WS-3 · Magnitude of Vectors

TTask

① Given vector $\vec{P} = \hat{i} - 2\hat{j} - 2\hat{k}$

It is a 3D vector. The Magnitude of 3D vector

$$\text{is} = \sqrt{x^2+y^2+z^2}$$

$$= \sqrt{1^2+(-2)^2+(-2)^2}$$

$$= \sqrt{1+4+4} = \sqrt{9} = 3$$

② Given $\vec{A} = 3\hat{i} + 5\hat{j} - 2\hat{k}$; $\vec{B} = -3\hat{j} + 6\hat{k}$

Given condition is $2\vec{A} + 7\vec{B} + 4\vec{C} = 0$

$$\Rightarrow 4\vec{C} = -2\vec{A} - 7\vec{B}$$

$$= -2[3\hat{i}+5\hat{j}-2\hat{k}] - 7[-3\hat{j}+6\hat{k}]$$

$$= -6\hat{i} - 10\hat{j} + 4\hat{k} + 21\hat{j} - 42\hat{k}$$

$$= -6\hat{i} + 11\hat{j} - 38\hat{k}$$

$$\Rightarrow \vec{C} = -\frac{6}{4}\hat{i} + \frac{11}{4}\hat{j} - \frac{38}{4}\hat{k}$$

$$\vec{C} = -1.5\hat{i} + 2.75\hat{j} - 9.5\hat{k}$$

③ Given $\vec{A} = 6\hat{i} - 2\hat{k}$

Given condition $\vec{A} - 2\vec{B} = -3(\vec{A} + \vec{B})$

$$\Rightarrow \vec{A} - 2\vec{B} = -3\vec{A} - 3\vec{B}$$

$$\Rightarrow \vec{A} + 3\vec{A} = 2\vec{B} - 3\vec{B}$$

$$\Rightarrow 4\vec{A} = -\vec{B}$$

$$\Rightarrow \vec{B} = -4(6\hat{i} - 2\hat{k}) = -24\hat{i} + 8\hat{k}$$

④ Given $|2\hat{i} + y\hat{j} + 3\hat{k}| = 5$ Given vector is 3D.

magnitude $= \sqrt{x^2+y^2+z^2}$

$$\Rightarrow \sqrt{2^2+y^2+3^2} = 5$$

$$\Rightarrow \sqrt{4+y^2+9} = 5$$ By squaring on both sides

$$\Rightarrow (\sqrt{13+y^2})^2 = 5^2 \Rightarrow 13 + y^2 = 25$$

$$\Rightarrow y^2 = 25 - 13 = 12 \Rightarrow y = \sqrt{12}$$

⑤ Given $\vec{P} = \hat{i} + 3\hat{j} - 7\hat{k}$ and $\vec{Q} = 5\hat{i} - 2\hat{j} + 4\hat{k}$

length of $\vec{PQ} = \vec{Q} - \vec{P} = 5\hat{i} - 2\hat{j} + 4\hat{k} - (\hat{i} + 3\hat{j} - 7\hat{k})$

$= 5\hat{i} - 2\hat{j} + 4\hat{k} - \hat{i} - 3\hat{j} + 7\hat{k}$

$= 4\hat{i} - 5\hat{j} + 11\hat{k}$

$|\vec{PQ}| = \sqrt{x^2 + y^2 + z^2} = \sqrt{4^2 + (-5)^2 + 11^2}$

$= \sqrt{16 + 25 + 121} = \sqrt{162}$.

⑥ Given vectors are $\vec{a} = 2\hat{i} + \hat{j} - 3\hat{k}$; $\vec{b} = 5\hat{i} + 3\hat{j} - 2\hat{k}$

Given condition is $3\vec{a} + 2\vec{b} - \vec{c} = 0$

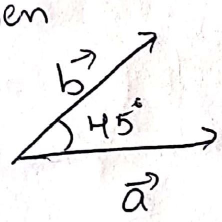
$\Rightarrow \vec{c} = 3\vec{a} + 2\vec{b}$

$\Rightarrow \vec{c} = 3(2\hat{i} + \hat{j} - 3\hat{k}) + 2(5\hat{i} + 3\hat{j} - 2\hat{k})$

