I task

(1)

using the Principle of conservation of energy

Loss in Gravitational P.E = gain in elastic P.E

$\Rightarrow mgy = \frac{1}{2}ky^2$. where 'y' is the total stretch from the unstretched position

substitute $k = \frac{mg}{x}$ in the above equation

$\therefore \cancel{m}\cancel{g}\,\cancel{y} = \frac{1}{2}\frac{\cancel{mg}}{x}y^{\cancel{2}}$ [From $F = kx$, $mg = kx$]

$\Rightarrow y = 2x$

[when the body slowly lowered, the force of gravity mg is balanced by the spring restoring force at equilibrium position. At this point stretch $x \rightarrow F_{\text{spring restoring force}} = kx$ $\therefore mg = kx$ $\Rightarrow k = \frac{mg}{x}$

(2)

Given $F = 10N$ $\Delta x = 1mm = 0.001m$

$\therefore F = k\Delta x \Rightarrow \Delta x = \frac{F}{k}$

$\Rightarrow \Delta x = \frac{10}{0.001} = 10^4 \, N/m$

work done $= \frac{1}{2}k(\Delta x)^2$ New elongation $\Delta x = 40mm = 40\times10^{-3}m$

$= \frac{1}{2}\times10^4\times(\cancel{10^{-1}}40\times10^{-3})^2$

$= \frac{1}{2}\times10^4\times16\times10^{-4}$

$= 8J$.

(3)

let 'k' be the spring constant.

when spring stretched by x cm, The P.E $U_1 = \frac{1}{2}kx^2$

Now the spring stretched to nx cm, the New P.E $U_2 = \frac{1}{2}k(nx)^2$

$\Rightarrow U_2 = \frac{1}{2}kn^2x^2 = n^2\frac{1}{2}kx^2$

$= \underline{n^2 U_1}$ $\quad U_1 = U =$ initial potential energy

(6)

let length of shorter Piece $= l_1 = x$.

longer Piece $= l_2 = 3x$.

$\therefore$ Total length $L = x + 3x = 4x$

$\Rightarrow x = \frac{L}{4} = l_1$

$l_2 = \frac{3L}{4}$

we know that $F = kx \Rightarrow k \propto \frac{F}{x} \Rightarrow k \propto \frac{1}{x}$.

$$k_{Long} = \frac{k}{l_2} = \frac{k}{\frac{3L}{4}} = \frac{4k}{3L} = \frac{4}{3}k$$

where $k =$ original spring constant $= \frac{k}{L}$

$k_{long} = \frac{4k}{3} \rightarrow C$

(7)

we know that elastic P.E of the spring

$U = \frac{1}{2}kx^2 \rightarrow U \propto x^2$

For $x_1 = 2cm \Rightarrow U_1 = u$ $\therefore \frac{U_2}{U_1} = \left[\frac{x_2}{x_1}\right]^2$

$$\Rightarrow \frac{U_2}{U} = \left(\frac{10}{2}\right)^2$$

$$\Rightarrow \frac{U_2}{U} = 5^2$$

$$\Rightarrow U_2 = 25U \rightarrow D$$

(8)

Given $k_1 = 1500\ N/m$ $k_2 = 3000\ N/m$.

$F =$ constant

Elastic P.E $U = \frac{1}{2} k x^2$ or $\frac{F^2}{2k}$

$U \propto \frac{1}{k} \Rightarrow \frac{U_1}{U_2} = \frac{k_2}{k_1} = \frac{3000}{1500} = \frac{2}{1} \rightarrow x$

(10)

In this force acting on a particle is same and along the path OP and OQP, initial and final position are same i.e. displacement is same.

$\therefore$ $W =$ Force $\times$ displacement

$W_1 = W_2$.

(11)

Forces do not store energy are called as Non conservative force. Potential energy is a non-conservative force because the work done by the body depends on the path connecting the starting point and ending point.

(13)

when the particle moves exactly once around any closed path, it's work equals the change in K.E of the particle. It work depends on the end points of motion, not on the path in between.

