + e^{\cos x}\right) = 2$

Let $x = \frac{\pi}{4}$

$LHS = e^{\frac{-1}{\sqrt{2}}}\left(e^{\sin\frac{\pi}{4}} + e^{\cos\frac{\pi}{4}}\right)$

$$= e^{-\frac{1}{\sqrt{2}}}\left(e^{\frac{1}{\sqrt{2}}} + e^{\frac{1}{\sqrt{2}}}\right)$$

$$= e^{-\frac{1}{\sqrt{2}}}\left(2 \cdot e^{\frac{1}{\sqrt{2}}}\right)$$

$$= 2 \cdot e^{0}$$

$$= 2 = RHS$$

Hence $x = (4m+1)\frac{\pi}{4}$, Ans: B

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11. Given $\sqrt{2}\tan^2 x - \sqrt{10}\tan x + \sqrt{2} = 0$

$$\Rightarrow \tan x = \frac{\sqrt{10} \pm \sqrt{10 - 8}}{2\sqrt{2}}$$

$$= \frac{\sqrt{10} \pm \sqrt{2}}{2\sqrt{2}}$$

$$= \frac{\sqrt{2}\cdot\sqrt{5} \pm \sqrt{2}}{2\sqrt{2}}$$

$$= \frac{\sqrt{5} \pm 1}{2}$$ ~~since $0 < x < \frac{\pi}{2}$~~

12. Given $\sin\theta = \cos\phi$

$\Rightarrow \sin\theta = \sin\left(\frac{\pi}{2} - \phi\right)$

$\Rightarrow \theta = n\pi + (-1)^n\left(\frac{\pi}{2} - \phi\right)$

let $n = 0 \Rightarrow \theta = \frac{\pi}{2} - \phi$

$\Rightarrow \theta + \phi = \frac{\pi}{2}$

let $n = 1 \Rightarrow \theta = \pi - \left(\frac{\pi}{2} - \phi\right)$

$\Rightarrow \theta = \frac{\pi}{2} + \phi$

$\Rightarrow \theta - \phi = \frac{\pi}{2}$

$\therefore \theta \pm \phi = \frac{\pi}{2}$

let $n = 2 \Rightarrow \theta = 2\pi + \frac{\pi}{2} - \phi$

$\Rightarrow \theta = \frac{5\pi}{2} - \phi \Rightarrow \theta + \phi = \frac{5\pi}{2}$

Given $\frac{1}{\pi}\left(\theta \pm \pi - \frac{\pi}{2}\right) = \frac{1}{\pi}\left(\frac{\pi}{2} - \frac{\pi}{2}\right) = 0$

Also $\frac{1}{\pi}\left(\frac{5\pi}{2} - \frac{\pi}{2}\right) = \frac{1}{\pi} \times 2\pi = 2$

$\therefore$ The possible values are 0 or 2

Ans: A, C

9

13. Given $\sin x + \cos x = \sqrt{y + \frac{1}{y}}$

We know $A.M \geq G.M$

$$\Rightarrow \frac{y + \frac{1}{y}}{2} \geq \sqrt{y \times \frac{1}{y}}$$

$$\Rightarrow y + \frac{1}{y} \geq 2$$

$$\Rightarrow \sqrt{y + \frac{1}{y}} \geq \sqrt{2}$$

i.e. The minimum value of $\sqrt{y + \frac{1}{y}}$ is $\sqrt{2}$

When $x = \frac{\pi}{4}$, $\sin\frac{\pi}{4} + \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

$$= \sqrt{2}$$

Also. when $y = 1$, $\sqrt{y + \frac{1}{y}} = \sqrt{2}$

$\therefore$ Ans : A, C

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14. Statement I: Given $9^{\cos x} - 2 \cdot 3^{\cos x} + 1 = 0$

$$\Rightarrow \left(3^{\cos x}\right)^2 - 2 \cdot 3^{\cos x} + 1 = 0.$$

Let $3^{\cos x} = t$

$$\Rightarrow t^2 - 2t + 1 = 0$$

$$\Rightarrow (t-1)^2 = 0$$

$$\Rightarrow t - 1 = 0$$

$$\Rightarrow t = 1$$

$$\Rightarrow 3^{\cos x} = 3^0$$

$$\Rightarrow \cos x = 0$$

$$\Rightarrow x = (2n+1)\frac{\pi}{2}, \quad n \in \mathbb{Z}$$

statement I is TRUE.

Statement II: Given $\sqrt{3} \sin x + \cos x = 4$

$$\Rightarrow \frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x = 2$$

$$\Rightarrow \sin x \cdot \cos\frac{\pi}{6} + \cos x \cdot \sin\frac{\pi}{6} = 2$$

$$\Rightarrow \sin\left(x + \frac{\pi}{6}\right) = 2$$

which is not possible, since the maximum value of $\sin\theta$ is 1.

$\therefore$ The given equation has no solutions.

Statement II is Not TRUE

Ans: C

Comprehension - I

Q. Given $7\cos^2\theta + 4\cos\theta - 1 = 0$

$$\Rightarrow \cos\theta = \frac{-4 \pm \sqrt{16+28}}{14}$$

$$\Rightarrow \cos\theta = \frac{-4 \pm \sqrt{44}}{14}$$

$$\Rightarrow \cos\theta = \frac{-4 \pm 2\sqrt{11}}{14}$$

$$\Rightarrow \cos\theta = \frac{-2 \pm \sqrt{11}}{7}$$

Now $\cos 2\theta = 2\cos^2\theta - 1$

$$= 2\left(\frac{-2 \pm \sqrt{11}}{7}\right)^2 - 1$$

$$= \frac{2}{49}\left(4 + 11 \pm 4\sqrt{11}\right) - 1$$

$$= \frac{30 \pm 8\sqrt{11} - 49}{49}$$

$$= \frac{8\sqrt{11} - 19}{49} = \frac{-19 \pm 8\sqrt{11}}{49} \rightarrow (1)$$

Given $p\cos^2 2\theta + q\cos 2\theta + r = 0$

$$\Rightarrow \cos 2\theta = \frac{-q \pm \sqrt{q^2 - 4pr}}{2p} \rightarrow (2)$$

Comparing (1) and (2)

