

8. PROPERTIES OF TRIANGLES-I

Class: IX ; MATHEMATICS ①

FOUNDATION+ SOLUTIONS

TEACHING TASK
JEE MAINS LEVEL

2025 26

Q1. Given $A : B : C = 2 : 3 : 5$

We have $A = \frac{2}{10} \times 180^\circ = 36^\circ$

$$B = \frac{3}{10} \times 180^\circ = 54^\circ$$

$$C = \frac{5}{10} \times 180^\circ = 90^\circ$$

Here, greatest angle $= C = 90^\circ$

and, least angle $= A = 36^\circ$

We have $\frac{C}{\sin 90^\circ} = \frac{a}{\sin 36^\circ}$

$$\Rightarrow \frac{c}{a} = \frac{\sin 90^\circ}{\sin 36^\circ} = \frac{1}{\left(\frac{\sqrt{10-2\sqrt{5}}}{4}\right)} = \frac{4}{\sqrt{10-2\sqrt{5}}}$$

$$\therefore c : a = 4 : \sqrt{10-2\sqrt{5}}$$

Ans: B

Q2. Given $A + C = 2B$

$$\Rightarrow A + B + C = 3B$$

$$\Rightarrow 180^\circ = 3B$$

$$\Rightarrow B = 60^\circ$$

$$\Rightarrow A + C = 120^\circ$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos 60^\circ = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\Rightarrow \frac{1}{2} = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\Rightarrow b^2 = a^2 - ac + c^2$$

$$\text{Now, } \frac{a+c}{\sqrt{a^2 - ac + c^2}} = \frac{a+c}{\sqrt{b^2}} \\ = \frac{a+c}{b}$$

$$\text{Now, } \frac{a+c}{b} = \frac{2R \sin A + 2R \sin C}{2R \sin B}$$

(2)

$$= \frac{\sin A + \sin C}{\sin B}$$

$$= \frac{2 \sin\left(\frac{A+C}{2}\right) \cos\left(\frac{A-C}{2}\right)}{\sin 60^\circ}$$

$$= \frac{2 \sin\left(\frac{120^\circ}{2}\right) \cos\left(\frac{A-C}{2}\right)}{\sin 60^\circ}$$

$$= \frac{2 \sin\left(\frac{120^\circ}{2}\right) \cos\left(\frac{A-C}{2}\right)}{\sin 60^\circ}$$

$$= 2 \cos\left(\frac{A-C}{2}\right)$$

Ans: A

03 Given $c^4 - 2(a^2 + b^2)c^2 + a^4 + a^2b^2 + b^4 = 0$

Let $a = b = c = 1$

$$\Rightarrow 1^4 - 2(1^2 + 1^2)1^2 + 1^4 + 1^2 \cdot 1^2 + 1^4 = 0$$

$$= 1 - 4 + 1 + 1 + 1 = 0$$

$\therefore$ The given eqn satisfies when $a = b = c$

$\therefore \triangle ABC$ is an equilateral triangle

$$\therefore \angle C = 60^\circ$$

Ans: C

04 Given $x^2 - 2\sqrt{3}x + 2 = 0$

Let a, b be two sides of the triangle

$$\therefore a + b = 2\sqrt{3}, \quad ab = 2$$

$$\text{Now } \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$= \cos \frac{\pi}{3} = \frac{(a+b)^2 - 2ab - c^2}{2ab}$$

$$\Rightarrow \frac{1}{2} = \frac{(2\sqrt{3})^2 - 2(2) - c^2}{2(2)}$$

(3)

$$\Rightarrow c = \sqrt{6}$$

$$\therefore \text{perimeter} = a + b + c$$

$$= 2\sqrt{3} + \sqrt{6}$$

Ans. B

05 Shortcut methodConsider $\triangle ABC$ is an equilateral triangle

$$\therefore \text{let } a = b = c = 1$$

$$\therefore (a+b+c)(b+c-a) = \lambda bc$$

$$\Rightarrow (1+1+1)(1+1-1) = \lambda \cdot 1 \cdot 1$$

$$\Rightarrow \boxed{3 = \lambda}$$

In the options: c) $0 < \lambda < 4$ satisfies

Ans. C

06 Shortcut method

$$\text{Given } \tan \frac{A}{2} \cdot \tan \frac{B}{2} + \tan \frac{B}{2} \cdot \tan \frac{C}{2} = \frac{2}{3}$$

This eqy satisfies when $A = B = C = 60^\circ$

$$\text{Since } LHS = \tan 30^\circ \cdot \tan 30^\circ + \tan 30^\circ \cdot \tan 30^\circ$$

$$= \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

 $\therefore$ i.e. Given triangle is an equilateral triangleHence $a + c = 2b$ satisfies equilateraltriangle property ($a = b = c$)

Ans. B

07. Without loss of generality, let us consider an equilateral triangle with unit side

$$\text{i.e. } a = b = c = 1. \text{ then}$$

$$\text{Altitudes} = \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{2} \cdot 1 = \frac{\sqrt{3}}{2}, \quad \Delta = \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4}$$

Consider $\frac{1}{\alpha} + \frac{1}{\beta} - \frac{1}{\gamma} = \frac{2ab}{(a+b+c)\Delta} \cdot \cos \frac{C}{2}$

(4)

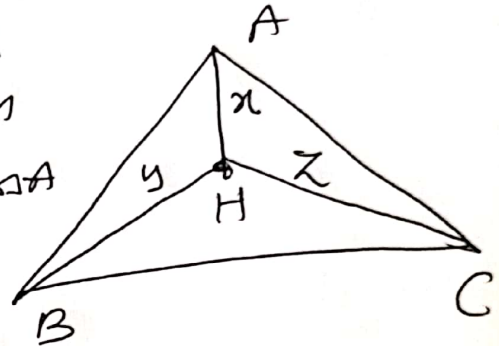
$$= \frac{2}{\sqrt{3}} + \frac{2}{\sqrt{3}} - \frac{2}{\sqrt{3}} = \frac{2 \cdot 1 \cdot 1}{(1 \cdot 1 \cdot 1) \left(\frac{\sqrt{3}}{4} \right)} \cdot \cos \left(\frac{60^\circ}{2} \right)$$

$$= 0$$

Ans. A

Q8

H $\rightarrow$ Orthocentre of $\triangle ABC$
 $\therefore$ The distance of H from the vertex A is $2R \cos A$



$$\begin{aligned} AH = x &= 2R \cos A \\ y &= 2R \cos B \\ z &= 2R \cos C \end{aligned}$$

$$\text{Now } \frac{a}{x} + \frac{b}{y} + \frac{c}{z} = \frac{a}{2R \cos A} + \frac{b}{2R \cos B} + \frac{c}{2R \cos C}$$

$$= \frac{2R \sin A}{2R \cos A} + \frac{2R \sin B}{2R \cos B} + \frac{2R \sin C}{2R \cos C}$$

$$= \tan A + \tan B + \tan C$$

In $\triangle ABC$, we have

$$\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$$

$$= \pi \tan A$$

$$= \pi \frac{\sin A}{\cos A}$$

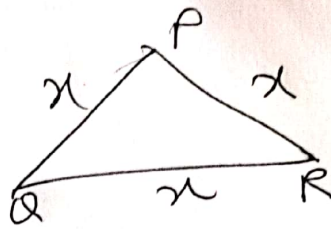
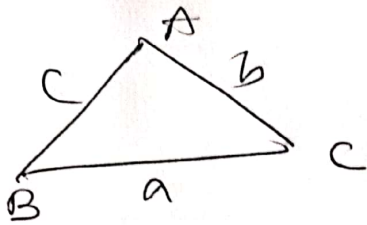
$$= \pi \cdot \frac{a}{2R \cos A}$$

$$= \pi \left(\frac{a}{x} \right)$$

$$\therefore \frac{a}{x} + \frac{b}{y} + \frac{c}{z} = \frac{a}{x} \cdot \frac{b}{y} \cdot \frac{c}{z}$$