(14), (15), (16)

At $x = 5m$

$U = 20 + (x-2)^2$

$= 20 + (5-2)^2$

$= 20 + 3^2 = 29J$

Given at $x = 5m$ $K.E = 20J$

$\therefore$ Total mechanical energy $= K.E + U = 20+29$

$= 49J$

Here the particle has maximum K.E, when P.E is minimum. howeve Total Energy $E = 49J$

At $x = 2$ $U_{min} = 20 + (2-2)^2 = 20J$

$E = K_{max} + U_{min}$

$\Rightarrow 49 = K.E_{max} + 20 \Rightarrow K.E_{max} = 49-20$

$= 29J$

For min P.E condition.
From

$U = 20 + (x-2)^2$

$= 20 + x^2 - 4x + 4$

$\frac{dU}{dx} = \frac{d}{dx}[24 + x^2 - 4x] = \frac{d}{dx}(20) + \frac{d}{dx}x^2 - 4\frac{d}{dx}x$

$= 0 + 2x - 4$

$\frac{dU}{dx} = 0 \Rightarrow 2x - 4 = 0$

$x = 2.$

limit of x is

for $U = 49J$

$20 + (x-2)^2 = 49$

$\Rightarrow (x-2)^2 = 29 \Rightarrow x - 2 = \sqrt{29}$

$\Rightarrow x - 2 = \pm 5.38$

$\therefore x = 7.38$ (or) -3.38 m

(17)

Given $W = 1J$; $x = 20\times10^{-2}m$

work done $= \frac{1}{2}kx^2$

$\Rightarrow 1 = \frac{1}{2}\times k\times(20\times10^{-2})^2$

$\Rightarrow 1 = \frac{k}{2}\times \overset{200}{\cancel{400}}\times10^{-4}$

$\Rightarrow k = \frac{10^4}{200} = 50\,N/m$

(18)

we know that $U = \frac{1}{2}kx^2$

$U \propto x^2$ Given $x_1 = 1cm \rightarrow U_1 = 0$

$\rightarrow U_2 = 250$

$\Rightarrow \frac{U_1}{U_2} = \left(\frac{x_1}{x_2}\right)^2$

$\Rightarrow \frac{4}{250} = \left(\frac{1}{x}\right)^2 \Rightarrow \frac{1}{x} = \sqrt{\frac{1}{25}} \Rightarrow x = 5cm$

(19)

Given $e_1 = 10cm \rightarrow U_1 = 4J$ $k = ?$

$= 10^{-1}m$

we know that

$U = \frac{1}{2}kx^2$

$\Rightarrow 4 = \frac{1}{2}k(10^{-1})^2$

$\Rightarrow 8 = k\times10^{-2} \Rightarrow k = 800\,N/m$

$= 8\times100\,N/m$

(20)

we know workdone in stretching a string

$W = \frac{1}{2}kx^2$ $x = same$

$W \propto k$ $W_B < W_A$

$F = kx$ so $\Rightarrow F \propto k$ $F_A > F_B$

LTask CUQ's

(1) A conservative force is a force with the Property that ~~so~~ the work done in moving a Particle between two points is independent of the Path taken and is equal to zero when path is closed.

So work done by a conservative force is independent of the path resulting in a given displacement

(2) A conservative force is work done move a particle from one point to another and is independent of the Path. work done over a closed loop is zero. for a conservative force.

(3) when the force depends only on initial and final position irrespective of the path taken. In any closed path, the work done by a conservative force is zero. The work done by a conservative is reversable.

(6) For a completely closed loop work done by conservative force is zero.

SAQ's

① If the spring is stretched by length x, then work done $= \frac{1}{2}kx^2$

so work done by each mass on the spring $= \frac{1}{4}kx^2$

$\therefore$ work done by spring on each mass $= -\frac{1}{4}kx^2$

Since the spring force and the displacement of masses are in opposite direction, work done is -ve.

② we know that

work done = change in energy.

$\Rightarrow W_{NC} = \Delta K.E + \Delta U$

$\Rightarrow W_{NC} = K_F - K_i + U_F - U_i$

$\Rightarrow 0 = \frac{1}{2}m\left(\frac{v}{2}\right)^2 - \frac{1}{2}mv^2 + \frac{1}{2}kx^2 - 0$

$\Rightarrow 0 = \frac{1}{4}\left[\frac{1}{2}mv^2\right] - \left[\frac{1}{2}mv^2\right] + \frac{1}{2}kx^2$