We, get $2p = 49 \Rightarrow p = \frac{49}{2} \rightarrow (3)$

$-q = -19 \Rightarrow q = 19 \rightarrow (4)$

$\sqrt{q^2 - 4pr} = 8\sqrt{11}$

$\Rightarrow q^2 - 4pr = 704$

$\Rightarrow 19^2 - 4 \cdot \frac{49}{2} \cdot r = 704$

$\Rightarrow r = \frac{-343}{98}$

$\Rightarrow r = -\frac{7}{2} \rightarrow (5)$

Given $p\cos^2 2\theta + q\cos 2\theta + r = 0$

$\Rightarrow \frac{49}{2}\cos^2 2\theta + 19\cos 2\theta - \frac{7}{2} = 0$

$\Rightarrow 49\cos^2 2\theta + 38\cos 2\theta - 7 = 0$

$\therefore p = 49, \quad q = 38, \quad r = -7$

15. $p = 49$ Ans: B

16. $q = 38$ Ans: C

17. $r = -7$ Ans: A

Comprehension - III

Trigonometric Equations (Continuation) 13

class 9th

Given $x\cos^3 y + 3x\cos y \cdot \sin^2 y = 14 \rightarrow (1)$

$x\sin^3 y + 3x \cdot \cos^2 y \cdot \sin y = 13 \rightarrow (2)$

Now, (1) + (2)

$x(\cos^3 y + 3\cos y \sin^2 y + 3\cos^2 y \cdot \sin y + \sin^3 y) = 27$

$\Rightarrow x(\cos y + \sin y)^3 = 27 \rightarrow (3)$

Now, (1) − (2)

$\Rightarrow x(\cos y - \sin y)^3 = 1 \rightarrow (4)$

Now, $\frac{(3)}{(4)} \Rightarrow \left(\frac{\cos y + \sin y}{\cos y - \sin y}\right)^3 = 27$

$\Rightarrow \frac{\cos y + \sin y}{\cos y - \sin y} = 3$

$\Rightarrow \cos y + \sin y = 3\cos y - 3\sin y$

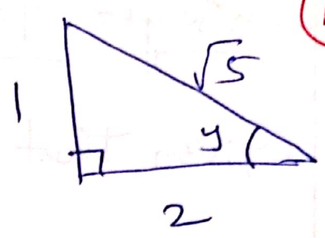
$\Rightarrow 4\sin y = 2\cos y$

$\Rightarrow \tan y = \frac{1}{2} \Rightarrow$

$\Rightarrow$ ~~$\tan y = \tan\alpha$~~

$\sin y = \frac{1}{\sqrt{5}}$

and $\cos y = \frac{2}{\sqrt{5}}$



21. Given $x \cos^3 y + 3x . \cos y . \sin^2 y = 14$

$\Rightarrow x \left(\frac{2}{\sqrt{5}}\right)^3 + 3x . \left(\frac{2}{\sqrt{5}}\right) \left(\frac{1}{\sqrt{5}}\right)^2 = 14$

$\Rightarrow \frac{8x}{5\sqrt{5}} + \frac{6x}{5\sqrt{5}} = 14$

$\Rightarrow 14x = 70\sqrt{5}$

$\Rightarrow x = 5\sqrt{5}$ Ans: A

22. The number of values of $y \in [0, 6\pi]$ are 6. Ans: D

23. Given $\sin^2 y + 2\cos^2 y$

$= \left(\frac{1}{\sqrt{5}}\right)^2 + 2\left(\frac{2}{\sqrt{5}}\right)^2$

$= \frac{1}{5} + \frac{8}{5} = \frac{9}{5}$ Ans: B

Integer Type

(15)

24 Given that $a\sin x + \cos 2x = 2a - 7$

$\Rightarrow a\sin x + 1 - 2\sin^2 x = 2a - 7$

$\Rightarrow 2\sin^2 x - a\sin x + 2a - 8 = 0$

$$\Rightarrow \sin x = \frac{a \pm \sqrt{a^2 - 4\cdot 2\cdot(2a-8)}}{4}$$

$$= \frac{a \pm \sqrt{a^2 - 16a + 64}}{4}$$

$$= \frac{a \pm \sqrt{(a-8)^2}}{4}$$

$$= \frac{a \pm (a-8)}{4}$$

$\therefore \sin x = \frac{a + a - 8}{4}$ or $\sin x = \frac{a - (a-8)}{4}$

$\Rightarrow \sin x = \frac{2a-8}{4}$

$\Rightarrow \sin x = \frac{a-4}{2}$

$\Rightarrow \sin x = \frac{8}{4} = 2$

which is not possible

We know, $-1 \le \sin x \le 1$

$\Rightarrow -1 \le \frac{a-4}{2} \le 1$

$\Rightarrow -2 \le a-4 \le 2$

$\Rightarrow 2 \le a \le 6$

$\therefore$ The maximum integral value of a is 6

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25 a) $4\sin^2 x + \sin^2 2x = 3$

$\Rightarrow 4\sin^2 x + 4\sin^2 x . \cos^2 x = 3$

$\Rightarrow \cancel{\frac{4\sin^2 x}{\cos^2 x} + \frac{4\sin^2 x . \cos^2 x}{\cos^2 x} = \frac{3}{\cos^2 x}}$

$\Rightarrow \cancel{4\tan^2 x + 4\sin^2 x = 3\sec^2 x}$

$\Rightarrow 4\tan^2 x + 4(1-\cos^2 x) = 3\sec^2 x$

$\Rightarrow 4\sin^2 x + 4\sin^2 x(1-\sin^2 x) = 3$

$\Rightarrow 4\sin^2 x + 4\sin^2 x - 4\sin^4 x = 3$

$\Rightarrow 8\sin^2 x - 4\sin^4 x = 3$

$\Rightarrow 4\sin^4 x - 8\sin^2 x + 3 = 0$

$\Rightarrow \underline{4\sin^4 x - 2\sin^2 x} - \underline{6\sin^2 x + 3} = 0$

$\Rightarrow 2\sin^2 x(2\sin^2 x - 1) - 3(2\sin^2 x - 1) = 0$

17

$\Rightarrow (2\sin^2 x - 1)(2\sin^2 x - 3) = 0$

$\Rightarrow 2\sin^2 x - 1 = 0$	$2\sin^2 x - 3 = 0$
$\Rightarrow \sin x = \pm \frac{1}{\sqrt{2}}$	$\Rightarrow \sin x = \pm \frac{\sqrt{3}}{2}$
$\Rightarrow x = \pm \frac{\pi}{4}$	$x = \pm \frac{\pi}{3}$

$: \tan \frac{\pi}{4} = 1$

$\therefore \tan x = 1$

$\Rightarrow x = \tan^{-1}(1)$

b) Given $4\cos^2 2x + 8\cos^2 x = 7$

let $x = \frac{\pi}{6}$

Now, LHS $= 4\cos^2 2\left(\frac{\pi}{6}\right) + 8\cos^2\left(\frac{\pi}{6}\right)$