Ans: A

Q9



5

Given a, b, c are in A.P

$$\Rightarrow 2b = a + c$$

$$\begin{aligned} \text{perimeter } \triangle ABC &= a + b + c \\ &= a + c + b \\ &= 2b + b \\ &= 3b \end{aligned}$$

$$\begin{aligned} \text{perimeter } \triangle PQR &= 3x \end{aligned}$$

Given $3b = 3x \Rightarrow b = x \rightarrow \textcircled{1}$

Area of $\triangle ABC = \frac{3}{5} \times \text{Area of } \triangle PQR$

$$\begin{aligned} &= \frac{3}{5} \times \frac{\sqrt{3}}{4} x^2 \\ &= \frac{3\sqrt{3}}{20} b^2 \end{aligned}$$

Ans: B

10 Given in $\triangle ABC$, a, b and A are given

We have $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$$\Rightarrow b^2 + c^2 - a^2 = 2bc \cos A$$

Also we have $c_1 + c_2 = 2b \cos A$
 $c_1 \cdot c_2 = b^2 - a^2$

$$\begin{aligned} \text{Now, } c_1 - c_2 &= \sqrt{(c_1 + c_2)^2 - 4c_1 c_2} \\ &= \sqrt{(2b \cos A)^2 - 4(b^2 - a^2)} \\ &= \sqrt{4b^2 \cos^2 A - 4b^2 + 4a^2} \\ &= \sqrt{4a^2 - 4b^2 \sin^2 A} \end{aligned}$$

(6)

$$= \sqrt{4a^2 - (2b \cos A)^2 \cdot \tan^2 A}$$

$$= \sqrt{4a^2 - (c_1 + c_2)^2 \cdot \tan^2 A}$$

Ans: A

TEE ADVANCED LEVEL

01. Given $c^2 x^2 - c(a+b)x + ab = 0 \rightarrow (1)$
 Given $\sin A$ and $\sin B$ are the roots of (1)
 $\therefore \sin A + \sin B = \frac{c(a+b)}{c^2}$

$$\sin A + \sin B = \frac{a+b}{c} \rightarrow (2)$$

$$= \frac{2R \sin A + 2R \sin B}{2R \sin C}$$

$$= \frac{\sin A + \sin B}{\sin C}$$

$$\Rightarrow \sin C = 1 \Rightarrow C = \frac{\pi}{2}$$

$\therefore \triangle ABC$ is an right angled triangle, $C = \frac{\pi}{2}$
 $\Rightarrow A+B = \frac{\pi}{2} \Rightarrow B = \frac{\pi}{2} - A$

$$(2) \Rightarrow \sin A + \sin B = \frac{a+b}{c}$$

$$\Rightarrow \sin A + \sin\left(\frac{\pi}{2} - A\right) = \frac{a+b}{c}$$

$$\therefore \sin A + \cos A = \frac{a+b}{c}$$

Ans: B, D

02. Given $\cos\left(\frac{A}{2}\right) = \sqrt{\frac{b+c}{2c}}$

$$\Rightarrow \cos^2\left(\frac{A}{2}\right) = \frac{b+c}{2c}$$

⑦

$$\Rightarrow 2 \cos\left(\frac{A}{2}\right) = \frac{b+c}{c}$$

$$\Rightarrow 1 + \cos A = \frac{b+c}{c}$$

$$\Rightarrow \cos A = \frac{b+c}{c} - 1 = \frac{b}{c}$$

$$\Rightarrow \frac{b^2 + c^2 - a^2}{2bc} = \frac{b}{c}$$

$$\Rightarrow a^2 + b^2 = c^2$$



$\therefore \triangle ABC$ is a right angled triangle

$$\text{Area of } \triangle ABC = \frac{1}{2} ab$$

Also, we know Circum radius = $\frac{1}{2} \times \text{hypotenuse}$
 $= \frac{c}{2}$ Ans. A, B

Q3 Statement I:

$$\begin{aligned} \text{Given } & \frac{2a}{b+c-a} + \frac{2b}{c+a-b} + \frac{2c}{a+b-c} \\ = & \left(\frac{2a}{b+c-a} + 1 \right) + \left(\frac{2b}{c+a-b} + 1 \right) + \left(\frac{2c}{a+b-c} + 1 \right) - 3 \\ = & \frac{a+b+c}{b+c-a} + \frac{a+b+c}{c+a-b} + \frac{a+b+c}{a+b-c} - 3 \rightarrow \textcircled{1} \end{aligned}$$

We know $AM \geq HM$

$$\begin{aligned} \Rightarrow & \frac{x+y+z}{3} \geq \frac{3}{\frac{1}{x} + \frac{1}{y} + \frac{1}{z}} \\ \Rightarrow & \frac{\frac{1}{b+c-a} + \frac{1}{c+a-b} + \frac{1}{a+b-c}}{3} \geq \frac{3}{(b+c-a) + (c+a-b) + (a+b-c)} \end{aligned}$$

$$\Rightarrow \frac{a+b+c}{a+b-c} + \frac{a+b+c}{b+c-a} + \frac{a+b+c}{a+c-b} \geq 9 \quad (8)$$

$\therefore$ The minimum value of this quantity is 9.

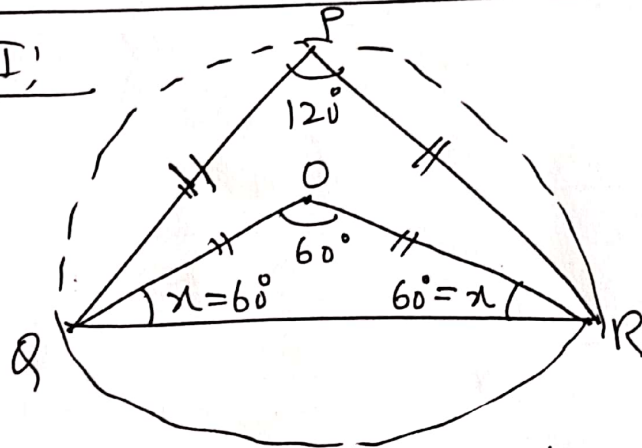
$\therefore$ The minimum value of expression (i)

$$= 9 - 3 = 6. \quad (\text{False})$$

Statement I: Conceptual (True)

Ans: D

04 Statement I:



$\therefore \angle Q = 120^\circ, \angle R = 120^\circ$ also given $\angle P = 120^\circ$
which is not possible (False)

Statement II: Conceptual (False)

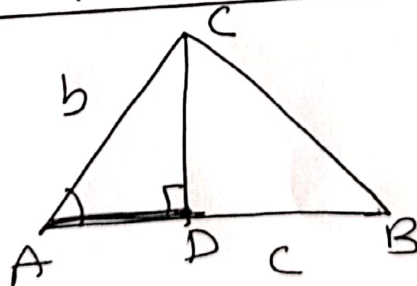
Ans: B

05 Statement I: Conceptual (True)

Statement II: Conceptual (True)

Ans: A

06 Assertion (A):



AD is called the projection side AC (b) on the side AB (c).

