WS-4 foundation 8th class

Task

①

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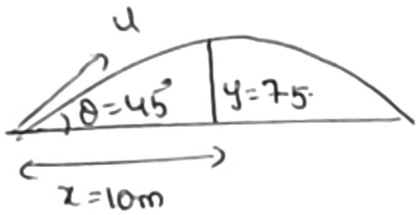

From $y = \tan\theta \, x - \frac{g}{2u^2\cos^2\theta} x^2$

$\Rightarrow 7.5 = \tan 45 \times 10 - \frac{10}{2u^2\cos^2 45}(10)^2$

$\Rightarrow 7.5 = 10 - \frac{5}{u^2 \times \frac{1}{2}} \times 100$

$\Rightarrow 7.5 = 10 - \frac{10}{u^2} \times 100$

$\Rightarrow \frac{1000}{u^2} = 10 - 7.5 \Rightarrow \frac{1000}{u^2} = 2.5 \Rightarrow u^2 = \frac{1000}{2.5} = \frac{10000}{25}$

$\Rightarrow u^2 = 400 \rightarrow u = 20\,m/s$

②

$\theta = 45^\circ$; $u = 20\sqrt{2}\,m/s$.

velocity after '3' sec $v = (u\sin\theta - gt)\hat{j} + u\cos\theta\,\hat{i}$

$\Rightarrow v = (20\sqrt{2}\sin 45 - 10\times 3)\hat{j} + 20\sqrt{2}\cos 45\,\hat{j}$

$= (20\sqrt{2}\,\frac{1}{\sqrt{2}} - 30)\hat{j} + 20\sqrt{2}\,\frac{1}{\sqrt{2}}\,\hat{j}$

$v = -10\hat{j} + 20\hat{i}$

2nd continuation

$$\vec{u} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$$

$$= 20\sqrt{2}\cos 45\,\hat{i} + 20\sqrt{2}\sin 45\,\hat{j}$$

$$= 20\sqrt{2} \times \frac{1}{\sqrt{2}}\,\hat{i} + 20\sqrt{2}\,\frac{1}{\sqrt{2}}\,\hat{j}$$

$$\vec{u} = 20\hat{i} + 20\hat{j}$$

$$\therefore \langle velocity \rangle = \frac{\vec{u}+\vec{v}}{2} = \frac{20\hat{i}+20\hat{j} + 20\hat{i} - 10\hat{j}}{2}$$

$$= \frac{40\hat{i} + 10\hat{j}}{2} = 20\hat{i} + 5\hat{j}$$

$$|velocity| = \sqrt{(20)^2 + 5^2} = \sqrt{400+25} = 20.62 \text{ m/s.}$$

(3)

Initial velocity vector $\vec{u} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

velocity after 't' sec $\vec{v} = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\hat{j}$

According to given question $\vec{u}$ and $\vec{v}$ are Perpendicular

i.e) $\vec{u}\cdot\vec{v} = 0$

$$\Rightarrow (u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}) \cdot (u\cos\theta\,\hat{i} + (u\sin\theta - gt)\hat{j}) = 0$$

$$\Rightarrow (u\cos\theta)(u\cos\theta) + u\sin\theta(u\sin\theta - gt) = 0$$

$$\Rightarrow u^2\cos^2\theta + u^2\sin^2\theta - u\sin\theta\, gt = 0$$

$$\Rightarrow u^2(\cos^2\theta + \sin^2\theta) - u\sin\theta\, gt = 0$$

$$\Rightarrow u^{\cancel{2}} = \cancel{u}\sin\theta\, gt \quad \Rightarrow u = g\sin\theta\, t$$

$$\Rightarrow t = \frac{u}{g\sin\theta}.$$

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(4)

$u = 20\sqrt{2}$ m/s ; $\theta = 45^\circ$

Initial velocity vector $\vec{u} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

$\Rightarrow \vec{u} = 20\sqrt{2}\cos 45\,\hat{i} + 20\sqrt{2}\sin 45\,\hat{j}$

$= 20\cancel{\sqrt{2}} \times \frac{1}{\cancel{\sqrt{2}}}\,\hat{i} + 20\cancel{\sqrt{2}} \times \frac{1}{\cancel{\sqrt{2}}}\,\hat{j}$