$= 4\cos^2 \frac{\pi}{3} + 8\cos^2 \frac{\pi}{6}$

$= 4\left(\frac{1}{2}\right)^2 + 8\left(\frac{\sqrt{3}}{2}\right)^2$

$= 1 + 6$

$= 7 =$ RHS

$\therefore \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$\therefore \tan x = \frac{1}{\sqrt{3}} \Rightarrow x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

c) $3(1+\sin x) = 1+\cos 2x$

$\Rightarrow 3(1+\sin x) = 2\cos^2 x$

$\Rightarrow 3(1+\sin x) = 2(1-\sin^2 x)$

$\Rightarrow 2\sin^2 x + 3\sin x + 1 = 0$

$\Rightarrow (2\sin x + 1)(\sin x + 1) = 0$

$\Rightarrow \sin x = -\frac{1}{2}$ or $\sin x = -1$

$\Rightarrow x = \sin^{-1}(-1)$

d) Given $4x^4 + x^6 + \sin^2 5x = 0$

$x = 0$, satisfies the above equation

Ans: a – p, b – r, c – q, d – s

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a) Given $7\cos^2 x + \sin x \cdot \cos x - 3 = 0$

Dividing with $\cos^2 x$ on Either side

$\Rightarrow 7 + \tan x - 3\sec^2 x = 0$

$\Rightarrow 7 + \tan x - 3(1+\tan^2 x) = 0$

$\Rightarrow 7 + \tan x - 3 - 3\tan^2 x = 0$

$\Rightarrow 3\tan^2 x - \tan x - 4 = 0$

$\Rightarrow (3\tan x - 4)(\tan x + 1) = 0$

$\Rightarrow \tan x = \frac{4}{3}$ or $\tan x = -1$

19

b) Given $2\sin^2\left(\frac{\pi}{2}\cdot\cos^2 x\right) = 1 - \cos(\pi \sin 2x)$

$\Rightarrow 2\sin^2\left(\frac{\pi}{2}\cdot\cos^2 x\right) = 2\sin^2\left(\frac{\pi}{2}\cdot\sin 2x\right)$

$\Rightarrow \frac{\pi}{2}\cdot\cos^2 x = \frac{\pi}{2}\cdot\sin 2x$

$\Rightarrow \cos^2 x = 2\sin x\cos x$

$\Rightarrow \cos x(\cos x - 2\sin x) = 0$

$\Rightarrow \cos x = 0$ or $\tan x = \frac{1}{2}$

c) Given $6\sec^2 x - 11\tan x - 2 = 0$

$\Rightarrow 6(1+\tan^2 x) - 11\tan x - 2 = 0$

$\Rightarrow 6\tan^2 x - 11\tan x + 4 = 0$

$\Rightarrow (2\tan x - 1)(3\tan x - 4) = 0$

$\Rightarrow \tan x = \frac{1}{2}$ or $\tan x = \frac{4}{3}$

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d) $\cancel{2\cos x - \sin x = \cos^2 x - 4\sin x}$

$\cancel{\text{let } x = \frac{\pi}{3}}$

Given $2\cos x - \sin x = \cos^2 x - 4\sin x$

20

$\Rightarrow 2\cos x - \cos^2 x = -3\sin x$

$\Rightarrow 2 - \cos x = -3\tan x$

$\Rightarrow 2 + 3\tan x = \cos x$

$\Rightarrow \dfrac{1}{2+3\tan x} = \sec x$

$\Rightarrow \dfrac{1}{(2+3\tan x)^2} = \sec^2 x$

$\Rightarrow \dfrac{1}{(2+3\tan x)^2} = 1+\tan^2 x$

$\Rightarrow (1+\tan^2 x)(2+3\tan x)^2 = 1$

For any value of x, the above equation does not satisfy.

(21)

27 a) Given $\sqrt{p}\cos x - 2\sin x = \sqrt{2} + \sqrt{2-p} \rightarrow (1)$

We know, the maximum value of

$$\sqrt{p}\cos x - 2\sin x = c + \sqrt{a^2+b^2}$$
$$= 0 + \sqrt{4+p}$$

Also, minimum value $= c - \sqrt{a^2+b^2}$
$$= -\sqrt{4+p}$$

$$\therefore \left|\sqrt{p}\cos x - 2\sin x\right| \leq \sqrt{4+p} \rightarrow (2)$$

also equation (1) has solutions if

$p \geq 0$ and $2-p \geq 0$ i.e. $p \leq 2$

Now from (2)

$$\left|\sqrt{2} + \sqrt{2-p}\right| \leq \sqrt{4+p}$$

$$\Rightarrow \left(\sqrt{2} + \sqrt{2-p}\right)^2 \leq 4+p$$

$$\Rightarrow p^2 + 2p - 4 \geq 0$$

$$\Rightarrow (p+1)^2 \geq 5$$

$$\Rightarrow |p+1| \geq \sqrt{5}$$

i.e. $p+1 \geq \sqrt{5}$ or $p+1 \leq -\sqrt{5}$

$\Rightarrow p \geq \sqrt{5}-1$ or $p \leq -1-\sqrt{5}$

We have $p \geq 0$, $p \leq 2$, $p \geq -1+\sqrt{5}$

and $p \leq -1-\sqrt{5}$.

The common solution of the above inequality is

$p \in (-1+\sqrt{5}, 2)$.

b) See the solution of Q.NO.24.

c) Given $a \sin\frac{x}{2} = \sin x + \sin\frac{3x}{2}$

d) Given $\sin^4 x + \cos^4 x + \sin 2x + a = 0$

$\Rightarrow (\sin^2 x)^2 + (\cos^2 x)^2 + \sin 2x + a = 0$

$\Rightarrow (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cdot \cos^2 x + \sin 2x + a = 0$

