

PROGRESSIONS-III ①

class: VIII, Mathematics

SOLUTIONS

TEACHING TASK

01. $\frac{a}{r}, a, ar \rightarrow A.P$
 $\Rightarrow \frac{a}{r}, 2a, ar \rightarrow A.P$
 $\Rightarrow 2 \times 2a = \frac{a}{r} + ar$
 $\Rightarrow 4 = \frac{1}{r} + r$
 $\Rightarrow 4r = 1 + r^2$
 $\Rightarrow r^2 - 4r + 1 = 0$

$$r = 2 \pm \sqrt{3}$$

Since G.P. is increasing
 r should be positive
 hence $r = 2 + \sqrt{3}$ and
 $r = 2 - \sqrt{3}$

Ans: c

02

$$a^x = b^y = c^z$$

$$\Rightarrow \log a^x = \log b^y = \log c^z = k$$

$$\Rightarrow x \log a = k, y \log b = k, z \log c = k$$

$$\Rightarrow \log a = \frac{k}{x} \rightarrow (1), \log b = \frac{k}{y} \rightarrow (2), \log c = \frac{k}{z} \rightarrow (3)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{\log a}{\log b} = \frac{\left(\frac{k}{x}\right)}{\left(\frac{k}{y}\right)} = \frac{y}{x}$$

$$\frac{(2)}{(3)} \Rightarrow \frac{\log b}{\log c} = \frac{\left(\frac{k}{y}\right)}{\left(\frac{k}{z}\right)} = \frac{z}{y}$$

x, y, z AP

$$\therefore \frac{\log a}{\log b} = \frac{\log b}{\log c} \Rightarrow \log_b a = \log_c b$$

$$\text{Since } \frac{y}{x} = \frac{z}{y}$$

Ans c



03

(2)



$$\text{Total distance} = 120 + 2 \times 120 \times \frac{4}{5} + 2 \times 120 \times \frac{4}{5} \times \frac{4}{5} + \dots$$

$$\dots + 2 \times 120 \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} + \dots$$

$$= 120 + 2 \times 120 \times \frac{4}{5} \left[1 + \frac{4}{5} + \left(\frac{4}{5}\right)^2 + \left(\frac{4}{5}\right)^3 + \dots \right]$$

$$= 120 + 2 \times 120 \times \frac{4}{5} \times \frac{1}{1 - \frac{4}{5}}$$

$$= 120 + 2 \times 120 \times \frac{4}{5} \times \frac{5}{1}$$

$$= 120 + 960$$

$$= 1080.$$

Ans: C

04 $a, ar, ar^2, ar^3, \dots, a^3, (ar)^3, (ar^2)^3, \dots$

$$\frac{a}{1-r} = 57 \rightarrow (1)$$

$$\frac{a^3}{1-r^3} = 9747 \rightarrow (2)$$

$$\frac{a^3}{(1-r)^3} = (57)^3 \rightarrow (1)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{\left[\frac{a^3}{(1-r)^3} \right]}{\left[\frac{a^3}{1-r^3} \right]} = \frac{(57)^3}{9747}$$

$$\Rightarrow \frac{1-r^3}{(1-r)^3} = 19$$

$$\Rightarrow \frac{(1-r)(1+r+r^2)}{(1-r)(1-r)^2} = 19$$

$$\frac{1+r+r^2}{1-2r+r^2} = 19$$

$$\Rightarrow 1+r+r^2 = 19-18r+19r^2$$

$$\Rightarrow 18r^2 - 39r + 18 = 0$$

$$\Rightarrow r = \frac{2}{3} \text{ or } \frac{3}{2}$$

Ans: B

05

$$a^2 + b^2, ab + bc, b^2 + c^2 \rightarrow G.P.$$

(3)