$\vec{u} = 20\hat{i} + 20\hat{j}$

velocity of a projectile at maximum height $= u\cos\theta\,\hat{i}$
$= 20\sqrt{2}\cos 45^\circ$
$= 20\hat{i}$

$\therefore \langle velocity \rangle = \frac{\vec{u}+\vec{v}}{2} = \frac{20\hat{i}+20\hat{j}+20\hat{i}}{2}$

$= \frac{40\hat{i}+20\hat{j}}{2} = 20\hat{i} + 10\hat{j}$

$|velocity| = \sqrt{20^2+10^2} = \sqrt{400+100} = \sqrt{500} = 10\sqrt{5}$ m/s.

(5)

Initial velocity $u = 20$ m/s. Range is maximum means angle of projection $\theta = 45^\circ$

As the ball ~~hits~~ before hits the ground, the second player has to catch it means time of journey for ball and player is same (i.e) $T = \frac{2u\sin\theta}{g} = \frac{2\times 20\times\sin 45}{10} = 4\times\frac{1}{\sqrt{2}} = 2\sqrt{2}$ sec

x is the distance travelled by the player $= (40-24)$
$= 16$m

$\therefore$ speed of player $= \frac{distance}{Time} = \frac{\cancel{16}^{8}}{\cancel{2}\sqrt{2}} = \frac{8}{\sqrt{2}} = 4\sqrt{2}$

Range $= \frac{u^2}{g} = \frac{20^2}{10} = 40$m

(6)

Initial velocity $\vec{u} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

velocity at highest point $\vec{v} = u\cos\theta\,\hat{i}$

change in velocity $dv = \vec{v} - \vec{u}$

$= \cancel{u\cos\theta}\,\hat{i} - \cancel{u\cos\theta}\,\hat{i} - u\sin\theta\,\hat{j}$

$= -u\sin\theta\,\hat{j}$

$|dv| = |-u\sin\theta\,\hat{j}| = u\sin\theta$

(7)

$u_A = v$; $u_B = \frac{v}{2}$ $\theta_B = 45^\circ$, $\theta_A = ?$

Given For A and B ranges are same

$R_A = R_B$

$\Rightarrow \frac{u_A^2 \sin 2\theta_A}{\cancel{g}} = \frac{u_B^2 \sin 2\theta_B}{\cancel{g}}$

$\Rightarrow v^2 \sin 2\theta_A = \left(\frac{v}{2}\right)^2 \sin 2(45^\circ)$

$\Rightarrow \cancel{v^2} \sin 2\theta_A = \frac{\cancel{v^2}}{4} \sin 90^\circ$ $\Rightarrow \sin 2\theta_A = \frac{1}{4}$

$\Rightarrow \theta_A = \frac{1}{2}\sin^{-1}\left(\frac{1}{4}\right)$

$\Rightarrow$

(9)

let the initial velocity $= u$; Area is maximum means Range of bullets is maximum which can act as radius for

$r = R_{max} = \frac{u^2}{g}$ Area $= \pi r^2$

$= \pi\left(\frac{u^2}{g}\right)^2$

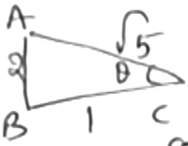
(8)

Given $R = 2H$

From $H = \frac{R}{4} \tan\theta$

$\Rightarrow H = \frac{2H}{4_2} \tan\theta$

$\Rightarrow 2 = \tan\theta$



$\sin\theta = \frac{2}{\sqrt{5}}$

$\cos\theta = \frac{1}{\sqrt{5}}$

$\text{Range} = u^2 \frac{\sin 2\theta}{g}$

$= u^2 \frac{2 \sin\theta \cos\theta}{g}$

$= \frac{2u^2}{g} \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}}$

$\text{Range} = \frac{4u^2}{5g}$

(10)

Range is maximum means $\theta = 45°$

At maximum height velocity $v = u\cos\theta = u\cos 45°$
$= \frac{u}{\sqrt{2}}$

change in velocity as it return to ground $= 2u\cos\theta$
$= 2 \times u \times \cos 45°$
$= 2\frac{u}{\sqrt{2}} = \sqrt{2}u$

From $v^2 - u^2 = 2as$ At half of max height
$S = \frac{H}{2} = \frac{u^2 \sin^2\theta}{2g}$