$\Rightarrow 1 - \frac{1}{2}(\sin 2x)^2 + \sin 2x + a = 0$

Let $\sin 2x = t$

$\Rightarrow 1 - \frac{t^2}{2} + t + a = 0$

$\Rightarrow 2 - t^2 + 2t + 2a = 0$

$\Rightarrow t^2 - 2t - 2 - 2a = 0$

$\Rightarrow t = \frac{2 \pm \sqrt{4 + 4(2+2a)}}{2}$

$\Rightarrow t = \frac{2 \pm 2\sqrt{3+2a}}{2}$

$\Rightarrow t = 1 \pm \sqrt{3+2a}$

24

$\Rightarrow \sin 2x = 1 - \sqrt{3+2a}$ ~~Since~~

~~$-1 \le \sin 2x \le 1$~~

Since $-1 \le \sin 2x \le 1$

$\Rightarrow -1 \le 1 - \sqrt{3+2a} \le 1$

$\Rightarrow -2 \le -\sqrt{3+2a} \le 0$

$\Rightarrow 2 \ge \sqrt{3+2a} \ge 0$

$\Rightarrow 0 \le \sqrt{3+2a} \le 2$

$\Rightarrow 0 \le 3+2a \le 4$

$\Rightarrow -3 \le 2a \le 1$

$\Rightarrow -\frac{3}{2} \le a \le \frac{1}{2}$

$\therefore a \in \left[-\frac{3}{2}, \frac{1}{2}\right]$

LEARNERS TASK

01. Given that $4\cos^2\theta = 3$

$$\Rightarrow \cos^2\theta = \frac{3}{4}$$

$$\Rightarrow \cos^2\theta = \left(\frac{\sqrt{3}}{2}\right)^2$$

$$\Rightarrow \cos^2\theta = \cos^2\frac{\pi}{6}$$

$$\Rightarrow \theta = n\pi \pm \frac{\pi}{6}$$

$$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

Ans: A

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02. $3\tan\theta = \cot\theta$

$$\Rightarrow 3\tan^2\theta = 1$$

$$\Rightarrow \tan^2\theta = \frac{1}{3} = \left(\frac{1}{\sqrt{3}}\right)^2$$

$$\Rightarrow \tan^2\theta = \tan^2\frac{\pi}{6}$$

$$\Rightarrow \theta = n\pi \pm \frac{\pi}{6}$$

$$\therefore \theta = \pm 30^\circ$$

Ans: A

26

Given $\cos m\theta = \sin n\theta$

$$\Rightarrow \cos m\theta = \cos\left(\frac{\pi}{2} - n\theta\right)$$

$$\Rightarrow m\theta = 2k\pi \pm \frac{\pi}{2} - n\theta \quad , k \in \mathbb{Z}$$

$$\Rightarrow m\theta + n\theta = 2k\pi \pm \frac{\pi}{2}$$

$$\Rightarrow (m+n)\theta = 2k\pi \pm \frac{\pi}{2}$$

$$\Rightarrow \theta = \frac{2k\pi \pm \frac{\pi}{2}}{m+n}$$

Ans: C

Given $\tan m\theta = \cot n\theta$

$$\cancel{\Rightarrow \tan m\theta = \tan\left(\frac{\pi}{2}\right)}$$

$$\cancel{\Rightarrow m\theta = k\pi + \frac{\pi}{2} - \theta}$$

$$\cancel{\Rightarrow m\theta + \theta = k\pi + \frac{\pi}{2}}$$

$$\cancel{\Rightarrow (m+1)}$$

$$\Rightarrow \tan m\theta = \tan\left(\frac{\pi}{2} - n\theta\right)$$

$$\Rightarrow m\theta = k\pi + \frac{\pi}{2} - n\theta \quad , k \in \mathbb{Z}$$

$$\Rightarrow (m+n)\theta = k\pi + \frac{\pi}{2}$$

$$\Rightarrow (m+n)\theta = (2k+1)\frac{\pi}{2}$$

27

$$\Rightarrow \theta = \frac{(2k+1)\pi}{2(m+n)}$$

Ans: B

05 Given $\sqrt{3}\sin\theta = \cos\theta$

$$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan\theta = \tan\frac{\pi}{6}$$

$$\Rightarrow \theta = n\pi + \frac{\pi}{6}, \quad n \in \mathbb{Z}$$

06 ~~Sln~~ Given $\sin 2x = 4\cos x$

$$\Rightarrow 2\sin x\cos x = 4\cos x$$

$$\Rightarrow \sin x\cos x - 2\cos x = 0$$

$$\Rightarrow \cos x(\sin x - 2) = 0$$

$\Rightarrow \cos x = 0$ or $\sin x = 2$ which is NOT possible

$$\therefore x = (2n+1)\frac{\pi}{2}, \quad n \in \mathbb{Z}$$

Ans. A

07\. Given $\tan\left(\frac{\pi}{2}\sin\theta\right) = \cot\left(\frac{\pi}{2}\cos\theta\right)$

$$\Rightarrow \tan\left(\frac{\pi}{2}\sin\theta\right) = \tan\left(\pm\frac{\pi}{2} - \frac{\pi}{2}\cos\theta\right)$$

$$\Rightarrow \frac{\pi}{2}\sin\theta = \pm\frac{\pi}{2} - \frac{\pi}{2}\cos\theta$$

$$\Rightarrow \sin\theta = \pm 1 - \cos\theta$$

$$\Rightarrow \sin\theta + \cos\theta = \pm 1$$

$$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \pm\frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) = \pm\frac{1}{\sqrt{2}}$$

Ans : B

08\. ~~tan~~ Given $\tan(\pi\cos x) = \cot(\pi\sin x)$

$$\Rightarrow \tan(\pi\cos x) = \tan\left(\pm\frac{\pi}{2} - \pi\sin x\right)$$

$$\Rightarrow \pi\cos x = \pm\frac{\pi}{2} - \pi\sin x$$

$$\Rightarrow \cos x + \sin x = \pm\frac{1}{2}$$

$$\Rightarrow \frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x = \pm\frac{1}{2\sqrt{2}}$$

$$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \pm\frac{1}{2\sqrt{2}}$$

Ans: C

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9. Given that $\tan(\cot x) = \cot(\tan x)$

$$\Rightarrow \tan(\cot x) = \tan\left(\frac{\pi}{2} - \tan x\right)$$

$$\Rightarrow \cot x = n\pi + \frac{\pi}{2} - \tan x \quad , n \in \mathbb{Z}$$

$$\Rightarrow \tan x + \cot x = n\pi + \frac{\pi}{2}$$

$$\Rightarrow \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = n\pi + \frac{\pi}{2}$$

$$\Rightarrow \frac{1}{\sin x \cdot \cos x} = \frac{2n\pi + \pi}{2}$$

$$\Rightarrow \frac{1}{\sin 2x} = \frac{2n\pi + \pi}{4}$$

$$\Rightarrow \sin 2x = \frac{4}{(2n+1)\pi}$$

Ans:- B

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10. $\sin\left(\frac{\pi}{4}\cot\theta\right) = \cos\left(\frac{\pi}{4}\tan\theta\right)$

$$\Rightarrow \cancel{\cos\left(\frac{\pi}{2} - \frac{\pi}{4}\cot\theta\right) = \cos\left(\frac{\pi}{4}\tan\theta\right)}$$