$$\Rightarrow (ab + bc)^2 = (a^2 + b^2)(b^2 + c^2)$$

$$\Rightarrow b^4 - 2ab^2c + a^2c^2 = 0$$

$$\Rightarrow (b^2 - ac)^2 = 0$$

$$\Rightarrow b^2 = ac$$

$$\Rightarrow a, b, c \text{ are in G.P.}$$

Ans: B

06

$$ar, ar^2, ar^3, \dots, ar^{10}$$

$$\frac{1}{ar}, \frac{1}{ar^2}, \frac{1}{ar^3}, \dots, \frac{1}{ar^{10}}$$

$$\frac{ar(1-r^{10})}{1-r} = 18 \rightarrow (1)$$

$$\frac{\frac{1}{ar}(1-\frac{1}{r^{10}})}{1-\frac{1}{r}} = 6$$

$$\Rightarrow \frac{1}{ar^{11}} \left(\frac{r^{10}-1}{r-1} \right) r = 6 \rightarrow (2)$$

$$\Rightarrow \frac{1}{ar^{11}} \times 18 = 6 \quad (\text{from (1)})$$

$$\Rightarrow ar^{11} = 3$$

$$\text{product} = ar \cdot ar^2 \cdot ar^3 \cdot \dots \cdot ar^{10}$$

$$= a^{10} \cdot r^{1+2+3+\dots+10}$$

$$= a^{10} \cdot r^{55}$$

$$= (a \cdot r^{11})^5$$

$$= (3)^5$$

$$= 243$$

Ans: B



07.

$$\left. \begin{aligned} a_p &= a + (p-1)d = A \rightarrow ① \\ a_q &= a + (q-1)d = AR \rightarrow ② \\ a_r &= a + (r-1)d = AR^2 \rightarrow ③ \end{aligned} \right\} \text{G.P.} \rightarrow ④$$

$$② - ① \Rightarrow (q-p)d = (r-1)A \rightarrow ④$$

$$③ - ② \Rightarrow (r-q)d = (r-1)AR \rightarrow ⑤$$

$$\frac{⑤}{④} \Rightarrow \frac{(r-q)d}{(q-p)d} = \frac{(r-1)AR}{(r-1)A}$$

$$\therefore r = \frac{r-A}{q-p} = \frac{q-r}{p-q}$$

Ans: D

08

$$\cancel{a_p = a + (p-1)d; a_q = a + (q-1)d; a_r = a + (r-1)d; a_s = a + (s-1)d}$$

08

$$a_p = a + (p-1)d \rightarrow A \rightarrow ①$$

$$a_q = a + (q-1)d \rightarrow AR \rightarrow ②$$

$$a_r = a + (r-1)d \rightarrow AR^2 \rightarrow ③$$

$$a_s = a + (s-1)d \rightarrow AR^3 \rightarrow ④$$

$$① - ② \Rightarrow (p-q) = \frac{A-AR}{d} \quad \left| \begin{array}{l} ② - ③ \Rightarrow q-r = \frac{AR-AR^2}{d} \\ = \frac{R(A-AR)}{d} \end{array} \right.$$

$$③ - ④ \Rightarrow r-s = \frac{AR^2-AR^3}{d} = \frac{R^2(A-AR)}{d}$$

$\therefore$ clearly $p-q, q-r, r-s \rightarrow \text{G.P.}$ Ans: B

09. $f(x), f(2x), f(4x) \rightarrow G.P.$
 $2x+1, 2(2x)+1, 2(4x)+1 \rightarrow G.P.$
 $2x+1, 4x+1, 8x+1 \rightarrow G.P.$

$$\Rightarrow (4x+1)^2 = (2x+1)(8x+1)$$

$$\Rightarrow x=0$$

Ans: C

10. $\frac{t_4}{t_6} = \frac{1}{4} \Rightarrow \frac{ax^3}{ax^5} = \frac{1}{4} \Rightarrow x = \pm 2$