$\Rightarrow 0 = -\frac{3}{8}mv^2 + \frac{1}{2}kx^2$

$\Rightarrow \frac{3}{8}mv^2 = \frac{1}{2}kx^2$

$\Rightarrow \frac{3}{4}mv^2 = kx^2$

$\Rightarrow k = \frac{3}{4k}mv^2$

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(3) we know that workdone by the conservative force = change in P.E

$$W_c = -(U_B - U_A)$$

$$= U_A - U_B$$

(6)

let the length of spring be l m

length of shorter path $l_1 = \frac{l}{3}$ m

length of longer part $l_2 = \frac{2l}{3}$ m

work done by the spring $= \frac{1}{2}kx^2$

here $k_2 < k_1$ because spring constant $k \propto \frac{1}{l}$

$\therefore$ The work done is greater for shorter path

(7)

Given $k = 5 \times 10^3$ N/m

stretched position $x_1 = 5$ cm

$\therefore \quad W_1 = \frac{1}{2}kx_1^2 = \frac{1}{2}(5 \times 10^3)(5 \times 10^{-2})^2$

$= 6.25$ J

For additional compression $W_2 = \frac{1}{2}kx_2^2$

$W_2 = \frac{1}{2} \times 5 \times 10^3 \times (10 \times 10^{-2})^2 = 25$ J

$\therefore$ The additional work done $= W_2 - W_1$

$= 25 - 6.25$

$= 18.75$ J

(8)

Since $W = \int F \cdot dr$.

clearly for forces (A) and (B), the integration do not require any information of path taken.

For (C): $W_c = \int \frac{3(x\hat{i}+y\hat{j})}{(x^2+y^2)^{3/2}} \cdot (dx\hat{i}+dy\hat{j})$

$$= 3\int \frac{x\,dx + y\,dy}{(x^2+y^2)^{3/2}}$$

Taking: $x^2+y^2 = t$

$\Rightarrow 2x\,dx + 2y\,dy = dt$

$\Rightarrow x\,dx + y\,dy = \frac{dt}{2}$ $\Rightarrow$ ~~$W = 3\int \frac{dt/2}{}$~~

$W = 3\int \frac{\frac{dt}{2}}{t^{3/2}} = \frac{3}{2}\int \frac{dt}{t^{3/2}}$. which is solvable.

$\therefore$ A, B, C are conservative force

(9)

From Diagram $(W_F)_{OAC} = \int (xy\,dx + x^2y\,dy)$

$$= \int_O^A (xy\,dx + x^2y\,dy) + \int_A^C (xy\,dx + x^2y\,dy)$$

on OA Path $y=0$; $dy=0$ and on AC Path $x=1$; $dx=0$

$$(W_F)_{OAC} = \int_O^A (0\cdot dx + 0\cdot dy) + \int_{y=0}^{4} (0 + 1 \times y\,dy)$$

$$\int_0^4 y\,dy = \left[\frac{y^2}{2}\right]_0^4 = \left[\frac{4^2}{2}\right] = 8\,J$$

$$(W_F)_{OBC} = \int_O^B (xy\,dx + x^2y\,dy) + \int_B^C (xy\,dx + x^2y\,dy)$$

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$\Rightarrow (W_F)_{OBC} = \int_O^B (0 \cdot dx + 0 \cdot dy) + \int_B^C (xy\,dx + x^2y\,dy)$

$$= 0 + \int_{x=0}^{1} (x(4)dx + x^2 \times 4(0)]\,dx =$$

$[y=4]$

$$= \int_0^1 4x\,dx = 4\left[\frac{x^2}{2}\right]_0^1 = 4\left(\frac{1}{2}\right) = 2J$$

$$(W_F)_{ODC} = \int_O^B [xy\,dx + x^2y\,dy] + \int_B^D [xy\,dx + x^2y\,dy]$$

$\underline{y = 4x^2}$ $\therefore$ $dy = 8x\,dx$ For Path OD $x=0$, $dy=0$

$$= \int_O^D (0 \cdot dx + 0\,dy) + \int_D^C [xy\,dx + x^2y\,dy]$$

For DC $x = 1$

$$= \int_0^1 x\,4x^2\,dx + x^2\,4x^2\,8x\,dx$$

$$= \int_0^1 4x^3\,dx + 32\int x^5\,dx$$

$$= 4\left[\frac{x^4}{4}\right]_0^1 + 32\left[\frac{x^6}{6}\right]_0^1$$

$$= 1 + \frac{32}{6} = \frac{38}{6} = \frac{19}{3} J$$

only A-correct