$\Rightarrow v_y^2 - u_y^2 = 2a_y \frac{H}{2}$

$\Rightarrow v_y^2 - (u\sin\theta)^2 = -2g \frac{u^2 \sin^2\theta}{2g \times 2}$

$\Rightarrow v_y^2 - u^2\sin^2\theta = -\frac{u^2\sin\theta}{2}$ $\Rightarrow v_y^2 = \frac{u^2\sin^2\theta}{2}$

$v_x = u_x = u\cos\theta$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\frac{u^2}{2}\sin^2\theta + u^2\cos^2\theta} = \sqrt{\frac{u^2}{2}(\sin 45)^2 + u^2(\cos 45)^2}$

$= \sqrt{\frac{u^2}{2} \times \frac{1}{2} + \frac{1}{2}u^2} \Rightarrow \frac{\sqrt{3}}{2}u$

(17), (18), (19)

$25 = \frac{u_p^2 \sin 2\theta}{g}$ $\theta = 45°$

$\Rightarrow 25 = \frac{u_p^2 \sin 2(45)}{10}$

$\Rightarrow 25 \times 10 = u^2 \sin 90°$

$\Rightarrow u_p^2 = 25 \times 10$

$\Rightarrow u_p = \sqrt{25 \times 10} = 5\sqrt{10}$ m/s

At A:-

At A and At P horizontal components of velocity are same

$u_A \cos\theta_A = u_p \cos\theta_B$

$\Rightarrow u_A \qquad = 5\sqrt{10} \cos 45°$

$= 5\sqrt{10} \times \frac{1}{\sqrt{2}}$

$\Rightarrow u_A \cos\theta_A = 5\sqrt{5} \rightarrow (1)$

At A Vertical component of velocity $u_A \sin\theta_A$

From $v^2 - u^2 = 2as$ $S = 12.5$

$\Rightarrow (u_A \cos\theta)^2 - (u_A \sin\theta)^2 = -2 \times g \times 12.5$

$\Rightarrow (5\sqrt{5})^2 - (u_A \sin\theta)^2 = -250$

$\Rightarrow 25 \times 5 - (u_A \sin\theta)^2 = -250$

$\Rightarrow (u_A \sin\theta)^2 = 250 + 125 = 375$

$u_A \sin\theta = 5\sqrt{15} \rightarrow (2)$

$\frac{(2)}{(1)} = \frac{u_A \sin\theta}{u_A \cos\theta} = \tan\theta = \frac{5\sqrt{15}}{5\sqrt{5}} = \sqrt{3} \rightarrow \theta = 60°$

From (1) $u_A \cos 60 = 5\sqrt{5}$ $\Rightarrow u_A \frac{1}{2} = 5\sqrt{5} \rightarrow u_A = 10\sqrt{5}$ m/s

Range $= \frac{u_p^2 \sin 2\theta}{g} = \frac{(5\sqrt{10})^2 \sin 2 \times \cancel{45}\, 60}{10} = \frac{500}{10} \times \sin 120$

$= 50 \times \frac{\sqrt{3}}{2} = 25\sqrt{3}$ m

Task

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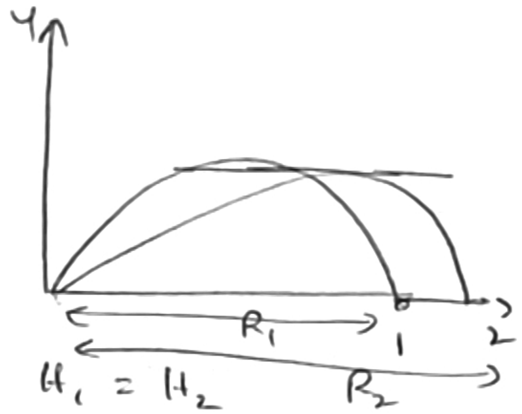


From fig it is clear that Maximum height reached by two Projectiles is same but Ranges are not same

$H_1 = H_2$

$\Rightarrow \dfrac{u_1^2 \sin^2\theta_1}{2\cancel{g}} = \dfrac{u_2^2 \sin^2\theta_2}{2\cancel{g}} \Rightarrow (u_1 \sin\theta_1)^2 = (u_2 \sin\theta_2)^2$