Let $\theta = \frac{\pi}{4}$

$$LHS = \sin\left(\frac{\pi}{4}\cot\frac{\pi}{4}\right)$$

$$= \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$RHS = \cos\left(\frac{\pi}{4}\tan\frac{\pi}{4}\right)$$

$$= \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

Hence $\theta = n\pi + \frac{\pi}{4}$ Ans: B

JEE MAINS LEVEL QUESTIONS

01 Given $\dfrac{\tan 2x - \tan x}{1 + \tan 2x \cdot \tan x} = 1$

$\Rightarrow \tan(2x - x) = 1$

$\Rightarrow \tan x = \tan\frac{\pi}{4}$

$\Rightarrow x = n\pi + \frac{\pi}{4}$ Ans: C

02 Given $\sin x \cdot \sin(60^\circ + x) \cdot \sin(60^\circ - x) = \frac{1}{8}$

We know $\sin x \cdot \sin(60^\circ + x) \cdot \sin(60^\circ - x) = \frac{1}{4}\sin 3x$

$\therefore \frac{1}{4}\sin 3x = \frac{1}{8}$

$\Rightarrow \sin 3x = \frac{1}{2}$

$\Rightarrow \sin 3x = \sin\frac{\pi}{6}$

$\Rightarrow 3x = n\pi + (-1)^n \cdot \frac{\pi}{6}$, $n \in Z$

$\Rightarrow x = \frac{n\pi}{3} + (-1)^n \cdot \frac{\pi}{18}$ Ans: B

04. Given $\cot^2\theta - \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)\cot\theta + 1 = 0$

let $\theta = \frac{\pi}{6}$

$LHS = \cot^2\theta - \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)\cot\theta + 1$

$= \cot^2\frac{\pi}{6} - \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)\cot\frac{\pi}{6} + 1$

$= (\sqrt{3})^2 - \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)(\sqrt{3}) + 1$

$= 3 - 3 - 1 + 1$

$= 0 = RHS$

Hence, the general solution, $\theta = n\pi + \frac{\pi}{6}$, $n \in Z$

04 Given $\sin\theta \cdot \sin(60^\circ + \theta) \cdot \sin(60^\circ - \theta) = \frac{1}{4}$

We know, $\sin\theta \cdot \sin(60^\circ + \theta) \cdot \sin(60^\circ - \theta) = \frac{1}{4}\sin 3\theta$

$\therefore \frac{1}{4}\sin 3\theta = \frac{1}{4}$

$\Rightarrow \sin 3\theta = 1$

$\Rightarrow \sin 3\theta = \sin \frac{\pi}{2}$

$\Rightarrow 3\theta = n\pi + (-1)^n \cdot \frac{\pi}{2}$

$\Rightarrow \theta = \frac{n\pi}{3} + (-1)^n \cdot \frac{\pi}{6}$, $n \in \mathbb{Z}$

Ans: B

05 $\sin x + \cos x = \sqrt{2} \cos \alpha$

$\Rightarrow \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = \cos \alpha$

$\Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \cos \alpha$

$\Rightarrow x - \frac{\pi}{4} = 2n\pi \pm \alpha$, $n \in \mathbb{Z}$

$\Rightarrow x = 2n\pi \pm \alpha + \frac{\pi}{4}$

$\Rightarrow x = 2n\pi + \frac{\pi}{4} \pm \alpha$ Ans: C

06 Given $\sin\left(x + \frac{\pi}{4}\right) = \sin 2x$

$\Rightarrow x + \frac{\pi}{4} = n\pi + (-1)^n \cdot 2x$, $n \in \mathbb{Z}$

$\Rightarrow x - (-1)^n 2x = n\pi - \frac{\pi}{4}$

$\Rightarrow x\left(1 - (-1)^n \cdot 2\right) = n\pi - \frac{\pi}{4}$

$$x = \frac{n\pi - \frac{\pi}{4}}{1-(-1)^n \cdot 2}, \quad n \in Z$$

Ans: (D)

07. Given that $81^{\sin^2 x} + 81^{\cos^2 x} = 30$

$\Rightarrow 81^{\sin^2 x} + 81^{1-\sin^2 x} = 30$

$\Rightarrow 81^{\sin^2 x} + \frac{81}{81^{\sin^2 x}} = 30$

$\Rightarrow t + \frac{81}{t} = 30$, where $81^{\sin^2 x} = t$

$\Rightarrow t^2 - 30t + 81 = 0$

$\Rightarrow (t-27)(t-3) = 0$

$\Rightarrow t = 27$

$\Rightarrow 81^{\sin^2 x} = 27$

$\Rightarrow 3^{4\sin^2 x} = 3^3$

$\Rightarrow \sin^2 x = \frac{3}{4}$

$\Rightarrow \sin x = \frac{\sqrt{3}}{2}$

$t = 3$

$\Rightarrow 81^{\sin^2 x} = 3$

$\Rightarrow 3^{4\sin^2 x} = 3^1$

$\Rightarrow 4\sin^2 x = 1$

$\Rightarrow \sin^2 x = \frac{1}{4}$

$\Rightarrow \sin x = \frac{1}{2}$

$\Rightarrow x = \frac{\pi}{6}$ or $\frac{\pi}{3}$ Since $0 \le x \le \frac{\pi}{2}$ (34)

Ans: A

Given $\sec^2(a+2)x + (a^2-1) = 0$

$\Rightarrow \sec^2(a+2)x - 1 + a^2 = 0$

$\Rightarrow \tan^2(a+2)x + a^2 = 0$

$\Rightarrow \tan^2(a+2)x = 0$ and $a^2 = 0$

$\Rightarrow a = 0$

$\Rightarrow \tan^2 2x = 0$

$\Rightarrow 2x = 0, \pi, -\pi$, Since $-\pi < x < \pi$

$\Rightarrow x = 0, \frac{\pi}{2}, -\frac{\pi}{2}$

So, the required ordered pairs are

$(0,0), (0, \frac{\pi}{2}), (0, -\frac{\pi}{2})$

$\therefore$ 3 possible ordered pairs Ans: C

Given $1 + \sin^2\theta = 3\sin\theta\cos\theta$

Divide with $\cos^2\theta$ on Either side

$\Rightarrow \sec^2\theta + \tan^2\theta = 3\tan\theta$

$\Rightarrow (1+\tan^2\theta) + \tan^2\theta = 3\tan\theta$

$\Rightarrow 2\tan^2\theta - 3\tan\theta + 1 = 0$

$\Rightarrow (2\tan\theta - 1)(\tan\theta - 1) = 0$

$\Rightarrow \tan\theta = \frac{1}{2}$ or $\tan\theta = 1$

$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{2}\right)$ $\Rightarrow \theta = \frac{\pi}{4}$