We have $\cos A = \frac{AD}{AC} = \frac{AD}{b} \Rightarrow AD = b \cos A$ (True)

Reason: Conceptual (True)

Ans: A

07. We know, $\Delta = \frac{1}{2}ax$

$$\Rightarrow ax = 2\Delta$$

Similarly $by = cz = 2\Delta$

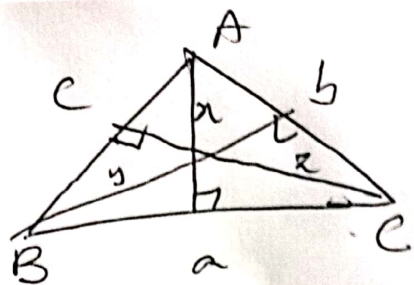
Given $\frac{bx}{c} + \frac{cy}{a} + \frac{az}{b} = \frac{a^2+b^2+c^2}{15}$

$$\Rightarrow \frac{ab^2x + bc^2y + ca^2z}{abc} = \frac{a^2+b^2+c^2}{15}$$

$$\Rightarrow \frac{2\Delta b^2 + 2\Delta c^2 + 2\Delta a^2}{abc} = \frac{a^2+b^2+c^2}{15}$$

$$\Rightarrow \frac{2\Delta(a^2+b^2+c^2)}{abc} = \frac{a^2+b^2+c^2}{15}$$

$$\Rightarrow K = \frac{abc}{2\Delta} = \frac{abc}{2\left(\frac{abc}{4R}\right)} = 2R \quad \text{Ans. C}$$



08

Let $\triangle ABC$ is an equilateral triangle with side 1 unit

$$\therefore \text{Also, we have } x = y = z = \frac{\sqrt{3}}{2}$$

Now, $\cot A + \cot B + \cot C = K \left(\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} \right)$

$$\Rightarrow \cot 60^\circ + \cot 60^\circ + \cot 60^\circ = K \left(\frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} + \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} + \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} \right)$$

$$\Rightarrow \frac{3}{\sqrt{3}} = \frac{12K}{3}$$

$$\Rightarrow K = \frac{\sqrt{3}}{4}$$

We know Area = $\Delta = \frac{\sqrt{3}}{4}a^2$

Ans. C

(10)

Let a, b, c

(10)

09.

Let $\triangle ABC$ is an equilateral $\triangle$

$$= \frac{c \sin B + b \sin C}{x} + \frac{a \sin C + c \sin A}{y} + \frac{b \sin A + a \sin B}{z}$$

$$= 3 \left(\frac{c \sin B + b \sin C}{x} \right)$$

$$= 3 \left(\frac{1 \cdot \sin 60^\circ + 1 \cdot \sin 60^\circ}{\left(\frac{\sqrt{3}}{2}\right)} \right) = 6$$

Ans: ~~B~~
D10 Let a, b, c be the sides of $\triangle ABC$

$$\therefore s = \frac{a+b+c}{2}$$

Also, $s, s-a, s-b, s-c$ will be positive

We have A.M. > G.M.

$$\frac{s + (s-a) + (s-b) + (s-c)}{4} > \left[s(s-a)(s-b)(s-c) \right]^{\frac{1}{4}}$$

$$\Rightarrow \frac{4s - (a+b+c)}{4} > \Delta^{\frac{1}{2}}$$

$$\Rightarrow \frac{2s}{4} > \Delta^{\frac{1}{2}}$$

$$\Rightarrow \frac{s}{2} > \Delta^{\frac{1}{2}}$$

$$\Rightarrow \frac{s^2}{4} > \Delta$$

$$= \Delta < \frac{s^2}{4}$$

Ans: C

11. Given $a+b+c = 6 \left(\frac{\sin A + \sin B + \sin C}{3} \right)$ (11)

$$\Rightarrow 2R \sin A + 2R \sin B + 2R \sin C = 2(\sin A + \sin B + \sin C)$$

$$\Rightarrow 2R = 2$$

$$\Rightarrow R = 1$$

We have $\frac{a}{\sin A} = 2R$

$$\Rightarrow \frac{1}{\sin A} = 2$$

$$\sin A = \frac{1}{2}$$

$$A = \frac{\pi}{6}$$

Ans: A

12 We have $\frac{a}{\sin A} = 2R \Rightarrow R = 5$

$$\Rightarrow \frac{5}{\sin 30^\circ} = 2R$$

$$\Rightarrow \frac{5}{\left(\frac{1}{2}\right)} = 2R$$

Ans: 5

13 $B = 30^\circ, C = 45^\circ \Rightarrow A = 180^\circ - (30^\circ + 45^\circ)$
 $A = 105^\circ$

We have $\frac{a}{\sin A} = 2R$

$$\Rightarrow \frac{\sqrt{3}+1}{\sin 105^\circ} = 2R$$

$$\frac{(\sqrt{3}+1)}{(2\sqrt{2})} = 2R$$

$$\Rightarrow R = \sqrt{2}$$

Now $\frac{c}{\sin C} = 2R$

$$\frac{c}{\sin 45^\circ} = 2\sqrt{2}$$

$$c = 2\sqrt{2} \times \sin 45^\circ$$

$$= 2\sqrt{2} \times \frac{1}{\sqrt{2}}$$

$$c = 2$$

Ans: 2

$$14 \ a) \ (a+b+c)(b+c-a) = \lambda bc$$

(12)

$$\Rightarrow \frac{(2s)(2s-2a)}{bc} = \lambda$$

$$\Rightarrow \frac{4(s)(s-a)}{bc} = \lambda$$

$$\Rightarrow \cos^2\left(\frac{A}{2}\right) = \frac{\lambda}{4}$$

$$\text{We know } 0 \leq \cos^2\left(\frac{A}{2}\right) \leq 1$$

$$\Rightarrow 0 \leq \frac{\lambda}{4} \leq 1$$

$$\Rightarrow 0 \leq \lambda \leq 4$$

$$\therefore \text{Greatest value} = 4 \text{ (P)}$$

$$b) \text{ Given } \tan A \cdot \tan B \cdot \tan C = 9$$

$$\text{We know } AM \geq G.M$$

$$\Rightarrow \frac{\tan A + \tan B + \tan C}{3} \geq (\tan A \cdot \tan B \cdot \tan C)^{\frac{1}{3}}$$

$$\Rightarrow \frac{K}{3} \geq (\tan A \cdot \tan B \cdot \tan C)^{\frac{2}{3}}$$

$$\Rightarrow \frac{K}{3} \geq 9^{\frac{2}{3}}$$

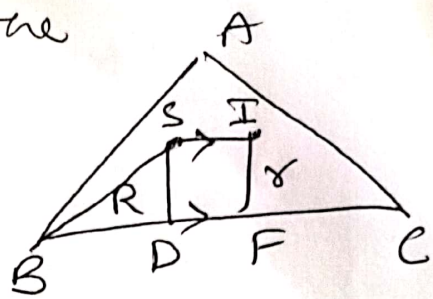
$$\Rightarrow \frac{K}{3} \geq 3 \cdot 9^{\frac{1}{3}}$$

$$\Rightarrow K \geq 9 \cdot 3^{\frac{1}{3}}$$

$$\therefore \text{Hence, Greatest value} = 9 \cdot 3^{\frac{1}{3}} \text{ (P)}$$

c) In $\triangle ABC$, let S be the Circumference, I be the Incentre.