$\Rightarrow u_1 \sin\theta_1 = u_2 \sin\theta_2$

Time of flight $T = \dfrac{2u\sin\theta}{g}$ (ie) $T \propto u\sin\theta$

$\therefore T_1 = T_2.$

Since $R_2 > R_1$

$\Rightarrow \dfrac{u_2^2 \sin 2\theta_2}{\cancel{g}} > \dfrac{u_1^2 \sin 2\theta_1}{\cancel{g}}$

$\Rightarrow u_2^2\, 2\sin\theta_2 \cos\theta_2 > u_1^2 \times 2\sin\theta_1 \cos\theta_1$

$\Rightarrow \cancel{2u_2\sin\theta_2}\, u_2\cos\theta_2 > \cancel{2u_1\sin\theta_1}\, u_1\cos\theta_1$

$\Rightarrow u_2\cos\theta_2 > u_1\cos\theta_1$

Since cos is a decreasing function. $\theta_1 > \theta_2$

$\therefore \underline{u_1 > u_2}$

b, d are correct

(16)

$\theta_1 = 60^\circ, \theta_2 = 30^\circ$ Given two angles are complimentary angles and also $u_1 = u_2$

$\therefore$ Ranges are same

$R_1 = R_2$ $H = \frac{u_y^2}{2g}$ $T = \frac{2u_y}{g}$

$\theta_1 > \theta_2$

$H \propto \sin^2\theta$ $T \propto \sin\theta$

$\Rightarrow \frac{H_1}{H_2} = \left[\frac{\sin\theta_1}{\sin\theta_2}\right]^2$ $\frac{T_1}{T_2} = \left(\frac{\sin\theta_1}{\sin\theta_2}\right)$

As $\sin\theta$ is increasing function.

$H_1 > H_2$ $T_1 > T_2$

$\therefore \frac{H_1}{T_1} > \frac{H_2}{T_2}$ & $\frac{H_1}{R_1} > \frac{H_2}{R_2}$

so a, c are correct

(19)

$\theta_A > \theta_B > \theta_C$

Time of flight $T = \frac{2u\sin\theta}{g}$

$T_A > T_B > T_C$

Because $T \propto \sin\theta$

T is equal for A & B

$u_x \times u_y = u^2 \sin\theta\cos\theta$

$= \frac{u^2}{2} \sin 2\theta$

$u_x u_y$ is equal for ~~A~~ B & C

$u_x = u\cos\theta$ $\cos\theta$ is decreasing function as θ increases

Since $\theta_A > \theta_B > \theta_C$

$\therefore u_{x_C} > u_{x_B} > u_{x\,A}$

$\frac{u_y}{u_x} = \tan\theta \rightarrow$ it is increasing function

$\therefore \left(\frac{u_y}{u_x}\right)_A > \left(\frac{u_y}{u_x}\right)_B > \left(\frac{u_y}{u_x}\right)_C$

$H_C < H_B \Rightarrow u_C^2 \sin^2\theta_C < u_B^2 \sin^2\theta_B$

$\therefore$ ~~$H_A > H_B$~~

(6)

Given equation $y = \sqrt{3}x - \frac{g}{2}x^2$

compare with $y = \tan\theta\, x - \frac{g}{2u^2\cos^2\theta}x^2$

$\therefore \tan\theta = \sqrt{3}$ $\qquad u^2\cos^2\theta = 1$

$\Rightarrow \theta = 60^\circ$ $\qquad \Rightarrow u^2 = \frac{1}{\cos^2\theta} \Rightarrow u = \frac{1}{\cos\theta} \Rightarrow u = \frac{1}{\cos 60}$

$\Rightarrow u = \frac{1}{\frac{1}{2}} = 2$ m/s

(7)

$u = 19.6$ m/s ; $\theta = 30^\circ$

Time of flight : $\frac{2u\sin\theta}{g} = \frac{2\times \cancel{19.6}^{2}\sin 30}{\cancel{9.8}} = 2\times 2\times\frac{1}{2} = 2$ sec.