Ans: B

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10 Given $2\cos x + \cos 2\lambda x = 3$

one of the possible case is

let $2\cos x = 2$ and $\cos 2\lambda x = 1$

$\Rightarrow \cos x = 1$

$\Rightarrow \cos x = \cos 0$

$\Rightarrow x = 2n\pi, n \in I$

~~If $\lambda \in Q$, the above equation has Infinitely~~

let $\cos 2\lambda x = 1$

~~If $x = 0$ i.e. $x \notin Q$, is the only solution for both the equations~~

If $\lambda \in Q$, there are infinitely many Solutions and, if $\lambda \notin Q$, then $x=0$ is the only one solution satisfying both the equations. 36

Hence, λ is an irrational number

Ans: C

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11. Given $\sin^4 x + \cos^4 x = 1$

$\Rightarrow \sin^4 x = 1 - \cos^4 x$

$\Rightarrow \sin^4 x = (1+\cos^2 x)(1-\cos^2 x)$

$\Rightarrow \sin^4 x = (1+\cos^2 x).\sin^2 x$

$\Rightarrow \sin^4 x - \sin^2 x(1+\cos^2 x) = 0$

$\Rightarrow \sin^2 x(\sin^2 x - 1 - \cos^2 x) = 0$

$\Rightarrow \sin^2 x = 0$ or | $\sin^2 x - \cos^2 x - 1 = 0$

$\Rightarrow \sin x = 0$ | $\cos^2 x - \sin^2 x = -1$

$\Rightarrow x = n\pi$ | $\cos 2x = -1$

$\Rightarrow \cos 2x = \cos(\pi)$

$\Rightarrow 2x = 2n\pi \pm \pi$

$\Rightarrow x = n\pi \pm \frac{\pi}{2}$

Ans A, D

12. **Statement I:** Given $\sqrt{3}\tan\theta - 1 = 0$

$$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan\theta = \tan\frac{\pi}{6}$$

$$\Rightarrow \theta = n\pi + \frac{\pi}{6}, \quad n \in \mathbb{Z}$$

Statement I is true

Statement II: Given $\tan x = k$

$$\Rightarrow x = n\pi + \alpha, \quad n \in \mathbb{Z}$$

∴ Statement II is true

Ans: A

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13. **Statement I:** Given $\sin\theta = \frac{1}{2}$ and $\tan\theta = \frac{1}{\sqrt{3}}$

$$\Rightarrow \sin\theta = \sin\frac{\pi}{6} \text{ and } \tan\theta = \tan\frac{\pi}{6}$$

General solution $= \left\{2n\pi + \frac{\pi}{6}, n \in \mathbb{Z}\right\}$

Statement I is TRUE

Statement II: Given $\cos\theta = \frac{-1}{\sqrt{2}}$, $\tan\theta = -1$

Now, $\cos\theta = \cos\frac{3\pi}{4}$ and $\tan\theta = \tan\frac{3\pi}{4}$

$\therefore$ The G.S. $= \left\{2n\pi + \frac{3\pi}{4}, n \in z\right\}$

$\therefore$ Statement II is TRUE   Ans. A

14. Given $2\sin^2\theta + 3\sin\theta + 1 = 0$

$\Rightarrow (2\sin\theta + 1)(\sin\theta + 1) = 0$

$\Rightarrow 2\sin\theta + 1 = 0$ or $\sin\theta + 1 = 0$

$\therefore \sin\theta = -\frac{1}{2}$

$\Rightarrow \sin\theta = \sin\left(-\frac{\pi}{6}\right)$

$\Rightarrow \theta = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$

$\Rightarrow \theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{5\pi}{6}$

$\sin\theta = -1$

$\Rightarrow \sin\theta = \sin\left(-\frac{\pi}{2}\right)$

$\Rightarrow \theta = n\pi + (-1)^n\left(-\frac{\pi}{2}\right)$

$\Rightarrow \theta = \frac{3\pi}{2}$

Total No. of solutions = 4.