Given $SI \parallel BC$



We have $SD = R \cos A$

But, from diagram $SD = IF = r$

$$\therefore r = R \cos A$$

$$\Rightarrow R \cos A = 4R \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$$

$$\Rightarrow \cos A = 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$$

~~$$\Rightarrow \cos A = -1 + \cos A + \cos B + \cos C$$~~

$$\Rightarrow \cos A = -1 + \cos A + \cos B + \cos C$$

$$\Rightarrow \cos B + \cos C = 1 \quad (r)$$

d) Given $a=5, b=4 \quad \cos(A-B) = \frac{31}{32}$

We know, $\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$

$$\cos(A-B) = \frac{1 - \tan^2 \left(\frac{A-B}{2} \right)}{1 + \tan^2 \left(\frac{A-B}{2} \right)}$$

$$\Rightarrow \frac{31}{32} = \frac{1 - x^2}{1 + x^2} \quad (\because x = \tan \left(\frac{A-B}{2} \right))$$

$$\Rightarrow x = \frac{1}{63}$$

$$\Rightarrow \tan \left(\frac{A-B}{2} \right) = \frac{1}{63}$$

We have, $\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b} \cot \frac{C}{2}$

(14)

$$\Rightarrow \frac{1}{\sqrt{63}} = \frac{5-4}{5+4} \cot \frac{C}{2}$$

$$\Rightarrow \cot \frac{C}{2} = \frac{9}{\sqrt{63}}$$

$$\Rightarrow \tan \frac{C}{2} = \frac{\sqrt{63}}{9}$$

We know, $\cos C = \frac{1 - \tan^2 \frac{C}{2}}{1 + \tan^2 \frac{C}{2}} = \frac{1 - \left(\frac{\sqrt{63}}{9}\right)^2}{1 + \left(\frac{\sqrt{63}}{9}\right)^2}$

$$\therefore \cos C = \frac{1}{8}$$

We know, $c^2 = a^2 + b^2 - 2ab \cos C$
 $= 25 + 16 - 2 \cdot 5 \cdot 4 \cdot \frac{1}{8}$
 $= 36$

$$\therefore c = 6 \text{ (s)}$$

Ans. p, q, r, s

15) a) $A = 60^\circ$
 $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$$\Rightarrow \cos 60^\circ = \frac{b^2 + c^2 - a^2}{2bc} \Rightarrow \frac{1}{2} = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\Rightarrow b^2 + c^2 = a^2 + bc \rightarrow \textcircled{1}$$

Now, $\frac{b}{c+a} + \frac{c}{a+b}$

$$\text{Now, } \frac{b}{c+a} + \frac{c}{a+b}$$

(15)

$$= \frac{ba + b^2 + c^2 + ca}{(c+a)(a+b)}$$

$$= a(b+a) + c(b+a)$$

$$= \frac{(a+b)(a+c)}{(c+a)(a+b)} = 1 \quad (s)$$

b) Given $C = 60^\circ$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\Rightarrow \cos 60^\circ = \frac{a^2 + b^2 - c^2}{2ab} \Rightarrow \frac{1}{2} = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\Rightarrow \textcircled{a} \quad a^2 + b^2 = ab + c^2 \rightarrow \textcircled{1}$$

Consider, $\frac{1}{a+c} + \frac{1}{b+c} - \frac{3}{a+b+c}$

$$= \frac{a+b+2c}{ab+ac+bc+c^2} - \frac{3}{a+b+c}$$

$$= \frac{(a+b+2c)(a+b+c) - 3(ab+bc+ca+c^2)}{(ab+bc+ca+c^2)(a+b+c)}$$

$$= \frac{[a^2 + ab + ac + ba + b^2 + bc + 2ac + 2bc + 2c^2] - 3ab - 3bc - 3ca - 3c^2}{(ab+bc+ca+c^2)(a+b+c)}$$

$$= \frac{a^2 + b^2 - c^2 - ab}{(ab+bc+ca+c^2)(a+b+c)}$$

$$= \frac{a^2 + b^2 - c^2 - ab}{(ab+bc+ca+c^2)(a+b+c)}$$

$$= \frac{0}{(ab+bc+ca)+c^2} = 0$$

$$\therefore \frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c} \quad (x)$$

c) Given $B = 60^\circ$

$$\text{Now, } \cos B = \frac{c^2 + a^2 - b^2}{2ca}$$

$$\Rightarrow \cos 60^\circ = \frac{c^2 + a^2 - b^2}{2ca}$$

$$\Rightarrow c^2 + a^2 - b^2 - ac = 0 \rightarrow (1)$$

Consider $\frac{1}{a+b} + \frac{1}{b+c} - \frac{3}{a+b+c}$

$$= \frac{(b+c)(a+b+c) + (a+b)(a+b+c) - 3(a+b)(b+c)}{(a+b)(b+c)(a+b+c)}$$

$$= 0$$

$$\therefore \frac{1}{a+b} + \frac{1}{b+c} = \frac{3}{a+b+c} \quad (x)$$

d) Given $C = 60^\circ$

$$\therefore \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\Rightarrow \frac{1}{2} = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\Rightarrow a^2 + b^2 - c^2 - ab = 0 \rightarrow (1)$$

(17)

$$\text{Now, } \frac{b}{c^2 - a^2} + \frac{a}{c^2 - b^2}$$

$$= \frac{bc^2 - b^3 + ac^2 - a^3}{(c^2 - a^2)(c^2 - b^2)}$$

$$= \frac{c^2(a+b) - (a^3 + b^3)}{(c^2 - a^2)(c^2 - b^2)}$$

$$= \frac{c^2(a+b) - (a+b)(a^2 - ab + b^2)}{(c^2 - a^2)(c^2 - b^2)}$$

$$= \frac{(a+b) [c^2 - a^2 + ab - b^2]}{(c^2 - a^2)(c^2 - b^2)}$$

$$= \frac{(a+b)(0)}{(c^2 - a^2)(c^2 - b^2)} = 0 \quad (t)$$

Ans: s, r, r, t

LEARNERS TASK (CUES)

01. $a=3, A=60^\circ$

We have $\frac{a}{\sin A} = 2R$

$$\Rightarrow \frac{3}{\sin 60^\circ} = 2R \Rightarrow R = \sqrt{3} \quad \text{Ans: B}$$