(8)

$u = 200$ m/s , $\theta' = 30^\circ$ with vertical

so with horizontal angle of Projection

$\theta = 90 - \theta' = 90 - 30 = 60^\circ$

$t = 3$ sec

horizontal distance covered $x = u\cos\theta\, t$

$\Rightarrow x = 200\times\cos 60\times 3$

$= 200\times\frac{1}{2}\times 3 =$

$= 300$m

(9)

$u = 20$ m/s. $\theta = 60^\circ$

$\vec{u} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$ $\qquad \Rightarrow \vec{u} = 10\hat{i} + 10\sqrt{3}\,\hat{j}$

$= 20\cos 60\,\hat{i} + 20\sin 60\,\hat{j}$

$= \cancel{20}^{10}\frac{1}{\cancel{2}}\,\hat{i} + \cancel{20}^{10}\frac{\sqrt{3}}{\cancel{2}}\,\hat{j}$

(5)

Acceleration of any Projectile is $\pm g$

so acceleration of one Projectile with another Projectile is 0

(4) The Given two angles $(45+\theta)$ & $(45-\theta)$ are complimentary angles so the ranges are same

$R_1 = \frac{u^2 \sin 2\theta_1}{g}$ $\quad$ $R_2 = \frac{u^2 \sin 2\theta_2}{g}$

$= \frac{u^2}{g} \sin 2(45+\theta)$ $\quad$ $= \frac{u^2 \sin 2(45-\theta)}{g}$

$= \frac{u^2}{g} \sin(90+2\theta)$ $\quad$ $= \frac{u^2 \sin(90-2\theta)}{g}$

$= \frac{u^2}{g} \cos 2\theta$ $\quad$ $= \frac{u^2}{g} \cos 2\theta$.

(1)

Range $\propto \sin 2\theta$ $\quad$ As θ changes from $0 \to 45°$

$2\theta \to 0 \to 90°$ so Range increases up to $\theta = 45°$
Because $\sin\theta$ is increasing function up to 90°

From $\theta > 45 \to 90°$ $\quad$ $2\theta \to 90 \to 180°$ Range decrease. here onwards because $\sin\theta$ is decreasing from $90 \to 180$.

(2)

$H \propto \sin^2\theta$ $\quad$ $\sin\theta$ is increasing function from $0 \to 90°$

so H also increases.

(10)

$H_{max} = \frac{u_y^2}{2g}$; $T = \frac{2u_y}{g}$ $\quad T \propto u_y$

$H \propto u_y^2$

$\Rightarrow H \propto T^2$ an Time flight doubled $T' = 2T$

$\Rightarrow \frac{H}{H'} = \left(\frac{T}{T'}\right)^2 \Rightarrow \frac{H}{H'} = \left[\frac{\cancel{T}}{2\cancel{T}}\right]^2 = \left[\frac{1}{2}\right]^2$

$\Rightarrow \frac{H}{H'} = \frac{1}{4} \Rightarrow \underline{H' = 4H}$

SAQ's

(11)

Given equation $y = \frac{x}{\sqrt{3}} - \frac{x^2}{60}$ m

compare with $y = \tan\theta\, x - \frac{g}{2u^2\cos^2\theta} x^2$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30°$.

(12)

Given $\theta = 30°$. $u = 50$ m/sec

$T = \frac{2u\sin\theta}{g} = \frac{2\times 50 \times \sin 30}{10} = \cancel{2}\times 5 \times \frac{1}{\cancel{2}} = 5$ sec

(13)

$u = 60$ m/s , $\theta = 30°$

After 3 sec velocity $v = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\hat{j}$

$= 60\cos 30\,\hat{i} + (60\sin 30 - 3\times 10)\hat{j}$

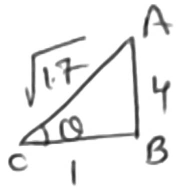
$= \cancel{60}^{30}\frac{\sqrt{3}}{\cancel{2}}\hat{i} + (\cancel{60}^{30}\times\frac{1}{\cancel{2}} - 30)\hat{j}$

$= 30\sqrt{3}\,\hat{i} + (30-30)\hat{j} = 30\sqrt{3}\,\hat{i}$ m/s

(14)

Given $H = R$

From : $H = \frac{R}{4}\,\mathrm{Tan}\,\theta$



$\Rightarrow \cancel{H} = \frac{\cancel{H}}{4}\,\mathrm{Tan}\,\theta \quad \Rightarrow \mathrm{Tan}\,\theta = 4$