15. a) Given $\cos x = -\frac{1}{2}$

Now $\cos\frac{8\pi}{3} = \cos\left(2\pi + \frac{2\pi}{3}\right)$

$= \cos\frac{2\pi}{3}$

$= \cos\left(\pi - \frac{\pi}{3}\right)$

$= -\cos\frac{\pi}{3}$

$= -\frac{1}{2}$

$\therefore\ x = \frac{8\pi}{3}$

b) Given $\sin x = \frac{\sqrt{3}}{2}$

Now $\sin\frac{8\pi}{3} = \sin\left(2\pi + \frac{2\pi}{3}\right)$

$= \sin\frac{2\pi}{3}$

$= \sin\left(\pi - \frac{\pi}{3}\right)$

$= \sin\frac{\pi}{6}$

$= \frac{\sqrt{3}}{2}$

Also, $\sin\frac{7\pi}{3} = \sin\left(2\pi + \frac{\pi}{3}\right)$

$= \sin\frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}$

hence $x = \frac{8\pi}{3}$ and $\frac{7\pi}{3}$

c) Given $\tan x = \frac{1}{\sqrt{3}}$

Now, $\tan\frac{19\pi}{6} = \tan\left(2\pi + \frac{7\pi}{6}\right)$

$= \tan\frac{7\pi}{6}$

$= \tan\left(\pi + \frac{\pi}{6}\right)$

$= \tan\frac{\pi}{6}$

$= \frac{1}{\sqrt{3}}$

hence, $x = \frac{19\pi}{6}$

d) Given $\cot x = -1$

$\Rightarrow \tan x = -1$

~~Now,~~ $\cancel{\tan\frac{11\pi}{4} = \tan\left(2\pi + \frac{5\pi}{8}\right)}$

$\cancel{= \tan\frac{5\pi}{8}}$

$\cancel{= \tan\left(\frac{\pi}{2} + \frac{\pi}{8}\right)}$

Now, $\tan\frac{11\pi}{4} = \tan\left(2\pi + \frac{3\pi}{4}\right)$

$= \tan\frac{3\pi}{4}$

$= \tan\left(\pi - \frac{\pi}{4}\right)$

$= -\tan\frac{\pi}{4}$

$= -1$

hence $x = \frac{11\pi}{4}$

Ans. a — r, b — (p, r), c — q, d — s

ADDITIONAL PRACTICICE QUESTIONS FOR STUDENTS

01. Given $K = \cos 20^\circ$

Now, $\cos x = 2K^2 - 1$

$= 2\cos^2 20^\circ - 1$

$= \cos 2\times 20^\circ$

$= \cos 40^\circ$

$\therefore \cos x = \cos 40^\circ$

$\Rightarrow x = 40^\circ$

$\cos x = \cos(360^\circ - 40^\circ)$

$\Rightarrow \cos x = \cos 320^\circ$

$\Rightarrow x = 320^\circ$

Ans: C

03 Given that $\tan\left(\theta + \frac{\pi}{4}\right) = 3 \cdot \tan 3\theta$.

$$\Rightarrow \frac{\tan\theta + \tan\frac{\pi}{4}}{1 - \tan\theta \cdot \tan\frac{\pi}{4}} = 3\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right)$$

$$\Rightarrow \frac{\tan\theta + 1}{1 - \tan\theta} = 3\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right)$$

$$\Rightarrow (1+\tan\theta)(1-3\tan^2\theta) = (9\tan\theta - 3\tan^3\theta)(1-\tan\theta)$$

$$\Rightarrow 1 - 3\tan^2\theta + \tan\theta - \cancel{3\tan^3\theta}$$
$$= 9\tan\theta - 9\tan^2\theta - \cancel{3\tan^3\theta} + 3\tan^4\theta$$

$$\Rightarrow 3\tan^4\theta - 6\tan^2\theta + 8\tan\theta - 1 = 0.$$

Sum of the roots $= \tan\alpha + \tan\beta + \tan\gamma + \tan\delta$

$$\tan\delta = \frac{0}{3} = 0.$$

Given $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$

Given this equation has real roots

$\therefore$ Discriminant ≥ 0

$$\Rightarrow \cos^2 p - 4(\cos p - 1)\sin p \geq 0$$

$$\Rightarrow \cos^2 p - 4\cos p \sin p + 4\sin p \geq 0$$

43

$\Rightarrow \cos^2 p - 4\cos p \sin p + 4\sin^2 p - 4\sin^2 p + 4\sin p \geq 0$

$\Rightarrow (\cos p - 2\sin p)^2 + 4\sin p(1 - \sin p) \geq 0$

here $(\cos p - 2\sin p)^2 \geq 0$ always.

$\therefore$ $4\sin p(1-\sin p)$ should be positive

here $\sin p$ is positive in $[0, \pi]$

also, $0 \leq 1 - \sin p \leq 2$

i.e. $(1-\sin p)$ is also positive in $[0, \pi]$

Hence, p belongs to $(0, \pi)$

Ans: D

4. Given $\sin^{50} x - \cos^{50} x = 1$

$\Rightarrow \sin^{50} x = 1 + \cos^{50} x$

The maximum value of $\sin^{50} x$ is $\underline{1}$.

also, $\cos^{50} x$ is always positive and must be equal to 0.

$\therefore \cos^{50} x = 0$

$\Rightarrow \cos x = 0$

$\Rightarrow x = (2n+1)\frac{\pi}{2} \Rightarrow x = n\pi + \frac{\pi}{2}$

Ans: C

Given $\cos^4 x - (a+2)\cos^2 x - (a+3) = 0$

44

Let $\cos^2 x = t$

$\Rightarrow t^2 - (a+2)t - (a+3) = 0$

$\Rightarrow t^2 - ((a+3)-1)t - (a+3) = 0$

$\Rightarrow t^2 + t - (a+3)t - (a+3) = 0$

$\Rightarrow t(t+1) - (a+3)(t+1) = 0$

$\Rightarrow (t+1)(t-(a+3)) = 0$

$\Rightarrow t+1 = 0$ or

$\Rightarrow \cos^2 x + 1 = 0$

$\Rightarrow \cos^2 x = -1$

which is not possible

$t-(a+3) = 0$

$\Rightarrow t = a+3$

$\Rightarrow \cos^2 x = a+3$

We know

$0 \le \cos^2 x \le 1$

$\Rightarrow 0 \le a+3 \le 1$

$\Rightarrow -3 \le a \le -2$

$\therefore a \in [-3,-2]$

Ans: A

06. Given $\sin x - \sqrt{3}\cos x = \frac{4m-6}{4-m}$

$\Rightarrow 2\left(\frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x\right) = \frac{4m-6}{4-m}$

$\Rightarrow \sin x \cos\frac{\pi}{3} - \cos x \cdot \sin\frac{\pi}{6} = \frac{2m-3}{4-m}$

$\Rightarrow \sin\left(x - \frac{\pi}{3}\right) = \frac{2m-3}{4-m}$

We know $-1 \le \sin\left(x - \frac{\pi}{3}\right) \le 1$

$\Rightarrow -1 \le \frac{2m-3}{4-m} \le 1$

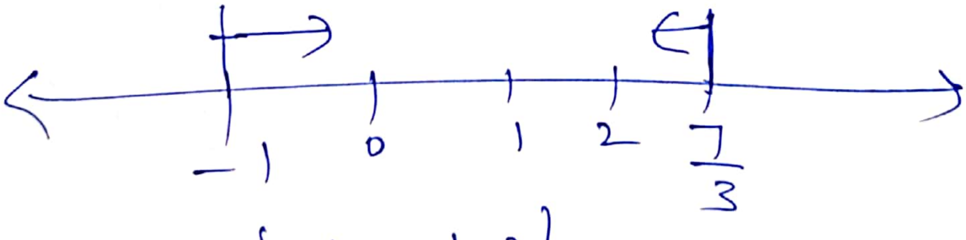
$\Rightarrow -1 \le \frac{2m-3}{4-m}$ and $\frac{2m-3}{4-m} \le 1$