02. $a=10\text{cm}, R=5\text{cm}$

$$\Rightarrow \frac{a}{\sin A} = 2R$$

$$\Rightarrow \frac{10}{\sin A} = 2 \times 5$$

$$\sin A = 1$$

$$A = 90^\circ \quad \text{Ans: D}$$

03 $A = 30^\circ, c = 90, c = 7\sqrt{3}$ (18)

$$\frac{c}{\sin C} = 2R$$

$$\Rightarrow \frac{7\sqrt{3}}{\sin 90^\circ} = 2R$$

$$\Rightarrow 2R = 7\sqrt{3} \rightarrow \textcircled{1}$$

$$\frac{a}{\sin A} = 2R$$

$$\frac{a}{\sin 30^\circ} = 7\sqrt{3}$$

$$\Rightarrow a = \frac{7\sqrt{3}}{2} \text{ Ans. C}$$

04 $a = 20, b = 21, \cos C = \frac{4}{5}$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\Rightarrow \frac{4}{5} = \frac{400 + 441 - c^2}{2 \cdot 20 \cdot 21} \Rightarrow c = 13 \text{ Ans: B}$$

05 $a = 2, b = 3, c = 4$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{9 + 16 - 4}{2 \cdot 3 \cdot 4} = \frac{7}{8} \text{ Ans. A}$$

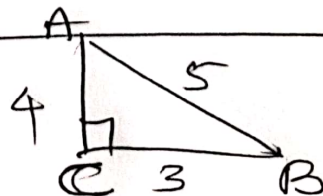
06 $\tan\left(\frac{A}{2}\right) = \frac{\Delta}{s(s-a)}$

$$= \frac{30}{15(15-5)} = \frac{1}{5} \text{ Ans. D}$$

07 $\tan\left(\frac{B}{2}\right) = \frac{\Delta}{s(s-b)} = \frac{330}{60(66-60)} = \frac{5}{6}$

$$\therefore \cot\left(\frac{B}{2}\right) = \frac{6}{5} \text{ Ans. B}$$

08 $a = 3, b = 4, c = 5$



$$\therefore C = 90^\circ$$

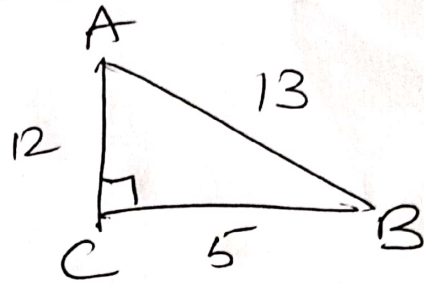
$$\cos \frac{C}{2} = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

Ans D

09

$$C = 90^\circ$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$



Ans A

10 Let us suppose $\triangle ABC$ is an equilateral $\triangle$ with $a = b = c = 1$

$$\text{Now, } \Delta ABC \sin \frac{A}{2} \cdot \sin \frac{B}{2} \cdot \sin \frac{C}{2}$$

$$= 1 \cdot 1 \cdot 1 \cdot \sin 30^\circ \cdot \sin 30^\circ \cdot \sin 30^\circ$$

$$= \frac{1}{8}$$

$$\text{Now, } \frac{\Delta^2}{S} = \frac{\left(\frac{\sqrt{3}}{4}\right)^2}{\left(\frac{3}{2}\right)} = \frac{3}{16} \cdot \frac{2}{3} = \frac{1}{8}$$

Ans: B

TEEMAINS LEVEL

01. Let $\triangle ABC$ is an equilateral $\triangle$

$$\therefore \sum \frac{\sin(B-C)}{bc} = \frac{\sum \sin(60^\circ - 60^\circ)}{bc} = 0$$

Ans: C

02 Let $\triangle ABC$ be an equilateral $\triangle$

$$b^2 \sin 2C + c^2 \sin 2B$$

$$= (1)^2 \sin 120^\circ + (1)^2 \sin 120^\circ = 2 \sin 120^\circ = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$$

Now, $4\Delta = 4 \times \frac{\sqrt{3}}{4} = \sqrt{3}$

(20)
Ans: A

03

$$a \cos A = b \cos B$$

$$\Rightarrow 2R \sin A \cos A = 2R \sin B \cos B$$

$$\Rightarrow \sin 2A = \sin 2B \quad \text{or} \quad \sin 2A = \sin (180^\circ - 2B)$$

$$\Rightarrow 2A = 2B$$

$$\Rightarrow A = B$$

$$\Rightarrow 2A = 180^\circ - 2B$$

$$\Rightarrow A + B = 90^\circ$$

$$\Rightarrow \angle C = 90^\circ$$

$\therefore \triangle ABC$ is right angled isosceles Ans: C

04 Given $\frac{b}{c+a} + \frac{c}{a+b} = 1$

$$\Rightarrow \frac{ba + b^2 + c^2 + ca}{(c+a)(a+b)} = 1$$

$$\Rightarrow \frac{b^2 + c^2 - a^2}{2bc} = \frac{1}{2}$$

$$\Rightarrow \cos A = \frac{1}{2} \Rightarrow A = 60^\circ \quad \text{Ans A}$$

05 $a = \sqrt{2}, b = \sqrt{6}, c = \sqrt{8}$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{6 + 8 - 2}{2 \cdot \sqrt{6} \cdot \sqrt{8}} = \frac{\sqrt{3}}{2}$$

$$\therefore A = 30^\circ$$

Similarly $B = 60^\circ, C = 90^\circ$

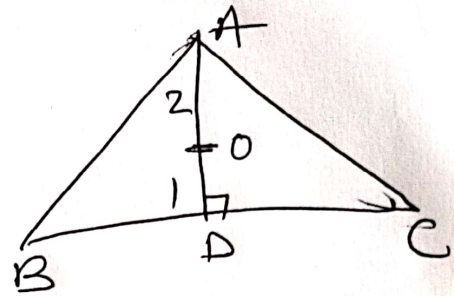
Ans: C

06 Let $\triangle ABC$ be an equilateral $\triangle$

(21)

Let $a=b=c=1$

Also $AB = \frac{\sqrt{3}}{2}$



We know, $AO:OD = 2:1$

$AO:OD = 2:1$

$AO = \frac{2}{3} \times AD$

$= \frac{2}{3} \times \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}$

$\therefore p = q = r = \frac{1}{\sqrt{3}}$

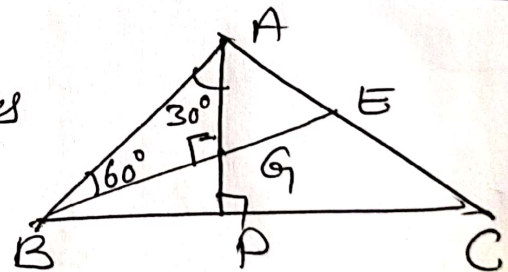
Now $ag, r = 1 \times \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{1}{3}$

