

# FUNCTIONS - I

①

Class IX, Mathematics.  
IIT FOUNDATION +  
SOLUTIONS

## TEACHING TASK

01.  $f(x) = \frac{1}{\sqrt{|x| - x}}$

$f$  is defined only when  $|x| - x > 0$   
 $\Rightarrow |x| > x$

This is possible only when  $x \in (-\infty, 0)$

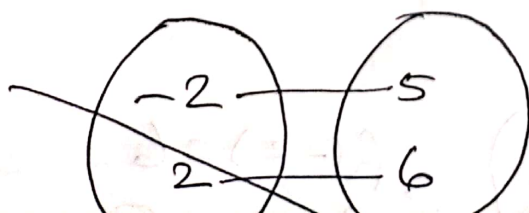
Ans. opt: A

02.  $n(A) = n$ ,  $n(B) = 2$

No. of surjection =  $2^n - 2 = 62$   
 $\Rightarrow n = 5$

Ans. opt: A

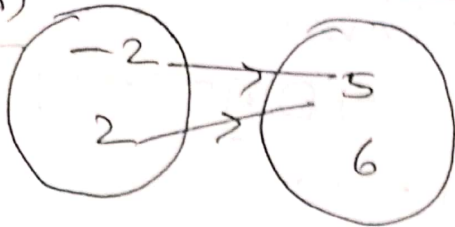
03.



here  $f(-2) = 5$   
 $f(2) = 6$

03

Case (i)



$$f(-2) = 5$$

$$f(2) = 5$$

(2)

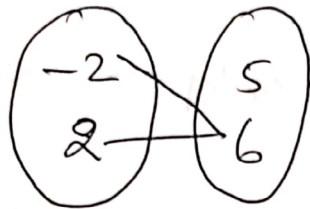
$$f(x) = ax + b$$

$$f(-2) = 5 \Rightarrow -2a + b = 5$$

$$\begin{array}{r} -2a + b = 5 \\ -2a + b = 5 \\ \hline a = 0 \end{array}$$

here  $a=0$ , this is not possible

Case (ii)



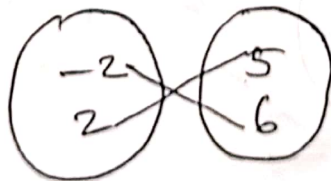
$$f(-2) = 6$$

$$f(2) = 6$$

$$\begin{array}{r} -2a + b = 6 \\ -2a + b = 6 \\ \hline a = 0 \end{array}$$

here  $a=0$ , this is not possible

Case (iii)

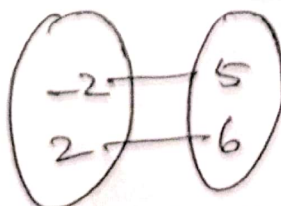


$$f(-2) = 5$$

$$f(2) = 6$$

$$\left. \begin{array}{l} -2a + b = 5 \\ 2a + b = 6 \end{array} \right\} \text{we can solve for } a \text{ \& } b$$

Case (iv)



$$f(-2) = 6$$

$$f(2) = 5$$

$$\left. \begin{array}{l} -2a + b = 6 \\ 2a + b = 5 \end{array} \right\} \text{we can solve for } a \text{ \& } b$$

hence there only 2 linear functions (3)

Ans: opt C

04.  $x_1, x_2, x_3 \rightarrow A.P.$  let  $f(x) = e^x$  or  $a^x$   
let  $1, 2, 3 \rightarrow A.P.$   
 $e^1, e^2, e^3 \rightarrow G.P.$  Ans: B