$$t_2 + t_5 = 216 \Rightarrow ax + ax^4 = 216$$

$$\Rightarrow a(2+16) = 216$$

$$\Rightarrow a = 12$$

Ans: A

11. $p, 1, \frac{1}{p}, \frac{1}{p^2}, \dots$

$$S_{\infty} = \frac{a}{1-r}$$

$$\Rightarrow \frac{9}{2} = \frac{p}{1-\frac{1}{p}}$$

$$\Rightarrow \frac{9}{2} = \frac{p^2}{p-1}$$

$$\Rightarrow 9p-9 = 2p^2$$

$$\Rightarrow 2p^2 - 9p + 9 = 0$$

$$p = 3, \frac{3}{2}$$

Ans: B, C

12. $3a_1 + 7a_2 + 3a_3 - 4a_5 = 0$

$$\Rightarrow 3a + 7ax + 3ax^2 - 4ax^4 = 0$$

$$\Rightarrow 3 + 7x + 3x^2 - 4x^4 = 0$$

$$\Rightarrow 4x^4 - 3x^2 - 7x - 3 = 0$$

(6)

$$12 \quad 3a_1 + 7a_2 + 3a_3 - 4a_5 = 0$$

$$\Rightarrow 7a_1 + 7a_2 + 7a_3 = 4a_1 + 4a_3 + 4a_5$$

$$\Rightarrow 7(a + ar + ar^2) = 4(a + ar^2 + ar^4)$$

$$\Rightarrow 7(1+r+r^2) = 4(1+r+r^2)$$

$\Rightarrow$

$$12 \quad 3a_1 + 7a_2 + 3a_3 - 4a_5 = 0$$

$$\Rightarrow 3a + 7(ar) + 3(ar^2) - 4(ar^4) = 0$$

$$\Rightarrow 3 + 7r + 3r^2 - 4r^4 = 0$$

$$\Rightarrow 4r^4 - 3r^2 - 7r - 3 = 0$$

By trial & Error method $r = \frac{3}{2}, -\frac{1}{2}$ are the solutions.

Ans: B, D

$$13 \quad ar^{m+n-1} = p \rightarrow (1); \quad ar^{m-n-1} = q \rightarrow (2)$$

$$(1) \times (2) \Rightarrow a^2 r^{m+n-1+m-n-1} = pq$$

$$\Rightarrow a^2 \cdot r^{2m-2} = pq$$

$$\Rightarrow (a r^{m-1})^2 = pq$$

$$\Rightarrow ar^{m-1} = \sqrt{pq}$$

$$\Rightarrow a_m = \sqrt{pq}$$

Statement II: Conceptual (true).

Ans: A

14. Statement I:

(7)

$$\begin{aligned}
 & a \times ar \times ar^2 \times \dots \times ar^{n-1} = \\
 & = a^n \cdot r^{1+2+\dots+n-1} \\
 & = a^n \cdot r^{\frac{(n-1)(n)}{2}} = \left(a \cdot r^{\frac{n-1}{2}} \right)^n \quad (\text{Total})
 \end{aligned}$$

Statement II: In a G.P. i th term from the

beginning $= ar^{i-1}$.

i th term from the end $= \frac{l}{r^{i-1}}$

product $= a \cancel{r^{i-1}} \times \frac{l}{\cancel{r^{i-1}}} = al$, which is

independent of i

Ans: A

17

$1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \dots$

$a=1, r=-\frac{1}{2}, n=20$ here $r < 1$

$$\therefore S_n = \frac{a(1-r^n)}{1-r} = \frac{1(1-(-\frac{1}{2})^{20})}{1-(-\frac{1}{2})} = \frac{2}{3} \left(1 - \left(\frac{1}{2} \right)^{20} \right)$$

Ans: C

18

$3, 3, 3, \dots$ 30 term

$$\begin{aligned}
 S_{30} &= na \\
 &= 30 \times 3 \\
 &= 90.
 \end{aligned}$$

Ans: D

19

$$G.M = \sqrt{(-36)(-25)}$$

$$= \sqrt{-1} \times \sqrt{-1} \times \sqrt{36 \times 25}$$

$$= i \times i \times 6 \times 5$$

$$= -30 \quad \text{since } i^2 = -1$$

Ans: B

20

$$G.M = \sqrt{\frac{1}{2} \cdot \frac{1}{8}}$$

$$= \pm \frac{1}{4}$$

Ans: C

21

$$\sum_{i=1}^{201} a_i = 625$$

$$\Rightarrow a_1 + a_2 + \dots + a_{201} = 625$$

$$\Rightarrow \frac{a(1-r^{201})}{1-r} = 625 \rightarrow (1)$$

$$\text{Now } \sum_{i=1}^{201} \frac{1}{a_i} = \frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_{201}} =$$