$\therefore ag, r + br, p + cr, q = 3 \times \frac{1}{3} = 1$

Now, $abc = 1 \cdot 1 \cdot 1 = 1$

Ans: B

07 Given $AD = 4$
Since, Centroid (G) divides
AD in the ratio 2:1



We have, $AG = \frac{2}{3} \times 4 = \frac{8}{3}$

Now, $\tan 60^\circ = \frac{AG}{BG}$

$\Rightarrow \sqrt{3} = \frac{8}{3BG}$

$\Rightarrow BG = \frac{8}{3\sqrt{3}}$

$Area \triangle ABG = \frac{1}{2} \times BG \times AG$
 $= \frac{1}{2} \times \frac{8}{3\sqrt{3}} \times \frac{8}{3}$
 $= \frac{32}{9\sqrt{3}}$

$Area \triangle ABC = 3 \times Area \triangle ABG$
 $= \frac{32}{3\sqrt{3}}$

Ans: C

08

$$(a+b+c)(b+c-a) = 3bc$$

(22)

$$\Rightarrow (2s)(2s-2a) = 3bc$$

$$\Rightarrow \frac{s(s-a)}{bc} = \frac{3}{4}$$

$$\Rightarrow \cos^2 \frac{A}{2} = \frac{3}{4}$$

$$\Rightarrow \cos \frac{A}{2} = \frac{\sqrt{3}}{2} \Rightarrow A = 60^\circ \quad \text{Ans. A}$$

09

Given $1 - \tan \frac{A}{2} \cdot \tan \frac{B}{2}$

$$= 1 - \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \cdot \frac{(s-a)(s-c)}{s(s-b)}$$

$$= 1 - \frac{s-c}{s}$$

$$= \frac{c}{s} = \frac{2c}{2s} = \frac{2c}{a+b+c}$$

Ans C

10

Given a, b, c are in AP

$$\Rightarrow 2b = a+c \rightarrow (1)$$

Now,

$$\frac{2 \sin \frac{A}{2} \cdot \sin \frac{C}{2}}{\sin \frac{B}{2}}$$

$$= \frac{2 \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \cdot \sqrt{\frac{(s-a)(s-b)}{ab}}}{\sqrt{\frac{(s-a)(s-c)}{ac}}}$$

$$\begin{aligned} &= \frac{2(s-b)}{b} \\ &= \frac{2s-2b}{b} \\ &= \frac{a+b+c-2b}{b} = \frac{a+c-b}{b} \\ &= \frac{2b-b}{b} = 1 \end{aligned}$$

Ans. A

11) Let a be the side of an equilateral Δ
 x be the side of regular hexagon

Given $3a = 6x$
 $\Rightarrow a = 2x \rightarrow (1)$

$$\begin{aligned} \text{Ratio of Areas} &= \frac{\sqrt{3}}{4} a^2 : 6 \times \frac{\sqrt{3}}{4} x^2 \\ &= \frac{a^2}{6} : x^2 \\ &= (2x)^2 : 6x^2 \\ &= 4 : 6 \\ &= 2 : 3 \end{aligned}$$

Ans. C

Please Note: In the
key, Ans: changed

JEE ADVANCED LEVEL

12. Given $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$

$$\Rightarrow \frac{\cos A}{2R \sin A} = \frac{\cos B}{2R \sin B} = \frac{\cos C}{2R \sin C}$$

$$\Rightarrow \cot A = \cot B = \cot C$$

$\Rightarrow A = B = C$
 ΔABC is an equilateral triangle

Given $a = 2 \text{ cm}$

(24)

$$\text{Area} = \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \cdot 2^2 = \sqrt{3}$$

Ans. A, B

13. Let $\triangle ABC$ be an equilateral triangle.

Let $a = b = c = 1 \text{ unit}$

$$\text{We have, } S = \frac{a+b+c}{2} = \frac{3}{2}$$

$$\therefore \Delta = \frac{\sqrt{3}}{4}, \quad R = \frac{1}{\sqrt{3}}$$

$$\begin{aligned} \text{Now, } \cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} \\ = \cot 30^\circ + \cot 30^\circ + \cot 30^\circ = 3 \times \sqrt{3} = 3\sqrt{3} \end{aligned}$$

$$\text{Now, } \frac{S^2}{\Delta} = \frac{\left(\frac{3}{2}\right)^2}{\left(\frac{\sqrt{3}}{4}\right)} = \frac{9}{4} \times \frac{4}{\sqrt{3}} = 3\sqrt{3}$$

$$\text{Also, } \frac{(a+b+c)^2}{abc} \cdot R$$

$$= \frac{(1+1+1)^2}{1 \cdot 1 \cdot 1} \cdot \frac{1}{\sqrt{3}} = \frac{9}{\sqrt{3}} = 3\sqrt{3}$$

Ans. A, C

14. Statement I,

$$4s(s-a)(s-b)(s-c) = a^2 b^2 c^2$$

$$= 2 \sqrt{s(s-a)(s-b)(s-c)} = ab$$

$$\Rightarrow 2\Delta = ab$$

$$\Rightarrow \Delta = \frac{1}{2} ab = \frac{1}{2} \times \text{base} \times \text{height}$$

$\triangle ABC$ is a right angled triangle (True)

Statement II;

(25)

$$\text{let } f(x) = \sin x$$

We have,

$$\begin{aligned} \frac{1}{3} f(A) + \frac{1}{3} f(B) + \frac{1}{3} f(C) &\leq f\left[\frac{1}{3}(A+B+C)\right] \\ &\leq f(60^\circ) \\ &\leq \sin 60^\circ \\ &\leq \frac{\sqrt{3}}{2} \end{aligned}$$

This inequality is achieved, when

$$A = B = C = \frac{\pi}{3}$$

Hence, $\triangle ABC$ is an equilateral $\triangle$ (True)
Ans. A

15 Statement I; Given, ~~$\triangle ABC$~~ the angles of

$\triangle ABC$ are in A.P.

$$\text{let } A = B = C = 60^\circ \quad (\text{Common difference } 0)$$

$$\text{Now, } \cos A + \cos B + \cos C$$

$$= \cos 60^\circ + \cos 60^\circ + \cos 60^\circ$$

$$= 3\left(\frac{1}{2}\right) = \frac{3}{2} \quad (\text{False})$$

Statement II; In $\triangle ABC$, Given A, B, C are in

A.P and $\cos A + \cos B + \cos C = 2$, which is
not possible for any angles of $\triangle ABC$
(False)

Ans. B

16. Given $a=2, b=4, C=60^\circ$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

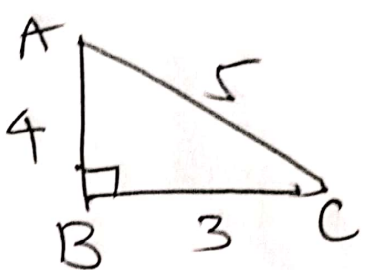
$$= 4 + 16 - 2 \cdot 2 \cdot 4 \cdot \frac{1}{2}$$

$$= 12$$

$$\therefore c = \sqrt{12}$$

Ans C

17.