05  $f(x) = \cos(\log x)$   
 $f\left(\frac{x^2}{y^2}\right) = \cos\left(\log\left(\frac{x^2}{y^2}\right)\right)$   
 $= \cos(\log x^2 - \log y^2)$   
 $f(x^2 y^2) = \cos(\log(x^2 y^2))$   
 $= \cos(\log x^2 + \log y^2)$   
 $f\left(\frac{x^2}{y^2}\right) + f(x^2 y^2) = \cos(\log x^2 - \log y^2) + \cos(\log x^2 + \log y^2)$   
 $= 2 \cos(\log x^2) \cdot \cos(\log y^2)$   
 $\frac{1}{2} \left[ f\left(\frac{x^2}{y^2}\right) + f(x^2 y^2) \right] = f(x^2) \cdot f(y^2)$   
 $\therefore f(x^2) \cdot f(y^2) - \frac{1}{2} \left[ f\left(\frac{x^2}{y^2}\right) + f(x^2 y^2) \right] = 0$  Ans: B

06  $f(x) \cdot g(y) + g(x) \cdot f(y)$   
 $= \frac{1}{2} (3^x + 3^{-x}) \cdot \frac{1}{2} (3^y - 3^{-y}) + \frac{1}{2} (3^x - 3^{-x}) \cdot \frac{1}{2} (3^y + 3^{-y})$   
 $= \frac{1}{4} \left[ \begin{array}{cc} 3^{x+y} & - 3^{x-y} \\ - 3^{x-y} & - 3^{-(x+y)} \end{array} + \begin{array}{cc} 3^{x+y} & + 3^{x-y} \\ - 3^{x-y} & - 3^{-(x+y)} \end{array} \right]$   
 $= \frac{2}{4} \left[ \begin{array}{cc} 3^{x+y} & - 3^{-(x+y)} \end{array} \right] = \boxed{f(x+y)} = g(x+y)$

Ans: B

67.  $y = f(x) = 10x - 7$

(4)

$$10x - 7 = y \Rightarrow x = \frac{y+7}{10}$$

$$\therefore f^{-1}(x) = \frac{x+7}{10} = g(x)$$

Ans: C

08

$$f(201) = -\left(\frac{201-1}{2}\right) = -100$$

Ans: C

$$\therefore f^{-1}(-100) = 201.$$

09.

Given  $\sin^{-1}(x) + \cos^{-1}(x) + \tan^{-1}(x)$

first we find the domain

$$\sin^{-1}x \rightarrow \text{domain } [-1, 1]$$

$$\cos^{-1}x \rightarrow \text{domain } [-1, 1]$$

$$\tan^{-1}x \rightarrow \text{domain } (-\infty, \infty)$$

$$\therefore \text{Domain of } f(x) = [-1, 1]$$

$$\text{Now } f(x) = \sin^{-1}x + \cos^{-1}x + \tan^{-1}x$$

$$f(x) = \frac{\pi}{2} + \tan^{-1}x$$

$$f'(x) = \frac{1}{1+x^2} > 0$$

$\therefore f$  is an increasing function

$$\therefore f(-1) = \frac{\pi}{2} + \tan^{-1}(-1) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$$f(1) = \frac{\pi}{2} + \tan^{-1}(1) = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$$

$$\therefore \text{Range} = \left[ \frac{\pi}{4}, \frac{3\pi}{4} \right]$$

Ans: B.

10. We have  $(a+b+c)^2 \geq 0$

(5)

$$\Rightarrow a^2 + b^2 + c^2 + 2(ab + bc + ca) \geq 0$$

$$\Rightarrow ab + bc + ca \geq -\frac{1}{2} \rightarrow \textcircled{1}$$

Also  $a^2 + b^2 + c^2 - (ab + bc + ca)$

$$= \frac{1}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2] \geq 0$$

$$= 1 - (ab + bc + ca) \geq 0$$

$$\Rightarrow ab + bc + ca \leq 1 \rightarrow \textcircled{2}$$

from  $\textcircled{1}$  and  $\textcircled{2}$   $-\frac{1}{2} \leq ab + bc + ca \leq 1$