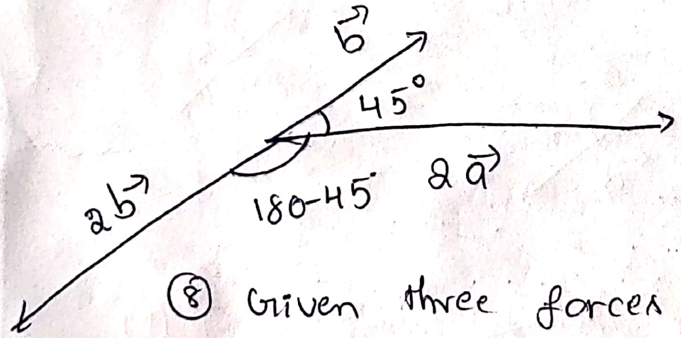
$= 6\hat{i} + 3\hat{j} - 9\hat{k} + 10\hat{i} + 6\hat{j} - 4\hat{k}$

$\Rightarrow \vec{c} = 16\hat{i} + 9\hat{j} - 13\hat{k}$

⑦ Given



Here magnitude of $\vec{a}$ & $\vec{b}$ is doubled also direction of $\vec{b}$ is reversed.



The angle between $2\vec{a}$ and $2\vec{b}$ is $180 - 45° = 135°$.

⑧ Given three forces are $F_1 = 3\hat{i} - 4\hat{j} + 2\hat{k}$

$F_2 = 2\hat{i} + 3\hat{j} - \hat{k}$

$F_3 = 2\hat{i} + 4\hat{j} - 5\hat{k}$.

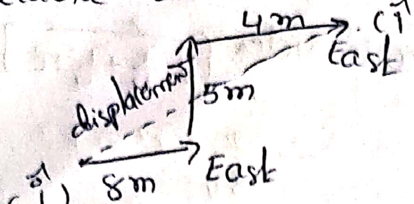
The resultant of three forces is $F_1 + F_2 + F_3$

$= 3\hat{i} - \cancel{4\hat{j}} + 2\hat{k} + 2\hat{i} + 3\hat{j} - \hat{k} + 2\hat{i} + \cancel{4\hat{j}} - 5\hat{k}$

$= 7\hat{i} + 3\hat{j} - 4\hat{k}$.

(9) The displacement of a Particle = Vector form

Because directions are given $\vec{S} = 8\hat{i} + 4\hat{i} + 5\hat{j}$



$\Rightarrow 12\hat{i} + 5\hat{j} \rightarrow$ 2D form

$|\vec{S}| = \sqrt{12^2+5^2} = \sqrt{144+25}$

$= \sqrt{169}$

$|\vec{S}| = 13$ m

(10) Given vector is $\vec{A} = 5\hat{i} + P\hat{j} + 4\sqrt{2}\,k$

It's magnitude $|\vec{A}| = 11$.

For any 3D vector like $x\hat{i} + y\hat{j} + z\hat{k}$ magnitude is $\sqrt{x^2+y^2+z^2}$

For given vector

$\therefore \sqrt{5^2+P^2+(4\sqrt{2})^2} = 11$

squaring on both sides

$\Rightarrow \left[\sqrt{25+P^2+32}\right]^2 = 11^2$

$\Rightarrow 57 + P^2 = 121 \Rightarrow P^2 = 121 - 57$

$\Rightarrow P^2 = 64 \Rightarrow P = \pm 8$.

Because $(-8)^2$ and $(-8)^2$ value is 64

(19) Given $\vec{a} = 4\hat{i} - 3\hat{j}$; $\vec{b} = 8\hat{i} - 6\hat{j}$

then $|\vec{b}| = \sqrt{8^2+(-6)^2} = \sqrt{64+36} = \sqrt{100} = 10$

magnitude of $\vec{a}+\vec{b} = |\vec{a}+\vec{b}| = |4\hat{i} - 3\hat{j} + 8\hat{i} - 6\hat{j}|$

$= |12\hat{i} - 9\hat{j}|$

$= \sqrt{12^2+(-9)^2} = \sqrt{144+81}$

$= \sqrt{225} = 15$

Magnitude of $\vec{a}-\vec{b} = |\vec{a}-\vec{b}| = |4\hat{i} - 3\hat{j} - 8\hat{i} + 6\hat{j}|$