$\Rightarrow -4+m \le 2m-3$ and $2m-3 \le 4-m$

$\Rightarrow 3m \le 7$

$\Rightarrow m \ge -1$

$\Rightarrow m \le \frac{7}{3}$



$\therefore m \in \{-1, 0, 1, 2\}$

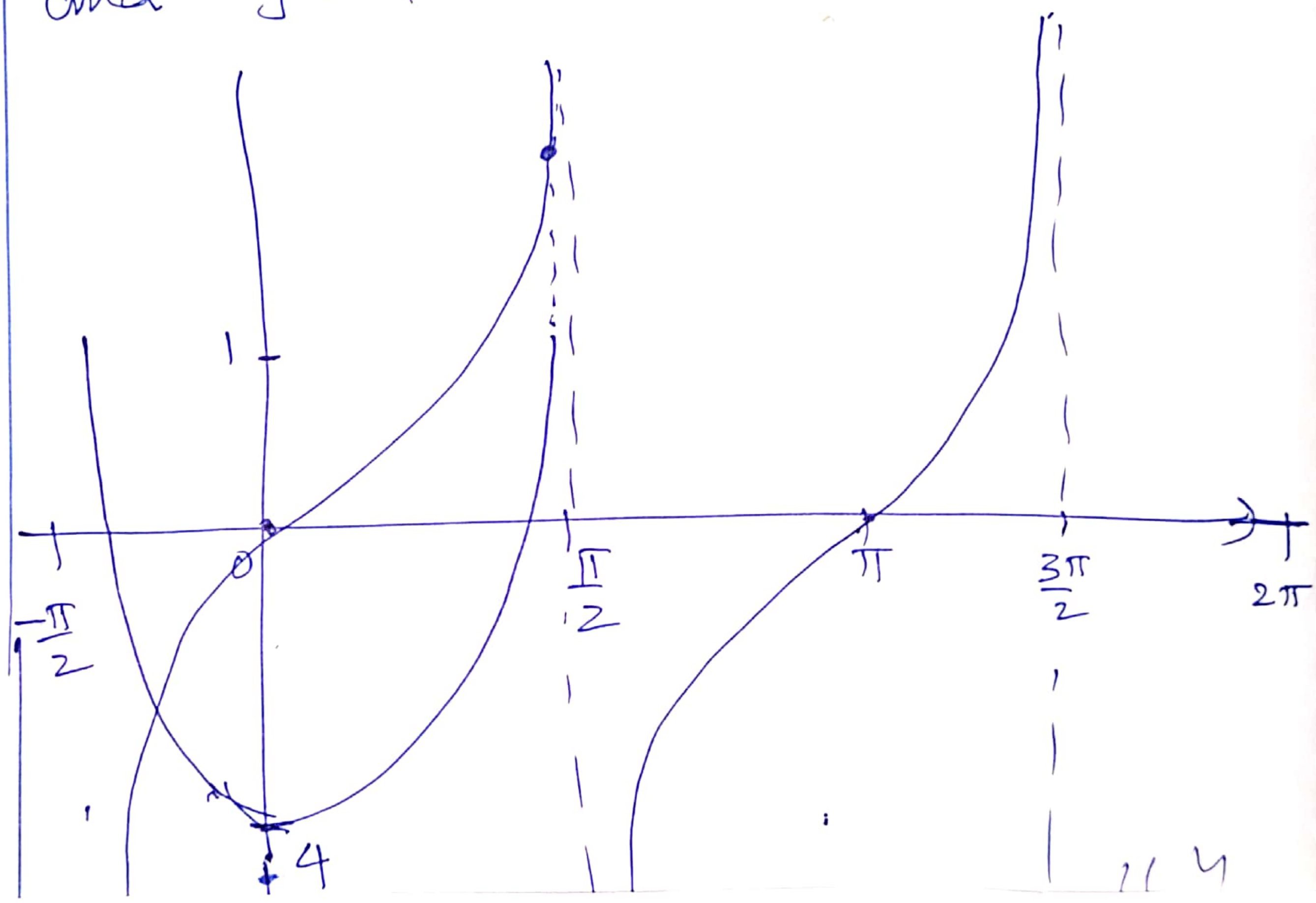
No. of integral values of m are 4

46

07 Given $3\sec\theta - 5 = 4\tan\theta$

Let us draw the graphs of $y = 3\sec\theta - 5$

and $y = 4\tan\theta$ in $[0, \pi]$



Clearly Both the graphs intersect at 2 points between $[0, \pi]$

47

Since, both the function are periodic function, in $[0, 4\pi]$, they intersect in 8 points.

Hence, they have 8 solutions in $[0, 4\pi]$

→ THE END ←

$= \left(\frac{\sqrt{3}}{}\right)^2$

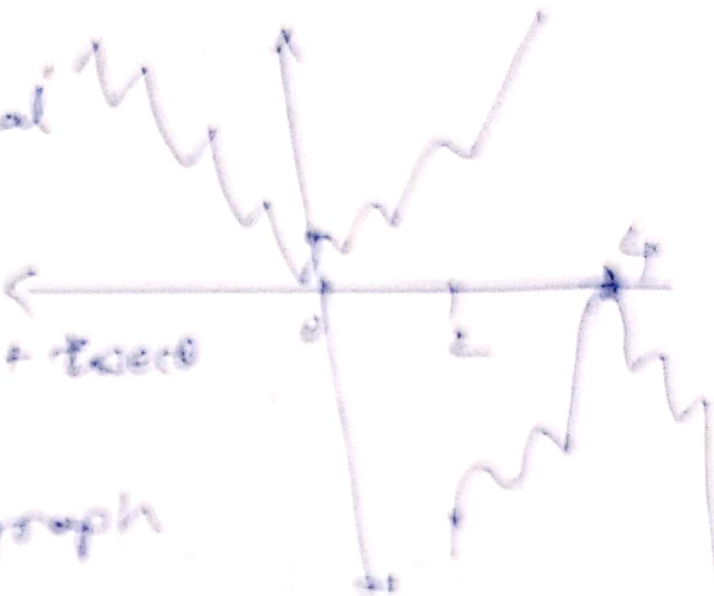
Comprehension - II

Given $\sec\theta + \csc\theta = a$,

Given $\theta \in (0, 2\pi) - \left\{\frac{\pi}{2}, \pi, \frac{3\pi}{2}\right\}$

Now, let us draw the graphs of

$y = \sec\theta + \csc\theta$ and $y = a$



Equation has two real distinct roots.

The period of $\sec\theta + \csc\theta$ is 2π.

According to the graph

$f(x) \to \infty$, so on

$\left(0, \frac{\pi}{2}\right)$ $f(x)$ is increasing.

At $\frac{\pi}{4}$ it has least value $2\sqrt{2}$, so in the given interval $|a| < 2\sqrt{2}$.

From the figure, we can say that $f(x) = 0$ has two distinct roots if $y = a$ cuts the graph $y = f(x)$ between $y = 2\sqrt{2}$ and $y = -2\sqrt{2}$.

i.e. $|a| < 2\sqrt{2}$