Ans: A

11.  $f(n) = 1! + 2! + 3! + \dots + n! \rightarrow \textcircled{1}$

$$f(n+1) = 1! + 2! + 3! + \dots + n! + (n+1)! \rightarrow \textcircled{2}$$

$$f(n+2) = 1! + 2! + 3! + \dots + (n+1)! + (n+2)! \rightarrow \textcircled{3}$$

$$\textcircled{2} - \textcircled{1} \Rightarrow f(n+1) - f(n) = (n+1)!$$

$$\textcircled{3} - \textcircled{2} \Rightarrow f(n+2) - f(n+1) = (n+2)!$$

$$= (n+2)(n+1)!$$

$$= (n+2) [f(n+1) - f(n)]$$

$$= (n+2) f(n+1) - \cancel{(n+2) f(n)}$$

$$\Rightarrow (n+3) f(n+1) - (n+2) f(n) = f(n+2)$$

$$\therefore f(n+2) = (n+3) f(n+1) - (n+2) f(n)$$

$$f(n+2) = p(n) f(n+1) + q(n) \cdot f(n)$$

$$\therefore p(x) = x+3, q(x) = -x-2$$

Ans: A, B

12. Consider option: c

$$f(x) = x^3 - 3x$$

$$\Rightarrow f\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$$

$$= x^3 + \frac{1}{x^3} + 3 \cdot x \cdot \frac{1}{x} \left(x + \frac{1}{x}\right) - 3\left(x + \frac{1}{x}\right)$$

$$= x^3 + \frac{1}{x^3}$$

13. Statement I:  $x^2 - 3x + 2 > 0$

$$\Rightarrow (x-1)(x-2) > 0$$

$$\Rightarrow (-\infty, 1) \cup (2, \infty)$$

Statement II:  $(x-a)(x-b) > 0$

$$\Rightarrow (-\infty, a) \cup (b, \infty) \quad \text{Ans: A}$$

14. Statement I:  $n(A) = 4, n(B) = 8$

No. of relations from A to B =  $2^{4 \times 8} = 2^{32}$

No. of functions from A to B =  $8^4 = 4096$

$\therefore$  No. of relations which are not functions

$$= 2^{32} - 4096 \neq 4096$$

clearly Statement I is false.

Statement II:

Many-one functions = total functions - one-one functions

$$= 4^3 - 4P_3 = 64 - 24 = 40$$

Hence, Statement II is True. Ans: C (7)

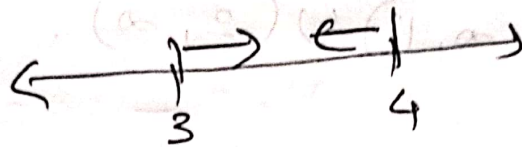
15. Domain of  $\sqrt{\frac{x-2}{x-3}} = (-\infty, 2] \cup (3, \infty)$   
Ans: D

16. Domain of  $\sqrt{\frac{3-x}{4-x}} =$

$$\frac{3-x}{4-x} \geq 0 \text{ and } 4-x \neq 0$$

$$\Rightarrow (3-x)(4-x) \geq 0 \text{ and } x \neq 4$$

$$\Rightarrow (x-4)(x-3) \leq 0$$



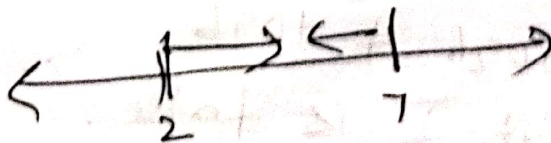
Domain =  $[3, 4)$

Ans: B

17.  $\frac{2-x}{x-7} \geq 0$  and  $x-7 \neq 0$

$$\Rightarrow (2-x)(x-7) \geq 0 \text{ and } x \neq 7$$

$$\Rightarrow (x-2)(x-7) \leq 0$$



Domain =  $[2, 7)$

Ans: B

18.  $x^2 + x + 1 > 0$  and  $x^2 + x + 1 \neq 1$  (8)  
 $\Delta < 0$   $x^2 + x \neq 0$   
 $\therefore \forall x \in \mathbb{R}, x^2 + x + 1 > 0$   $x(x+1) \neq 0$   
 $x \neq 0$  or  $x+1 \neq 0$   
 $\Rightarrow x \neq -1$

hence Domain =  $\mathbb{R} - \{0, -1\}$  Ans: B

19.  $f(x) = \log(x^2 + x + 1)$

Domain  $\rightarrow x^2 + x + 1 > 0 \forall x \in \mathbb{R}$ .