$= |-4\hat{i} + 3\hat{j}| = \sqrt{(-4)^2+3^2}$

$= \sqrt{16+9} = \sqrt{25} = 5$

(17) Given vector is $\left|\frac{1}{2}\hat{i}+\frac{1}{2}\hat{j}+c\hat{k}\right| = 1$.

$$\therefore \sqrt{(\tfrac{1}{2})^2+(\tfrac{1}{2})^2+c^2} = 1$$

By squaring on both sides $\left[\sqrt{\frac{1}{4}+\frac{1}{4}+c^2}\right]^{\cancel{2}} = 1^2$

$$= \frac{\cancel{2}}{\cancel{8}\,4} + c^2 = 1$$

$$\Rightarrow \frac{1}{4} + c^2 = 1 \Rightarrow c^2 = 1-\frac{1}{4}$$

$$\Rightarrow c^2 = \frac{3}{4}$$

$$\Rightarrow c = \frac{\sqrt{3}}{2}$$

(18) Given vector is $3\hat{i}-12\hat{j}-4\hat{k}$

It is a 3D vector whose magnitude

$$= \sqrt{3^2+(-12)^2+(-4)^2}$$

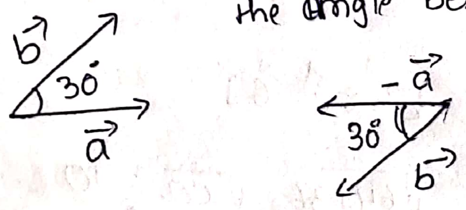
$$= \sqrt{9+144+16} = \sqrt{169} = 13$$

L Task [CUQ's]

(6) Given angle between $\vec{a}$ and $\vec{b}$ is $30°$

If it is represented graphically then

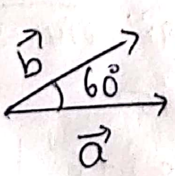
the angle between $-\vec{a}$ and $-\vec{b}$ is



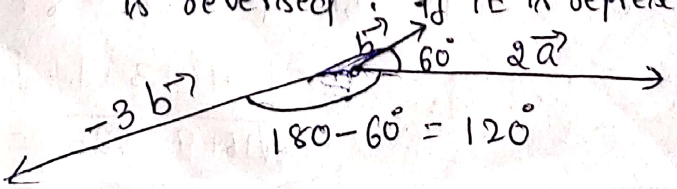
$\therefore$ The angle remain same i.e $30°$ because both $\vec{a}$ and $\vec{b}$ are reversed ~~of~~ their directions

(7) Given angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$ ie $60°$

Graphical representation is



Here the magnitude of $\vec{a}$ is doubled ie $2\vec{a}$ and $\vec{b}$ is tripled ie ($3\vec{b}$) also $\vec{b}$ direction is reversed ; If it is represented graphically



SAQ's

(11) Given $\vec{A} = 2\hat{i}+\hat{j}$ and $\vec{B} = \hat{j}-\hat{k}$ then

$$5A+2B = 5(2\hat{i}+\hat{j})+2(\hat{j}-\hat{k})$$
$$= 10\hat{i}+5\hat{j}+2\hat{j}-2\hat{k}$$
$$= 12\hat{i}+7\hat{j}-2\hat{k}$$

Magnitude of $(5A+2B) = \sqrt{(12)^2+7^2+(-2)^2}$

$$= \sqrt{144+49+4} = \sqrt{153} = 12.36$$

(12) Given vector is $2\hat{i}+3\hat{j}+4\hat{k}$

For a vector $x\hat{i}+y\hat{j}+z\hat{k}$ length in xy plane is

$$\sqrt{x^2+y^2}$$

$\therefore$ For $2\hat{i}+3\hat{j}+4\hat{k}$ length in xy plane is

$$= \sqrt{2^2+3^2} = \sqrt{13}$$

(13) By acting four forces on the Particle, It is at rest.