$$\therefore \angle B = 90^\circ$$

Ans C

18 Let $\triangle ABC$ is an equilateral $\triangle$ with $A=B=C=60^\circ, a=b=c=1$ unit

$$\text{Now, } a^2 + b^2 + c^2 = 1^2 + 1^2 + 1^2 = 3$$

$$\text{Now, } 4\Delta (\cot A + \cot B + \cot C)$$

$$= 4 \cdot \frac{\sqrt{3}}{4} (\cot 60^\circ + \cot 60^\circ + \cot 60^\circ)$$

$$= \sqrt{3} \left(3 \times \frac{1}{\sqrt{3}} \right) = 3$$

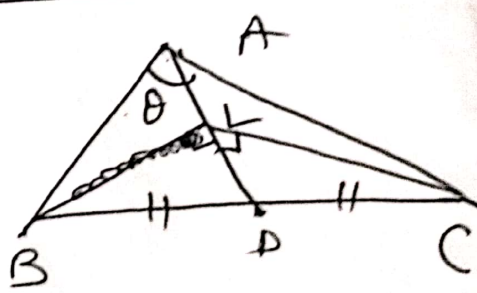
Ans. A

19

We know,

$$\tan \left(\frac{A-B}{2} \right) = \frac{1 - \cos(A+B)}{1 + \cos(A+B)}$$

$$= \frac{1 - \frac{4}{5}}{1 + \frac{4}{5}}$$



$$= \frac{1}{9}$$

$$\therefore \tan\left(\frac{A-B}{2}\right) = \frac{1}{3}$$

$$\therefore \tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b} \cot\left(\frac{C}{2}\right)$$

$$\Rightarrow \frac{1}{3} = \frac{6-3}{6+3} \cdot \cot\left(\frac{C}{2}\right)$$

$$\Rightarrow \frac{C}{2} = \frac{\pi}{4}$$

$$\Rightarrow C = \frac{\pi}{2}$$

$\therefore \triangle ABC$ is a right angle triangle Ans B

20 Area $\triangle ABC = \frac{1}{2} \times 6 \times 3$
 $= 9$

Ans. A

21 $AD^2 = 3^2 + 3^2$ | $\therefore AB = 3\sqrt{2}$
 $= 18$

Ans: C

22 Given $\cos A + \sin A - \frac{2}{\cos B + \sin B} = 0$

$$\Rightarrow \cos A + \sin A = \frac{2}{\cos B + \sin B}$$

$$\Rightarrow (\cos A + \sin A)(\cos B + \sin B) = 2$$

$$\Rightarrow \cos A \cos B + \cos A \sin B + \sin A \cos B + \sin A \sin B = 2$$

$$\Rightarrow \cos(A-B) + \sin(A+B) = 2$$

(28)

One of the possibility is

$$A-B=0$$

$$A+B=90^\circ$$

$$\text{Solving } \Rightarrow A=45^\circ, B=45^\circ \Rightarrow C=90^\circ$$

$$\text{Now, } \left(\frac{a+b}{c}\right)^4 = \left(\frac{2R\sin A + 2R\sin B}{2R\sin C}\right)^4$$

$$= \left(\frac{\sin A + \sin B}{\sin C}\right)^4$$

$$= \left(\frac{\sin 45^\circ + \sin 45^\circ}{\sin 90^\circ}\right)^4$$

$$= (\sqrt{2})^4 = 4.$$

Ans. 4

23 Given $a+b-c=2$; $2ab-c^2=4$

$$\text{Now } 2ab-c^2=2^2$$

$$\Rightarrow 2ab-c^2 = (a+b-c)^2$$

$$\Rightarrow 2ab-c^2 = a^2+b^2+c^2+2ab-2bc-2ca$$

$$\Rightarrow a^2+c^2-2ac + b^2+c^2-2bc=0$$

$$\Rightarrow (a-c)^2 + (b-c)^2 = 0$$

$$\Rightarrow a-c=0 \text{ \& } b-c=0$$

$$\Rightarrow a=b=c$$

$\triangle ABC$ is an equilateral $\triangle$

Now $a+b-c=2$

$\Rightarrow a+a-a=2$

$\Rightarrow a=2$

(Area of $\triangle ABC$)² = $\left(\frac{\sqrt{3}}{4} a^2\right)^2$

= $\left(\frac{\sqrt{3}}{4} \cdot 2^2\right)^2$

= 3

Ans. 3

23. NOTE: TO solve all these problems, let us consider $\triangle ABC$ is an equilateral triangle, with $a=b=c=1$ unit

$\Delta = \frac{\sqrt{3}}{4}$ and $S = \frac{1+1+1}{2} = \frac{3}{2}$, $R = \frac{1}{\sqrt{3}}$

a) $a^2 \sin 2C + b^2 \sin 2A = 1^2 \sin 120^\circ + 1^2 \sin 120^\circ$
 $= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \sqrt{3}$

Now, $4\Delta = 4 \times \frac{\sqrt{3}}{4} = \sqrt{3}$ (P)

b) $R^2 (\sin 2A + \sin 2B + \sin 2C) = \left(\frac{1}{\sqrt{3}}\right)^2 \cdot 3 \times \sin 120^\circ = \frac{\sqrt{3}}{2}$

Now, $2\Delta = 2 \times \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$ (Q)

c) $c \cos^2 \frac{A}{2} + a \cos^2 \frac{C}{2} = 1 \cdot \cos^2 30^\circ + 1 \cdot \cos^2 30^\circ$
 $= \frac{3}{4} + \frac{3}{4} = \frac{3}{2} = S$ (R)

d) $a \sin^2 \frac{A}{2} + c \sin^2 \frac{C}{2} = 1 \cdot \sin^2 30^\circ + 1 \cdot \sin^2 30^\circ$
 $= \left(\frac{1}{4}\right) + \frac{1}{4} = \frac{1}{2}$

$S - b = \frac{3}{2} - 1 = \frac{1}{2}$ (S)

Ans: P, Q, R, S

25 a) $(a-b)^2 \cos \frac{C}{2} + (a+b)^2 \sin \frac{C}{2}$ (30)

$$= (a-b)^2 \left(\frac{1+\cos C}{2} \right) + (a+b)^2 \left(\frac{1-\cos C}{2} \right)$$

$$= \frac{1}{2} [a^2+b^2-2ab + a^2+b^2+2ab] + \cos C \left[\frac{a^2+b^2-2ab}{2} - \frac{a^2+b^2-2ab}{2} \right]$$

$$= \frac{1}{2} [2a^2+2b^2] + \cos C (-4ab)$$

$$= a^2+b^2-4ab \left(\frac{a^2+b^2-c^2}{2ab} \right)$$

$$= c^2 - (P)$$

b) Consider $\triangle ABC$ is an equilateral triangle with side $a=b=c=1$.

$$\frac{1^2-1^2}{1^2} \sin^2 A + \frac{1^2-1^2}{1^2} \sin^2 B = 0.$$

$$\text{option (S): } \frac{b^2-a^2}{4R} = \frac{1^2-1^2}{4R} = 0 \quad (S)$$

c) $\frac{1 \cdot 1 \cdot \cos 60^\circ + 1 \cdot 1 \cdot \cos 60^\circ + 1 \cdot 1 \cdot \cos 60^\circ}{\cos 60^\circ + \cos 60^\circ + \cos 60^\circ} = \frac{\left(\frac{3}{2}\right)}{\left(\frac{3}{2}\right)} = \frac{1}{1} = 1$
 $= \Delta (X)$

d) $\frac{\cos 60^\circ + \cos 60^\circ}{1+1} + \frac{\cos 60^\circ}{1} = \frac{1}{2} + \frac{1}{2} = 1 = \frac{1}{b} \quad (V)$