Since  $\Delta < 0 \Rightarrow x^2 + x + 1 > 0 \forall x \in \mathbb{R}$ .

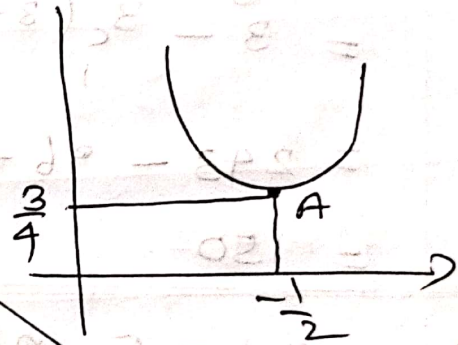
$\therefore$  Domain =  $\mathbb{R}$ .

$A = \left( \frac{-b}{2a}, \frac{4ac - b^2}{4a} \right)$

$= \left( -\frac{1}{2}, \frac{3}{4} \right)$

Lowest point =  $\frac{3}{4}$

Hence Range =  $\left[ \frac{3}{4}, \infty \right)$



19.  $f(x) = \log(x^2 + x + 1)$  since  $\Delta < 0$ .

$x^2 + x + 1 > 0 \forall x \in \mathbb{R}$

Minimum value =  $\frac{4ac - b^2}{4a} = \frac{3}{4}$ .

Minimum value of  $\log(x^2 + x + 1) = \log\left(\frac{3}{4}\right)$

Hence Range =  $\left[ \log\left(\frac{3}{4}\right), \infty \right)$

Ans: A



$$20. f(x) = \frac{2^x}{x+2}$$

(9)

$$\Rightarrow x+2 \neq 0$$

$$\Rightarrow x \neq -2$$

$$\therefore \text{Domain} = \mathbb{R} - \{-2\}$$

Ans: D

$$21. n(x) = 5, n(y) = 3$$

$$\text{No. of Surjections} = n^m - nC_1(n-1)^m + nC_2(n-2)^m - \dots$$

$$\text{here } m=5, n=3$$

$$= 3^5 - 3C_1(3-1)^5 + 3C_2(3-2)^5 - 3C_3(3-3)^5$$

$$= 243 - 96 + 3$$

$$= 150$$

$$22. n(A) = 5, n(B) = 2$$

$$\text{No. of Surjections} = 2^5 - 1$$

$$= 2^5 - 1$$

$$= 31$$

$$23. f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$$

$$= \cos[9.859]x + \cos[-9.859]x$$

Since  $\pi = 3.14$

$$= \cos 9x + \cos(-10)x$$

$$= \cos 9x + \cos 10x$$

$$a) f\left(\frac{\pi}{2}\right) = \cos \frac{9\pi}{2} + \cos 10 \cdot \frac{\pi}{2}$$

$$= 0 - 1 = -1$$



$$\begin{aligned}
 \text{b) } f\left(\frac{\pi}{4}\right) &= \cos 9\frac{\pi}{4} + \cos 10\frac{\pi}{4} \quad (10) \\
 &= \cos 375^\circ + \cos 225^\circ \\
 &= \cos 15^\circ + \cos 45^\circ \\
 &= \frac{\sqrt{3}+1}{2\sqrt{2}} - \frac{1}{\sqrt{2}} \\
 &= \frac{\sqrt{3}+1-2}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } f(\pi) &= \cos 9\pi + \cos 10\pi \\
 &= \cos(2 \times 2\pi - \pi) + \cos(2 \times 5\pi) \\
 &= \cos \pi + \cos 0 \\
 &= -1 + 1 \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } f(0) &= \cos 0 + \cos 0 \\
 &= 1 + 1 \\
 &= 2
 \end{aligned}$$