$\sin\theta = \frac{4}{\sqrt{17}}$

$$H_{max} = \frac{u^2 \sin^2\theta}{2g} = \frac{u^2}{2g}\left[\frac{4}{\sqrt{17}}\right]^2 = \frac{u^2}{2g} \times \frac{16}{17} = \frac{8u^2}{17g}$$

(15)

$\theta = 60^\circ$

$K.E = K = \frac{1}{2} m u^2$

At maximum height velocity $v = u\cos\theta$

$v = u\cos 60 = \frac{u}{2}$

$$K.E_H = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\frac{u}{2}\right)^2 = \frac{\frac{1}{2} m u^2}{4}$$

$$K.E_H = \frac{K}{4}$$

(16)

Dangerous distance means maximum Range of a ball.

i.e) $R_{max} = \frac{u^2}{g} = \frac{(20)^2}{g} = \frac{400}{10} = 40m$

(17)

Given $H = 10m$ From $H = \frac{R}{4} \tan\theta$ $\frac{u^2}{2g}$

$\Rightarrow 10 = \frac{u^2}{2 \times 10} \Rightarrow u^2 = 200m$

maximum horizontal distance = maximum Range

$= \frac{u^2}{g} = \frac{200}{10} = 20m$

(18)

Max Range of grass hopper $= \frac{u^2}{g} = 0.3m$

Range is maximum for $\theta = 45^\circ$

$u^2 = 0.3g \Rightarrow u^2 = 0.3 \times 10 = 3$

$\Rightarrow u = \sqrt{3}$

Horizontal component of velocity $= u\cos\theta$

$= \sqrt{3}\cos 45 = \sqrt{\frac{3}{2}}$

(19)

$H_1 = 20m \rightarrow \theta_1 = \theta$

$H_2 = 10m \rightarrow \theta_2 = 90 - \theta$

The given two angles are complimentary angles so their range is same

$H_1 + H_2 = \frac{u^2 \sin^2\theta_1}{2g} + \frac{u^2 \sin^2\theta_2}{2g}$

$\Rightarrow 20 + 10 = \frac{u^2}{2g}[\sin^2\theta + \sin^2(90-\theta)]$

$\Rightarrow 30 = \frac{u^2}{2g}[\sin^2\theta + \cos^2\theta]$

$\Rightarrow 30 \times 2 = \frac{u^2}{g}(1)$

$\Rightarrow 60 = \frac{u^2}{g} \Rightarrow$ Range

$R = 60m$

Advanced

(8), (9), (1)

Given $u = 60\ m/s$; $\theta = 30^\circ$

Initial velocity vector $\vec{u} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

$\vec{u} = 60\cos 30\,\hat{i} + 60\sin 30\,\hat{j}$

$= \cancel{60}^{30} \times \frac{\sqrt{3}}{\cancel{2}}\,\hat{i} + \cancel{60}^{30}\,\frac{1}{\cancel{2}}\,\hat{j}$

$= 30\sqrt{3}\,\hat{i} + 30\,\hat{j}$

velocity after 3 sec $\vec{v} = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}$

$= 60\cos 30\,\hat{i} + (60\sin 30 - 10\times 3)\,\hat{j}$

$= \cancel{60}^{30}\,\frac{\sqrt{3}}{\cancel{2}}\,\hat{i} + (\cancel{60}^{30}\,\frac{1}{\cancel{2}} - 30)\,\hat{j}$

$= 30\sqrt{3}\,\hat{i} + (30-30)\,\hat{j} = 30\sqrt{3}\,\hat{i}$

Displacement after '2' sec $\vec{r} = u\cos\theta\, t\,\hat{i} + (u\sin\theta\, t - \frac{1}{2} g t^2)\,\hat{j}$

$= 60\cos 30 \times 2\,\hat{i} + (60\sin 30\times 2 - \frac{1}{\cancel{2}}\times 10\times \cancel{2^2})\,\hat{j}$

$= 60\,\frac{\sqrt{3}}{\cancel{2}}\times\cancel{2}\,\hat{i} + (60\times\frac{1}{\cancel{2}}\times\cancel{2} - 10\times 2)\,\hat{j}$

$= 60\sqrt{3}\,\hat{i} + (60-20)\,\hat{j}$

$= 60\sqrt{3}\,\hat{i} + 40\,\hat{j}.$