$\therefore F_1+F_2+F_3+F_4 = 0.$ Given $\vec{F_1} = \hat{i}+\hat{k}$

$\vec{F_2} = 2\hat{j}+3\hat{k}$

$\vec{F_3} = 3\hat{i}$

$\vec{F_4} = 3\hat{j}-4\hat{i}-5\hat{k}$

$\Rightarrow \cancel{\hat{i}}+\hat{k}+2\hat{j}+3\hat{k}+\cancel{3\hat{i}}+3\hat{j}-\cancel{4\hat{i}}-5\hat{k}=0$

$\Rightarrow 5\hat{j}-\hat{k}=0$

$\therefore$ the body is in yz plane.

(15) Initial point coordinates are $(1,1,2)$ — $x_1\ y_1\ z_1$

Final point coordinates are $(2,1,3)$ — $x_2\ y_2\ z_2$

$\therefore$ Then the vector is $= (x_2-x_1)\hat{i}+(y_2-y_1)\hat{j}+(z_2-z_1)\hat{k}$

$$= (2-1)\hat{i}+(1-1)\hat{j}+(3-2)\hat{k}$$
$$= \hat{i}+0(\hat{j})+\hat{k} = \hat{i}+\hat{k}$$

$\therefore$ the length of the vector is $= \sqrt{(1)^2+(1)^2} = \sqrt{2}.$

(16) Initial Point of a vector is $P = (1, 2, -1)$ $x_1\ y_1\ z_1$

Terminal Point $Q = (3, 2, 1)$ $x_2\ y_2\ z_2$

$\therefore$ Then the vector PQ is $= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

$= (3-1)\hat{i} + (2-2)\hat{j} + (2-(-1))\hat{k}$

$= 2\hat{i} + 0\times\hat{j} + 3\hat{k}$

$= 2\hat{i} + 3\hat{k}$.

(17) Given vector is $0.2\hat{i} + 0.3\hat{j} + z\hat{k}$ which is a 3D vector

Given magnitude of $0.2\hat{i} + 0.3\hat{j} + z\hat{k}$ is 1.

For any 3D vector like $x\hat{i} + y\hat{j} + z\hat{k}$, Magnitude is $\sqrt{x^2+y^2+z^2}$

$\therefore |0.2\hat{i} + 0.3\hat{j} + z\hat{k}| = 1$

$\Rightarrow \sqrt{(0.2)^2 + (0.3)^2 + z^2} = 1$ By squaring on both sides we get

$\Rightarrow \cancel{\sqrt{0.04 + 0.09 + z^2}}^{\,2} = 1^2$

$\Rightarrow 0.13 + z^2 = 1 \Rightarrow z^2 = 1 - 0.13 \Rightarrow \underline{z^2 = 0.87}$

(18) Initial position of a Particle $= (1, 2, 3)$ m $x_1\ y_1\ z_1$

Final position of a Particle $= (5, 4, 2)$ m $x_2\ y_2\ z_2$

$\therefore$ The displacement vector $= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

$= (5-1)\hat{i} + (4-2)\hat{j} + (2-3)\hat{k}$

$= 4\hat{i} + 2\hat{j} - \hat{k}$

(19) let initial position vector $\vec{r_1} = 2\hat{i} + 3\hat{j} + 5\hat{k}$ m

Final Position vector $\vec{r_2} = \hat{i} - 2\hat{j} + \hat{k}$ m

$\therefore$ The displacement vector $\vec{s} = \vec{r_2} - \vec{r_1} = \hat{i} - 2\hat{j} + \hat{k} - 2\hat{i} - 3\hat{j} - 5$

$\vec{s} = -\hat{i} - 5\hat{j} - 4\hat{k}$

(20) Ttask Problem (3) model.

If $\vec{F_1} = 8\hat{i}+6\hat{j}$ N and $\vec{F_2} = 4\hat{i}-8\hat{j}$ N

(24) $\vec{F_1}+\vec{F_2} = 8\hat{i}+6\hat{j}+4\hat{i}-8\hat{j}$

$= 12\hat{i}-2\hat{j}$

(25) $\vec{F_1}-\vec{F_2} = 8\hat{i}+6\hat{j}-4\hat{i}+8\hat{j}$

$\Rightarrow 4\hat{i}+14\hat{j}$

(26) The magnitude of resultant means $|\vec{F_1}+\vec{F_2}|$

$|12\hat{i}-2\hat{j}| = \sqrt{(12)^2+(-2)^2} = \sqrt{144+4} = \sqrt{148}$