Ans:  $\rho, -, \rho, \gamma$ .

$$\begin{aligned}
 24 \text{ a) } 1+x &> 0 \\
 \Rightarrow x &> -1
 \end{aligned}$$

$$\text{Domain} = (-1, \infty)$$

$$\text{b) } |x| - x > 0$$

$$\Rightarrow |x| > x \quad \text{Domain} = (-\infty, 0)$$

$$\text{c) } 1 - |x| > 0$$

$$\Rightarrow |x| < 1 \quad \text{Domain} = [-1, 1]$$

$$\text{d) } f(x) = \sqrt{-x^2} \quad \text{Domain} = \phi$$

Ans:  $-, S, \gamma$

# LEARNERS TASK

11

CUQ's

01.  $A \times B = \{(1, x), (1, y), (2, x), (2, y), (3, x), (3, y)\}$

Ans: B

02. No. of relations =  $2^{m \times n}$

Ans: C

03. No. of functions =  $n^m$

Ans: A

04.  $f(x) = 3x^4 + 6x^3 - 7x^2 + 2$

Ans: C

05. Conceptual

Ans: B

06.  ${}^n P_m (n \geq m)$

Ans: B

07.  $f(A) \neq B$

Ans: A

08. No. of Bijections =  $n!$

Ans: D

09.  $f(x) = 2x^2 + 3x - 5$

$f(1) = 2(1)^2 + 3(1) - 5$

$= 0$

Similarly other values follow.

Range =  $\{0, 9, 22, 39, 60\}$

Ans: A

10. i) when  $x$  is very large and positive,  
 $-x$  approaches to 0.



- 10 (i) when  $x$  is very large and positive,  
 $10^{-x}$  approaches 0 (12)
- (ii) when  $x$  is very large and negative,  
 $10^{-x}$  approaches infinity
- (iii) when  $x=0$ ,  $10^{-x}=1$ .
- $\therefore$  Range =  $(0, \infty)$  Ans: C

### JEE MAIN LEVEL QUESTIONS

01.  $f(2) + f(3) = (2^2 - 5 \cdot 2 + 6) + (3^2 - 5 \cdot 3 + 6)$   
 $= (4 - 10 + 6) + (9 - 15 + 6)$   
 $= 0 + 0$   
 $= 0$  Ans: C

02.  $f(1+x) = x^2 + 1$   
 Let  $1+x = y \Rightarrow x = y-1$   
 $\therefore f(y) = (y-1)^2 + 1$   
 $= y^2 - 2y + 2 \therefore f(x) = x^2 - 2x + 2$   
 $f(2-h) = (2-h)^2 - 2(2-h) + 2$   
 $= 4 + h^2 - 4h - 4 + 2h + 2$   
 $= h^2 - 2h + 2$  Ans: B

03.  $n(A) = 100$ ,  $n(B) = 26$   
 No. of constant functions =  $n(B) = 26$  Ans: D

04.  $n(A) = n(B) = 4$   
 No. of Bijections =  $4!$  Ans: C

05

$$f(x) = 2x + |x|$$

(13)

$$f(x) = \begin{cases} 3x & x \geq 0 \\ x & x < 0 \end{cases}$$

Now given  $f(3x) - f(-x) - 4x$

$$f(3x) = \begin{cases} 9x & x \geq 0 \\ 3x & x < 0 \end{cases} \Rightarrow \phi$$

$$f(-x) = \begin{cases} -3x & x \leq 0 \\ -x & x > 0 \end{cases}$$

Now, to calculate  $f(3x) - f(-x) - 4x$

Case (i)  $x > 0$

$$f(3x) = 9x, \quad f(-x) = -x$$

$$f(3x) - f(-x) - 4x = 9x + x - 4x = 6x$$

Case (ii)  $x < 0$

$$f(3x) - f(-x) - 4x = 3x - (-3x) - 4x = 2x$$

$$\therefore f(3x) - f(-x) - 4x = \begin{cases} 6x & \text{if } x \geq 0 \\ 2x & \text{if } x < 0 \end{cases}$$

