Class: VIII, MATHEMATICS

FOUNDATION

3. DISTANCE BETWEEN TWO POINTS

TEACHING TASK (JEE MAINS)

01. $A(at_1^2, 2at_1)$, $B(at_2^2, 2at_2)$

$x_1 \quad y_1 \quad x_2 \quad y_2$

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(at_2^2-at_1)^2+(2at_2-2at_1)^2}$$

$$= \sqrt{a^2(t_2^2-t_1)^2+4a^2(t_2-t_1)}$$

$$= \sqrt{a^2(t_2+t_1)^2(t_2-t_1)^2+4a^2(t_2-t_1)}$$

$$= a(t_2-t_1)\sqrt{(t_2+t_1)^2+4}$$

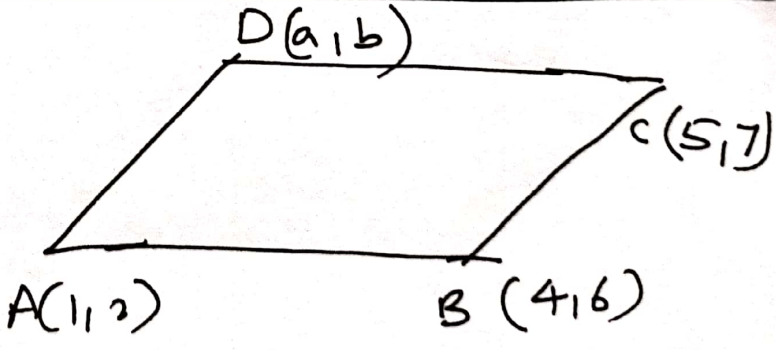
Ans. A

02

$a = 1+5-4 = 2$

$b = 2+7-6 = 3$

$\therefore D(a,b) = (2,3)$



Ans: C

03 $P(at^2, 2at)$, $Q\left(\frac{a}{t^2}, \frac{2a}{t}\right)$, $S(a, 0)$

$$SP = \sqrt{(a-at^2)^2 + (0-2at)^2}$$

$$= \sqrt{a^2(1-t^2)^2 + 4a^2t^2}$$

$$= a\sqrt{(1-t^2)^2 + 4t^2} = a(1+t^2)$$

$$SQ = \sqrt{\left(\frac{a}{t^2} - a\right)^2 + \left(\frac{2a}{t} - 0\right)^2}$$

$$= \sqrt{a^2\left(\frac{1}{t^2} - 1\right)^2 + \frac{4a^2}{t^2}}$$

$$= a\sqrt{\left(\frac{1}{t^2} - 1\right)^2 + \frac{4}{t^2}} = a\sqrt{\left(\frac{1}{t^2} + 1\right)^2}$$

$$= a\left(\frac{1}{t^2} + 1\right)$$

$$\therefore \frac{1}{SP} + \frac{1}{SQ} = \frac{1}{a(1+t^2)} + \frac{1}{a\left(\frac{1}{t^2}+1\right)}$$

$$= \frac{1}{a(1+t^2)} + \frac{t^2}{a(1+t^2)}$$

$$= \frac{1}{a}\left(\frac{1+t^2}{1+t^2}\right) = \frac{1}{a}$$

Ans: A

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04 Let $C(x, y)$, $A(3, 4)$, $B(7, 7)$

Option Verification method

Let $C(-5, -2)$

$$AC = \sqrt{(3+5)^2 + (4+2)^2}$$

$$= \sqrt{64+36} = \sqrt{100} = 10$$

Ans D

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05 $A(1,1)$, $B(-2,7)$, $C(3,-3)$

clearly $AB = 3\sqrt{5}$, $BC = 5\sqrt{5}$, $AC = 2\sqrt{5}$

Also, $AB + AC = 3\sqrt{5} + 2\sqrt{5} = 5\sqrt{5} = BC$

$\therefore$ A, B, C are collinear

Ans: C

06 $A(a\sin\alpha, -b\cos\alpha)$, $B(-a\cos\alpha, b\sin\alpha)$

$$AB = \sqrt{(a\sin\alpha + a\cos\alpha)^2 + (b\sin\alpha + b\cos\alpha)^2}$$

$$= \sqrt{(a^2+b^2)\sin^2\alpha + (a^2+b^2)\cos^2\alpha + 2(a^2+b^2)\sin\alpha\cos\alpha}$$

$$= \sqrt{(a^2+b^2)(\sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha)}$$