$$= \frac{1}{a} \left(\left(\frac{1}{r} \right)^{201} - 1 \right)$$

$$= \frac{1}{a} \left(\frac{1-r^{201}}{1-r} \right) \cdot \frac{1}{r^{200}}$$

$$= \frac{1}{a} \cdot \frac{625}{a} \cdot \frac{1}{r^{200}}$$

$$= \frac{625}{(a \cdot r^{100})^2} = \frac{625}{(25)^2} = 1$$

$$\text{Since } a r^{100} = 25$$

Ans: 1



$$\frac{a}{r}, a, ar \rightarrow G.P.$$

$$\Rightarrow \text{product} \Rightarrow \frac{a}{r} \cdot a \cdot ar = -216$$

(9)

$$\Rightarrow a^3 = -216$$

$$\Rightarrow a^3 = (-6)^3$$

$$\Rightarrow \boxed{a = -6}$$

$$\Rightarrow \cancel{a} \left(\frac{1}{r} + 1 + r \right) = -a \Rightarrow \frac{1}{r} + 1 + r = -1$$

$$\Rightarrow \text{Again } \frac{a}{r} \cdot a + a \cdot ar + \frac{a}{r} \cdot ar = b$$

$$\Rightarrow a^2 \left(\frac{1}{r} + r + 1 \right) = b$$

$$\Rightarrow 36(-1) = b \Rightarrow \boxed{b = -36}$$

$$\therefore \frac{b}{a} = 6$$

$$\begin{aligned} \text{a) } a, b, c \text{ A.P.} &\Rightarrow 2b = a + c \\ a, x, b \text{ G.P.} &\Rightarrow x^2 = ab \\ b, y, c \text{ G.P.} &\Rightarrow y^2 = bc \end{aligned}$$

$$\begin{aligned} \text{Now } x^2 + y^2 &= ab + bc \\ &= b(a + c) \\ &= b(2b) = 2b^2 \end{aligned}$$

$\therefore x^2, b^2, y^2$ are in A.P.

$$\text{b) } a, y, z \text{ G.P.} \Rightarrow y^2 = az \rightarrow \textcircled{1}$$

$$\Rightarrow a^x = b^y = c^z = K$$

$$\Rightarrow x = \log_a K = \frac{\log K}{\log a}$$

$$y = \frac{\log K}{\log b}, \quad z = \frac{\log K}{\log c}$$

$$\text{Now } \textcircled{1} \Rightarrow \left(\frac{\log K}{\log b} \right)^2 = \frac{\log K}{\log a} \cdot \frac{\log K}{\log c}$$

$$\Rightarrow (\log b)^2 = \log a \cdot \log c$$

$$\Rightarrow \log a, \log b, \log c \rightarrow \text{A.P.}$$

$$\text{c) } a = 3, \quad ar^{n-1} = 96, \quad \frac{a(r^n - 1)}{r - 1} = 189.$$

$$\Rightarrow r^{n-1} = 32 \Rightarrow \frac{r^n - 1}{r - 1} = 63$$

$$\Rightarrow r^n = 32r \Rightarrow \frac{32r - 1}{r - 1} = 63 \Rightarrow r = 2$$

$$\text{Now } 2^n = 32 \times 2 = 64 \Rightarrow n = 6$$

$$\text{d) } S_x = \frac{a}{1-r} = 2 \Rightarrow \frac{1}{1-r} = 2 \Rightarrow r = \frac{1}{2}$$