$$= 2|x| + 4x$$

$$= 2[|x| + 2x]$$

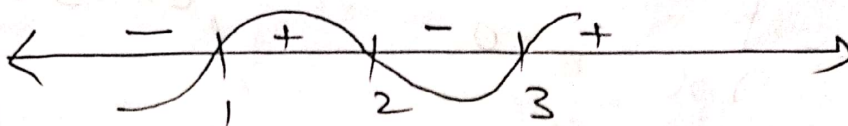
$$= 2f(x)$$

Ans: D



$$06. (x-1)(x-2)(x-3) \geq 0$$

14



$$\text{Domain} = [1, 2] \cup [3, \infty)$$

Ans: D

$$07. f(-3) = (-3)^2 + 1 = 10$$

$$f(2) = 2(2) - 1 = 3$$

$$f(5) = 4(5) + 3 = 23$$

$$f(1) = 2(1) - 1 = 1$$

$$\therefore \frac{f(-3) + f(2) + f(5)}{f(1)} = \frac{10 + 3 + 23}{1} = 36$$

Ans: B

$$08. f(x) + 2 \cdot f\left(\frac{1}{x}\right) = 3x \rightarrow \textcircled{1}$$

$$f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x} \rightarrow \textcircled{2}$$

$$\textcircled{2} \times 2 \Rightarrow 2f\left(\frac{1}{x}\right) + 4f(x) = \frac{6}{x}$$

$$\textcircled{1} \Rightarrow \underline{2f\left(\frac{1}{x}\right) + f(x) = 3x}$$

$$3f(x) = \frac{6}{x} - 3x$$

$$\Rightarrow f(x) = \frac{2}{x} - x$$

$$\text{Now } f(x) = f(-x)$$

$$\Rightarrow \frac{2}{x} - x = \frac{-2}{x} + x$$

$$\Rightarrow \frac{4}{x} - 2x = 0$$

$$\Rightarrow \frac{4}{x} - 2x = 0$$

$$\Rightarrow 2 - x^2 = 0$$

$$x = \pm\sqrt{2}$$

Ans: C

09.

$$-1 \leq x \leq 1$$

$$-5 \leq 5x \leq 5$$

$$-1 \leq 5x+4 \leq 9$$

Ans: B

(15)

10

$$f(x) = \sin^2 x + \cos^4 x$$

$$= \sin^2 x + (1 - \sin^2 x)^2$$

$$= \sin^4 x - \sin^2 x + 1$$

$$= \sin^2 x (\sin^2 x - 1) + 1$$

$$= -\sin^2 x \cdot \cos^2 x + 1$$

$$= -\frac{1}{4} (2 \sin x \cos x)^2 + 1$$

$$= -\frac{1}{4} \sin^2 2x + 1$$

We know  $0 \leq \sin^2 2x \leq 1$

$$0 \geq -\frac{1}{4} \sin^2 2x \geq -\frac{1}{4}$$

$$\frac{3}{4} \leq -\frac{1}{4} \sin^2 2x + 1 \leq 1$$

$$\Rightarrow \frac{3}{4} \leq f(x) \leq 1$$

$$\therefore \text{Range} = \left[ \frac{3}{4}, 1 \right]$$

Ans: A

11.