$$= \sqrt{(a^2+b^2)(\sin\alpha + \cos\alpha)^2}$$

$$= \sqrt{a^2+b^2}\,.\,(\sin\alpha + \cos\alpha)$$

Ans: C

07 $A(2,1)$, $B(1,-2)$, $P(x,y)$

$PA = PB \Rightarrow PA^2 = PB^2$

$\Rightarrow (x-2)^2 + (y-1)^2 = (x-1)^2 + (y+2)^2$

$\Rightarrow x + 3y = 0$

Ans: D

08 $x^2 + 2ax - b^2 = 0$

x_1 x_2

$x_1 + x_2 = -2a$

$x_1 \cdot x_2 = -b^2$

$y^2 + 2py - q^2 = 0$

y_1 y_2

$y_1 + y_2 = -2p$

$y_1 \cdot y_2 = -q^2$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

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$$= \sqrt{(x_1+x_2)^2 - 4x_1x_2 + (y_1+y_2)^2 - 4y_1y_2}$$

$$= \sqrt{(-2a)^2 - 4(-b^2) + (-2p)^2 - 4(-q^2)}$$

$$= \sqrt{4a^2 + 4b^2 + 4p^2 + 4q^2}$$

$$= 2\sqrt{a^2+b^2+p^2+q^2}$$ Ans.. B

09. $A(3,4)$, $B(-2,3)$ → Equilateral triangle
(x_1, y_1) (x_2, y_2)

$$\text{Third vertex} = \left(\frac{x_1+x_2 \pm \sqrt{3}(y_2-y_1)}{2}, \frac{y_1+y_2 \mp (x_2-x_1)}{2}\right)$$

$$= \left(\frac{3-2 \pm \sqrt{3}(3-4)}{2}, \frac{4+3 \mp \sqrt{3}(-2-3)}{2}\right)$$

$$= \left(\frac{1 \pm \sqrt{3}(-1)}{2}, \frac{7 \mp \sqrt{3}(-5)}{2}\right)$$

$$= \left(\frac{1 \mp \sqrt{3}}{2}, \frac{7 \pm 5\sqrt{3}}{2}\right)$$

$$= \left(\frac{1-\sqrt{3}}{2}, \frac{7+5\sqrt{3}}{2}\right), \left(\frac{1+\sqrt{3}}{2}, \frac{7-5\sqrt{3}}{2}\right)$$ Ans. D

10. $P(K-1, 2)$, $A(3, K)$, $B(K, 5)$

$$\Rightarrow PA = PB$$

$$\Rightarrow PA^2 = PB^2$$

$$\Rightarrow [(K-1-3)^2 + (2-K)^2] = [(K-1-K)^2 + (2-5)^2]$$

$$\Rightarrow K = 1, 5$$ Ans: B

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11. $A(5,7)$, $B(2,x)$

Given $AB = 5$

$\Rightarrow AB^2 = 25$

$\Rightarrow (5-2)^2 + (7-x)^2 = 25$

$\Rightarrow x = 3, 11$

Ans. A, C

12. $A(-4,0)$ (x_1, y_1), $B(4,0)$ (x_2, y_2) $\Rightarrow$ Equilateral triangle.

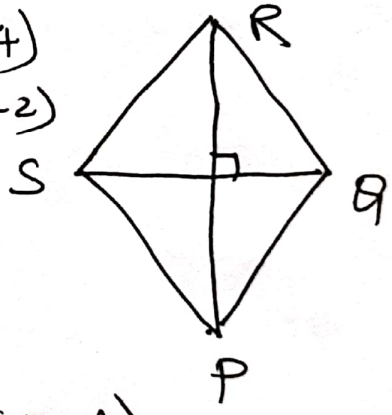
$$\text{Third vertex} = \left(\frac{x_1 + x_2 \pm \sqrt{3}(y_2 - y_1)}{2}, \frac{y_1 + y_2 \mp \sqrt{3}(x_2 - x_1)}{2} \right)$$

$$= \left(\frac{-4 + 4 \pm \sqrt{3}(0-0)}{2}, \frac{0 + 0 \mp \sqrt{3}(4+4)}{2} \right)$$

$$= (0, \pm 4\sqrt{3})$$

Ans. A, B

13. Statement I: $P(2,-1)$, $Q(3,4)$, $R(-2,3)$, $S(-3,-2)$



$PR = \sqrt{(-2-2)^2 + (3+1)^2}$

$= 4\sqrt{2}$

Similarly $SQ = 6\sqrt{2}$

$\therefore PR \neq SQ \Rightarrow$ Rhombus (True)