Ans: t, r, n, S

$$a) (\sqrt{2}+1) + 1 + (\sqrt{2}-1) + \dots \infty \quad (11)$$

$$S_{\infty} = \frac{\sqrt{2}+1}{1 - \left(\frac{1}{\sqrt{2}+1}\right)} = \frac{(\sqrt{2}+1)^2}{\sqrt{2}} = \frac{3+2\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}+4}{2}$$

$$b) \frac{1}{2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^4} + \frac{1}{2^5} + \frac{1}{3^6} + \dots \infty$$

$$= \left(\frac{1}{2} + \frac{1}{2^3} + \frac{1}{2^5} + \dots \right) + \left(\frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6} + \dots \right)$$

$$= \frac{\frac{1}{2}}{1 - \frac{1}{2}} + \frac{\frac{1}{3^2}}{1 - \frac{1}{9}}$$

$$= 1 + \frac{1}{8} = \frac{9}{8}$$

$$c) 6^{\frac{1}{2}} \times 6^{\frac{1}{4}} \times 6^{\frac{1}{8}} \times \dots =$$

$$= 6^{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots}$$

$$= 6^{\frac{\frac{1}{2}}{1 - \frac{1}{2}}} = 6$$

$$d) \frac{a}{1-r} = 20 \rightarrow (1) \quad \frac{a^2}{1-r^2} = 100 \rightarrow (2)$$

$$\Rightarrow \frac{a}{1-r} \cdot \frac{a}{1+r} = 100$$

$$\Rightarrow 20 \times \frac{a}{1+r} = 100$$

$$\Rightarrow \frac{a}{1+r} = 5 \rightarrow (2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{1+r}{1-r} = 4$$

$$\Rightarrow r = \frac{3}{5}$$

Ans: s, t, p, q

LEARNER'S TASK

(12)

CUQ'S

01.

$$t_{n+1} = ar^n$$

Ans: B

02.

$$r = \frac{t_n}{t_{n-1}} = \frac{t_{n+1}}{t_n} = \frac{t_{n-2}}{t_{n-3}}$$

Ans: D

03. Conceptual (all the terms are in G.P.)

Ans: D

04. Conceptual

Ans: A

05. $a, b, c \rightarrow G.P.$

$$\Rightarrow b^2 = ac, \quad \frac{b}{a} = \frac{c}{b}, \quad \frac{a}{b} = \frac{b}{c}$$

Ans: D

06. Conceptual

Ans: D

~~07~~ $S_n = \frac{a(1-r^n)}{1-r}$

Ans: B

~~08~~

07. $S_n = \frac{a(1-r^n)}{1-r}$

Ans: B

08 $S_n = \frac{a(1-r^n)}{1-r}$

Ans: B

09. $S_\infty = \frac{a}{1-r}$

Ans: A

10. $S_n = na$

Ans: A

JEE MAINS LEVEL

B

01. $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2 \rightarrow$ 5 terms of G.P.

$$a = 4, \text{ product} = \frac{a}{r^2} \times \frac{a}{r} \times a \times ar \times ar^2 =$$

$$= a^5$$

$$= 4^5$$

$$= 1024$$

Ans. A

02.

Clearly 1, 4, 16, 64, 256 are in G.P.
Ans: C

03 $\frac{a}{r} + a + ar = 70 \rightarrow (1)$

$$\frac{4a}{r}, 5a, 4ar \rightarrow \text{A.P.}$$

$$\Rightarrow 10a = \frac{4a}{r} + 4ar$$

$$\Rightarrow 10 = \frac{4}{r} + 4r$$

$$\Rightarrow 2r^2 - 5r + 2 = 0$$

$$\Rightarrow \boxed{r = \frac{1}{2}, 2}$$

$$\text{sub in (1)} \Rightarrow \boxed{a = 20}$$

Sum

$$\frac{4a}{r} + 5a + 4ar$$

$$\frac{4(20)}{2} + 5(20) + 4(20)(2)$$

$$= 40 + 100 + 160$$

$$= 300$$

04.

3, ~~27~~, —, 81, G.P.

option verification.

let 9, 27 be the numbers

hence, 3, 9, 27, 81 are A.P.