$$f(x+y) = f(x) + f(y)$$

$$\Rightarrow f(x) = kx \quad k \in \mathbb{R} - \{0\}$$

$$\Rightarrow f(1) = k \cdot 1 = 2 \quad \Rightarrow k = 2$$

$$\therefore f(x) = 2x$$

Ans: B

12

$$f(x+y) = f(x) + f(y) \quad (16)$$

$$\Rightarrow f(x) = kx$$

$$\Rightarrow f(1) = k \cdot 1 = 7 \Rightarrow k = 7$$

$$\therefore f(x) = 7x$$

$$\sum_{x=1}^n f(x) = f(1) + f(2) + f(3) + \dots + f(n)$$

$$= 7 \cdot 1 + 7 \cdot 2 + 7 \cdot 3 + \dots + 7 \cdot n$$

$$= 7(1+2+3+\dots+n)$$

$$= \frac{7 \cdot n(n+1)}{2}$$

Ans: D

13. Statement I:Domain of  $f = \mathbb{R}^+$ Domain of  $g = \mathbb{N}$ 

hence the given functions are equal functions. (False)

Statement II: Conceptual (True)

Ans: D

14. Statement I:

$$f(x) = \frac{1-x^2}{1+x^2}$$

$$f(\tan \theta) = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

$$= \cos 2\theta \quad (\text{True})$$

Statement II: False

Ans: C



15. Statement I Conceptual (False) (17)

Statement II

$$\text{No. of onto functions} = n^m - n \cdot {}_1 C_1 (n-1)^m +$$

$${}_2 C_2 (n-2)^m + \dots$$

$$= 3^4 - 3 \cdot {}_1 C_1 (3-1)^4 + 3 \cdot {}_2 C_2 (3-2)^4$$

$$= 81 - 48 + 3$$

$$= 36 \quad \text{(False)} \quad \text{Ans: B}$$

16  $f(x) = \cos 3x$

$$f(-x) = \cos 3(-x) = \cos 3x = f(x) \quad \text{Ans: A}$$

17  $f(x) = x^2 + x + 1$

$$g(-x) = (-x)^2 + (-x) + 1 = x^2 - x + 1 \neq g(x)$$

$g(x)$  is neither even nor odd. Ans: C

18  $g \circ f(x) = g(f(x))$

$$= g(1-x^4)$$

$$= (1-x^4)^2 + 1$$

$$= 1 - 2x^4 + x^8 + 1$$

$$= x^8 - 2x^4 + 2$$

$$g \circ f(-x) = x^8 - 2x^4 + 2 = g \circ f(x)$$

$g \circ f(x)$  is even function. Ans: A

19.  $A = \{a, e, i, o\} \Rightarrow n(A) = 4$  (15)  
 $B = \{2, 3, 5, 7\} \Rightarrow n(B) = 4$   
 No. of Bijections =  $4! = 24$  Ans: B

20.  $n(A) = 3$  Ans: B  
 No. of bijections =  $3! = 6$

21. Let  $f(x) = x$   
 $f(x + f(x)) = f(x + x) = f(2x) = 2x$   
 $x + f(x) = x + x = 2x$   
 which is linear function

Let  $f(x) = -x$   
 ~~$f(-x - x) = f(-2x) = 2x$~~   
 $f(x + f(x)) = f(x - x) = f(0) = 0$   
 $x + f(x) = x - x = 0$   
 here, these only two linear functions  
 satisfying the given condition. Ans: 2

22.  $n(A) = 5, n(B) = 3$   
 No. of Many to one-one function = total  
 No. of functions - one-one functions  
 $= \binom{5}{3} - 0$   
 $= 243$  Ans: 243

23

$$f(x) = \frac{1 + \log_e^x}{\log_e^x}$$

(19)