Ans: P, S, V, Q

ADDITIONAL PRACTICE QUESTIONS

(31)

01. Let $a=1$, $b=2004$
Let the third side be c , which is an integer

We have $a+b > c$

$$\Rightarrow 1+2004 > c$$

$$\Rightarrow \cancel{2004} c < 2005 \rightarrow \textcircled{1}$$

Also $b-a < c$

$$\Rightarrow 2004-1 < c$$

$$\Rightarrow 2003 < c \rightarrow \textcircled{2}$$

$$\therefore \textcircled{1}, \textcircled{2} \Rightarrow 2003 < c < 2005$$

$$\therefore c = 2004$$

$$\text{Perimeter} = a+b+c = 1+2004+2004 = 4009.$$

Ans: B

02 $a \cos^2 \frac{C}{2} + c \cos^2 \frac{A}{2} = \frac{3b}{2}$

$$\Rightarrow a \cdot \frac{s(s-c)}{ab} + c \cdot \frac{s(s-a)}{bc} = \frac{3b}{2}$$

$$\Rightarrow s(s-c) + s(s-a) = \frac{3b^2}{2}$$

$$\Rightarrow 2s^2 - s(a+c) = \frac{3b^2}{2}$$

$$\Rightarrow 2s^2 - s(2s-b) = \frac{3b^2}{2}$$

$$= sb = \frac{3b^2}{2}$$

$$\Rightarrow 2sb = 3b^2$$

$$\Rightarrow (a+b+c)b^2 = 3b^2$$

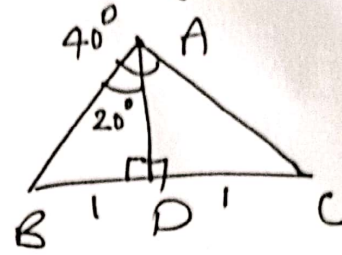
$$\Rightarrow a+c=2b$$

$$\Rightarrow a, b, c \text{ are in A.P.}$$

Ans: A

03 Given a regular polygon of 9 sides is (32)
inscribed in an equilateral triangle

$$\text{We have } \alpha = \frac{360^\circ}{9} = 40^\circ$$



In the diagram

$AB = \text{radius of the circle}$

$$\sin 20^\circ = \frac{BD}{AB} \Rightarrow \sin 20^\circ = \frac{1}{AB}$$

$$\Rightarrow AB = \csc 20^\circ = \csc \frac{\pi}{9}$$

Ans. A

04 Let $\triangle ABC$ be an equilateral triangle, with each side 1 unit
 $\therefore a = b = c = 1 \text{ unit}$

$$\begin{aligned} \therefore (b-c) \cot \frac{A}{2} + (c-a) \cot \frac{B}{2} + (a-b) \cot \frac{C}{2} \\ = (1-1) \cot \frac{A}{2} + (1-1) \cot \frac{B}{2} + (1-1) \cot \frac{C}{2} = 0 \end{aligned}$$

Ans. D

05 Let $B = 30^\circ$, $C = 45^\circ$

$$a = BC = \sqrt{3} + 1$$

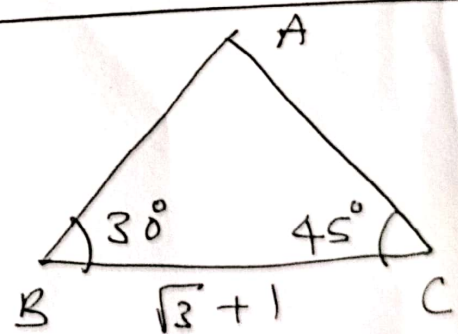
Clearly, $A = 105^\circ$

$$\text{We have, } \frac{a}{\sin A} = 2R$$

$$\Rightarrow \frac{\sqrt{3} + 1}{\sin 105^\circ} = 2R$$

$$\Rightarrow 2R = \frac{(\sqrt{3} + 1)}{\left(\frac{\sqrt{3} + 1}{2\sqrt{2}}\right)}$$

$$\Rightarrow R = \sqrt{2}$$



Now, $\frac{b}{\sin B} = 2R$

(23)

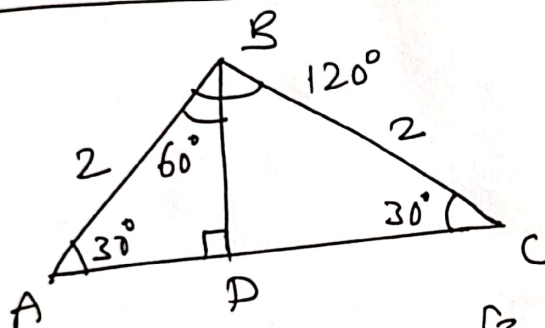
$$\Rightarrow \frac{b}{\sin 30^\circ} = 2\sqrt{2}$$

$$\Rightarrow b = \sqrt{2}$$

Now, $\Delta ABC = \frac{1}{2}ab \sin C$
 $= \frac{1}{2} \cdot (\sqrt{2}+1)(\sqrt{2}) \cdot \sin 45^\circ$
 $= \frac{\sqrt{3}+1}{2}$

Ans: A

06



$$\sin 60^\circ = \frac{AD}{AB} \Rightarrow \frac{\sqrt{3}}{2} = \frac{AD}{2}$$

$$\Rightarrow AD = \sqrt{3}$$

$$\therefore b = AC = 2AD = 2\sqrt{3}$$

$\Delta ABC = \frac{1}{2}ab \sin C$
 $= \frac{1}{2} \cdot 2 \cdot 2\sqrt{3} \sin 30^\circ$
 $= 2\sqrt{3} \times \frac{1}{2} = \sqrt{3}$

Ans: B

07

Given $\sin \frac{A}{2} \cdot \cos \frac{B}{2} = \left(\frac{a+c-b}{2c} \right) k$

$$\Rightarrow \sqrt{\frac{(s-b)(s-c)}{bc}} \cdot \sqrt{\frac{s(s-b)}{ac}} = \frac{(2s-2b)}{2c} \cdot k$$

$$\Rightarrow \frac{s-b}{c} \sqrt{\frac{s(s-c)}{ab}} = \frac{s-b}{c} \cdot K$$

(34)

$$\Rightarrow K = \cos \frac{C}{2}$$

Ans. D

08 Given $\frac{2\cos A}{a} + \frac{\cos B}{b} + \frac{2\cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$

$$\Rightarrow \frac{2(b^2+c^2-a^2)}{2abc} + \frac{c^2+a^2-b^2}{abc} + \frac{2(a^2+b^2-c^2)}{2abc} = \frac{a^2+b^2}{abc}$$

$$\Rightarrow a^2 = b^2 + c^2$$

$\Rightarrow \triangle ABC$ is a right angle triangle
with $\angle A = 90^\circ$

Ans. D

09 Statement I: Conceptual (True)

Statement II: Conceptual (True)

Ans. A

$\Rightarrow$ THE END $\Leftarrow$