Ans: D

05

$$\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$$

$$\text{let } n = -\frac{1}{2}$$

$$= \frac{a^{-\frac{1}{2}+1} + b^{-\frac{1}{2}+1}}{a^{-\frac{1}{2}} + b^{-\frac{1}{2}}}$$

$$= \frac{\sqrt{a} + \sqrt{b}}{\left(\frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}}\right)}$$

 $= \sqrt{ab}$, which is G.M
between a and b

Ans: C

06

$$ar^5 = -\frac{1}{32} \rightarrow \textcircled{1} \quad ar^8 = \frac{1}{256} \rightarrow \textcircled{2}$$

$$\frac{\textcircled{2}}{\textcircled{1}} \Rightarrow \frac{ar^8}{ar^5} = \frac{\left(\frac{1}{256}\right)}{\left(-\frac{1}{32}\right)} = -\frac{1}{8}$$

$$\Rightarrow r^3 = \left(-\frac{1}{2}\right)^3 \Rightarrow \boxed{r = -\frac{1}{2}}$$

$$Q_1 = \text{Also } ar^5 = -\frac{1}{32}$$

$$\Rightarrow a\left(-\frac{1}{2}\right)^5 = -\frac{1}{32}$$

$$\Rightarrow \boxed{a=1}$$

$$\begin{aligned} \text{Now } a_{11} &= ar^{10} \\ &= 1 \cdot \left(-\frac{1}{2}\right)^{10} \\ &= \frac{1}{1024} \end{aligned}$$

Ans: B

07. Sum of three numbers = 26

(15)

$$\text{i.e. } a_1 + a_2 + a_3 = 26$$

$$\text{Also } a_1 a_2 + a_2 a_3 + a_1 a_3 = 156$$

option verification. let us take the numbers 2, 6, 18

$$\text{Now, } 2 + 6 + 18 = 26$$

$$\text{Also } 2 \cdot 6 + 6 \cdot 18 + 18 \cdot 2 = 156$$

Ans: A

Q8 ~~Given~~ $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots \text{ up to } \infty = \frac{\pi^2}{6}$

Now, $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots \text{ up to } \infty$
 $= \frac{1}{1^2} + 1$

Q8 $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots \text{ up to } \infty = \frac{\pi^2}{6}$

Now $S = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$

$$= \left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots \infty \right) - \left(\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \dots \infty \right)$$

$$= \frac{\pi^2}{6} - \frac{1}{4} \left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots \infty \right)$$

$$= \frac{\pi^2}{6} - \frac{1}{4} \times \left(\frac{\pi^2}{6} \right)$$

$$= \frac{\pi^2}{6} \left(1 - \frac{1}{4} \right)$$

$$= \frac{\pi^2}{6} \times \frac{3}{4} = \frac{\pi^2}{8}$$

Ans: E

09.

$$S_{\infty} = 4, \quad ar = \frac{3}{4}$$

$$\frac{a}{1-r} = 4$$

$$\Rightarrow \frac{ar}{r-r^2} = 4$$

$$\Rightarrow \frac{\left(\frac{3}{4}\right)}{r-r^2} = 4$$

$$\frac{3}{r-r^2} = 16$$

(16)

$$\Rightarrow 16r - 16r^2 = 3$$

$$\Rightarrow 16r^2 - 16r + 3 = 0$$

$$\Rightarrow 16r^2 - 12r - 4r + 3 = 0$$

$$\Rightarrow 4r(4r-3) - 1(4r-3) = 0$$

$$\Rightarrow (4r-3)(4r-1) = 0$$

$$\Rightarrow r = \frac{3}{4}, \quad \boxed{\frac{1}{4}}$$

$$\text{Now } ar = \frac{3}{4}$$

$$\Rightarrow a \cdot \frac{1}{4} = \frac{3}{4}$$

$$\Rightarrow \boxed{a=3}$$

Ans: D

$$10. \quad a + ar + ar^2 + \dots = 20$$

$$\frac{a}{1-r} = 20$$

$$\Rightarrow r = \frac{20-a}{20}$$

$$\text{Now, } a^2, a^2r^2, a^2r^4, \dots \infty$$

$$S_{\infty} = \frac{a^2}{1-r^2} = 100$$

$$\Rightarrow a^2 = 100 \left(1 - \left(\frac{20-a}{20} \right)^2 \right)$$

$$\Rightarrow a^2 = 100 \left(\frac{400 - 400 + 40a - a^2}{400} \right)$$

$$\Rightarrow 4a^2 = 40a - a^2$$

$$\Rightarrow 5a^2 = 40a \Rightarrow a = 8$$

Ans: D



11. $a = x^{-4}$, $ax = x^n$, $ax^7 = x^{52}$

(17)