$$= \frac{1 + \log_e^x}{\log_e^x} = 1 + \frac{1}{\log_e^x}$$

$$\therefore f(2014) = 1$$

Ans: 1

24

a)  $|x| - x \neq 0$

$$\Rightarrow |x| \neq x \quad \forall x \in (-\infty, 0)$$

b)  $|x|$

Domain =  $\mathbb{R}$ 

c)  $\sec x$

Domain =  $\mathbb{R} - \left\{ (2n+1)\frac{\pi}{2} \right\}$ 

d)  $\sqrt{x}$

Domain =  $[0, \infty)$ 

Gas: t, s, r, v

25

a)  $-1 \leq \cos x \leq 1$

$$-2 \leq 2 \cos x \leq 2$$

$$2 \geq -2 \cos x \geq -2$$

$$3 \geq 1 - 2 \cos x \geq -1$$

25

$$a) -1 \leq \cos x \leq 1$$

$$-2 \leq 2 \cos x \leq 2$$

$$2 \geq -2 \cos x \geq -2$$

$$-2 \leq -2 \cos x \leq 2$$

$$-1 \leq 1 - 2 \cos x \leq 3$$

$$-1 \geq \frac{1}{1 - 2 \cos x} \geq \frac{1}{3}$$



$$\text{Range} = (-\infty, -1] \cup \left[\frac{1}{3}, \infty\right)$$

$$b) -1 \leq \cos 2x \leq 1$$

$$-3 \leq 3 \cos 2x \leq 3$$

$$-2 \leq 1 + 3 \cos 2x \leq 4$$

$$\text{Range} = [-2, 4]$$

$$c) 5 \sin x + 12 \cos x - 13$$

$$\text{max. value} = c + \sqrt{a^2 + b^2}$$

$$= -13 + \sqrt{25 + 144}$$

$$= -13 + 13$$

$$= 0$$

$$\text{min. value} = -13 - 13$$

$$= -26$$

$$\text{Range} = [0, -26] = [-26, 0]$$

$$d) |x - 2|$$

$$\text{Range} = [0, \infty)$$

Ans: x, y, p, t

(20)

# ADDITIONAL PRACTICE QUESTIONS

(21)

01. No. of constant functions =  $n(B) = n$

Ans: B

02.  $n(A) = 10, n(B) = 20$

No. of functions =  $20^{10}$

Ans: D

03.  $n(A) = 4, n(B) = 6$

No. of one-one functions =  ${}^6P_4$

Ans: C

04.  $n(A) = 106$

No. of Bijections =  $106!$

Ans: C

05.  $f(x) = |x|, g(x) = [x-3]$

$$\begin{aligned} \therefore g(f(x)) &= g(|x|) \\ &= [|x| - 3] \end{aligned}$$

Given

$$-\frac{8}{5} < x < \frac{8}{5}$$

$$\Rightarrow 0 < |x| < \frac{8}{5}$$

$$\Rightarrow -3 < |x| - 3 < -\frac{7}{5}$$

$$\Rightarrow [-3] < [|x| - 3] < \left[-\frac{7}{5}\right]$$

05

$$f(x) = |x|, \quad g(x) = [x-3]$$

(22)

$$S(f(x)) = g(|x|)$$

$$= [|x| - 3]$$

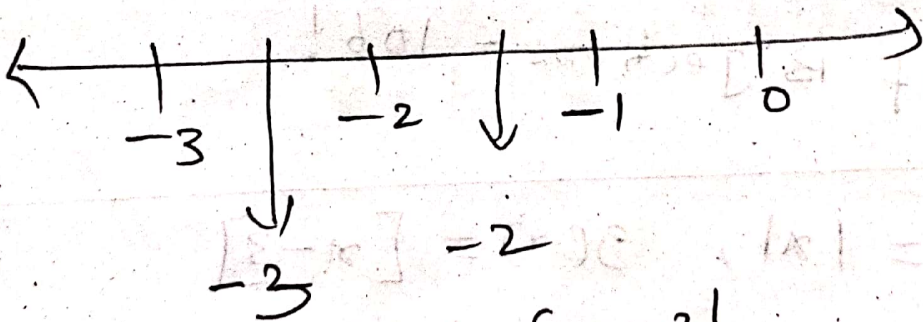
Given

$$-\frac{8}{5} < x < \frac{8}{5}$$

$$0 < |x| < \frac{8}{5}$$

$$-3 < |x| - 3 < -\frac{7}{5}$$

$$-3 < |x| - 3 < -1.4$$



$$\therefore [|x| - 3] = \{-3, -2\}$$

Ans = C

⇒ THE END ←