$$\begin{aligned} ax &= x^n \\ \Rightarrow x^{-4}x &= x^n \\ \Rightarrow x &= x^{n+4} \end{aligned} \quad \left| \begin{aligned} \text{Now } ax^7 &= x^{52} \\ \Rightarrow x^{-4} \cdot (x^{n+4})^7 &= x^{52} \\ \Rightarrow x^{-4} \cdot x^{7n+28} &= x^{52} \\ \Rightarrow x^{7n+24} &= x^{52} \\ \Rightarrow 7n+24 &= 52 \\ \Rightarrow n &= 8 \text{ or } \frac{8}{2} \end{aligned} \right.$$

Ans: B, C

12. $1 + 3 + 9 + \dots$

$a = 1$, $r = 3$

$$\frac{1(3^n - 1)}{3 - 1} = 364$$

$$\Rightarrow 3^n - 1 = 728$$

$$\Rightarrow 3^n = 729$$

$$\Rightarrow n = 6 \text{ or } \frac{12}{2}$$

Ans: A, B

13. Statement I: a, b, c are in G.P.

Let $a = 1$, $b = 2$, $c = 4$.

$$\begin{aligned} \text{Now } \frac{b-a}{b-c} + \frac{b+a}{b+c} &= \frac{2-1}{2-4} + \frac{2+1}{2+4} \\ &= \frac{1}{-2} + \frac{3}{6} = 0. \text{ (True)} \end{aligned}$$

Statement II: $S_n = \frac{a(1-r^n)}{1-r}$ when $r < 1$ (False)

Ans: C

14. Statement I: 6, x , y , z , 54 are in G.P. (18)

$$a=6, ar^4=54$$

$$\Rightarrow 6 \cdot r^4 = 54$$

$$\Rightarrow r^4 = 9$$

$$\Rightarrow r = \left(\frac{9}{6}\right)^{\frac{1}{4}} = \sqrt{3}$$

$$\text{Now } z = ar^3$$

$$= 6 \cdot (\sqrt{3})^3 = 6 \times 3\sqrt{3} = 18\sqrt{3} \text{ (True)}$$

Ans: A

Statement II: Conceptual (True)

17

$$S_{\infty} = \frac{a}{1-r}$$

$$= \frac{1}{1-\frac{1}{2}} = 2$$

Ans: C

18

$$S_{\infty} = \frac{1}{3}$$

$$\Rightarrow \frac{a}{1-r} = \frac{1}{3}$$

$$\Rightarrow \frac{a}{1+\frac{1}{2}} = \frac{1}{3}$$

$$\Rightarrow \frac{2a}{3} = \frac{1}{3}$$

$$\Rightarrow a = \frac{1}{2}$$

Ans: A

19

$$S_n = \frac{18-a}{r-1} = \frac{32 \cdot 2 - 2}{2-1} = 62$$

Ans: C

$$S_n = \frac{13}{27}$$

(19)

$$\Rightarrow \frac{a - 18}{1 - r} = \frac{13}{27}$$

$$\Rightarrow \frac{a - \frac{1}{27} \cdot \frac{1}{3}}{1 - \frac{1}{3}} = \frac{13}{27}$$

$$\Rightarrow a = \frac{1}{3}$$

Ans: B

$$a, ar, ar^2, \dots, ar^{n-1}$$

Given $t_n = 2(t_{n+1} + t_{n+2} + \dots)$

$$t_n = 2(ar^n + ar^{n+1} + \dots)$$

$$\Rightarrow t_n = 2ar^n(1 + r + r^2 + \dots \infty)$$

$$\Rightarrow t_n = 2ar^n \times \frac{1}{1-r}$$

$$\Rightarrow ar^{n-1} = \frac{2ar^n}{1-r}$$

$$\Rightarrow \frac{r}{1-r} = \frac{1}{2}$$

$$\Rightarrow 2r = 1-r$$

$$\Rightarrow 3r = 1$$

$$\Rightarrow r = \frac{1}{3}$$

$$\Rightarrow r = 0.33$$

Integral part = 0.

Ans: 0

22

(20)

$$\frac{a+b}{2} = 13 \quad \& \quad \sqrt{ab} = 12$$

$$\Rightarrow a+b=26 \quad \& \quad ab=144$$

$$\begin{aligned} \text{Now } (a-b)^2 &= (a+b)^2 - 4ab \\ &= (26)^2 - 4 \times 144 \\ &= 100 \end{aligned}$$

$$\Rightarrow a-b=10.$$

Ans: 10

23

$$a) \quad t_n = ar^{n-1}$$

$$b) \quad r = \frac{t_n}{t_{n-1}}$$

$$c) \quad S_{\infty} = \frac{a}{1-r}$$

$$d) \quad S_n = \frac{a(r^n - 1)}{r - 1}$$

Ans: t, r, S

a) $ax^3 = x$, $ax^9 = y$, $ax^{15} = z$

(21)

$$y^2 = (ax^9)^2 = a^2 \cdot x^{18}$$

$$xz = ax^3 \cdot ax^{15} = a^2 \cdot x^{18}$$

$$\therefore y^2 = xz \Rightarrow x, y, z \text{ are in G.P.}$$

b) $2+x, 14+x, 62+x \rightarrow \text{G.P.}$

$$\Rightarrow (14+x)^2 = (2+x)(62+x)$$

$$\Rightarrow x = 2$$

c) $x, 2y, 3z$ are in A.P.

$$\Rightarrow 4y = x + 3z \rightarrow (1)$$

$$x, y, z \text{ are in G.P.} \\ \Rightarrow y^2 = xz$$

Now, $\frac{y}{x} = \frac{z}{y} = r$ Also $\frac{z}{x} = r^2$

Now $4 \cdot \frac{y}{x} = \frac{x}{x} + 3 \cdot \frac{z}{x}$

$$r = \frac{1}{3} \text{ or } 1$$

$$\Rightarrow 4r = 1 + 3r^2$$

$$\Rightarrow 3r^2 - 4r + 1 = 0$$

d) Let $a=1, b=2, c=4, d=8$ are in G.P.
 $a^n + b^n, b^n + c^n, c^n + d^n$

Let $n=1$ $a+b, b+c, c+d$

$$1+2, 2+4, 4+8$$

$$3, 6, 12 \rightarrow \text{G.P.}$$

Ans: S, P, G, S

Teaching Task (JEE Advanced)

15. Assertion:

$$\begin{aligned} a_{m+n} &= a \cdot r^{m+n-1} \\ &= a \cdot r^{m-1} \cdot r^n \\ &= a \cdot r^n \quad (\text{True}) \end{aligned}$$

Reason: $a_n = a \cdot r^{n-1}$ (False)

Ans: C

16

Assertion: $S_{100} = \frac{a(1-r^n)}{(1-r)}$

$$\begin{aligned} &= \frac{10(1-(0.5)^{100})}{1-0.5} \\ &= \frac{10(1-\frac{1}{2^{100}})}{\frac{1}{2}} \end{aligned}$$

$$\begin{aligned} &= 20(1-\frac{1}{2^{100}}) \\ &= 20 \quad (\text{True}) \end{aligned}$$

Since $\frac{1}{2^{100}} \rightarrow 0$

Reason: Conceptual (True)

Ans: A

Learner's Task (JEE Advanced)

15. Assertion: $r = \frac{2}{-2} = -1$ (True)

Reason: Conceptual (True)

Ans: A

16. Assertion: Conceptual (True)

Reason: Conceptual (True)

Ans: A