Statement II: Conceptual (True)

Ans. A

14. Statement I: $A(5,2)$, $B(7,9)$, $C(9,16)$

$AB = \sqrt{53}$, $BC = \sqrt{53}$, $AC = 2\sqrt{53}$

$\therefore AB + BC = AC \Rightarrow A, B, C$ collinear (True)

Statement II: Conceptual (True)

Ans. A

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15 Assertion: $A(1,2), B(2,3), C(3,5)$

$AB = \sqrt{(2-1)^2+(3-2)^2} = \sqrt{1+1} = \sqrt{2}$

$BC = \sqrt{(3-2)^2+(5-3)^2} = \sqrt{1+4} = \sqrt{5}$

$CA = \sqrt{(3-1)^2+(5-2)^2} = \sqrt{4+9} = \sqrt{13}$

clearly, A, B, C are not collineal (True)

Reason: Conceptual (True)

Ans. A

16 Assertion: $A(2,3), B(4,7), C(6,11)$

$AB = \sqrt{(4-2)^2+(7-3)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$BC = \sqrt{(6-4)^2+(11-7)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$CA = \sqrt{(6-2)^2+(11-3)^2} = \sqrt{16+64} = \sqrt{80} = 4\sqrt{5}$

clearly $AB+BC = AC \Rightarrow$ A, B, C are collinear

Reason: Given $y = 2x-1$

$(2,3) \Rightarrow 3 = 2(2)-1 = 3$ (True)

$(4,7) \Rightarrow 7 = 2(4)-1 = 7$ (True)

$(6,11) \Rightarrow 11 = 2(6)-1 = 11$ (True)

Ans: A

17 $A(5,7), B(8,-3)$ and $C(14,-1)$

$AB = \sqrt{52}, \quad BC = \sqrt{52}, \quad AC = 2\sqrt{52}$

Clearly $AB+BC = CA$

Ans: A

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18. $A(-1,3)$, $B(2,p)$ and $C(5,-1)$
(x_1,y_1) (x_2,y_2) (x_3,y_3)

$\text{Area} = \frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right| = 0$

$\Rightarrow p = 1$

Ans: D

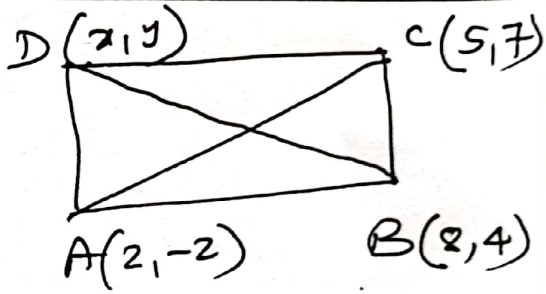
19. Conceptual

Ans: D

20. $MN = \sqrt{(6-2)^2+(7-3)^2} = \sqrt{16+16} = \sqrt{32} = \sqrt{16\times 2}$
$= 4\sqrt{2}$

Ans: D

21. mid-point of BD = mid-point of AC



$\Rightarrow \frac{x+8}{2} = \frac{2+5}{2}$

$\Rightarrow x = -1$

$\Rightarrow 2x = -2$

$\Rightarrow 2x+3 = -2+3 = 1$

Ans: 1

22. $A(x,0)$, $B(4,3)$

$AB = 5 \Rightarrow AB^2 = 25$

$\Rightarrow (x-4)^2+(0-3)^2 = 25$

$\Rightarrow (x-4)^2 = 25-9 = 16$

$\Rightarrow x-4 = \pm 4$

$\Rightarrow x = 8, 0$

Ans: 8, 0

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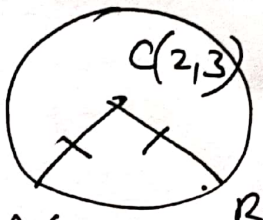
23. (8)

a) $\sqrt{(-1+6)^2+(-5-7)^2} = \sqrt{25+64} = \sqrt{89}$ (_)

b) $CA = CB$

$\Rightarrow CA^2 = CB^2$



$\Rightarrow (4-2)^2+(3-3)^2 = (x-2)^2+(5-3)^2$

$\Rightarrow x = 2$ (p)

c) $PA = PB \Rightarrow PA^2 = PB^2$

$\Rightarrow (x-2)^2+(0+5)^2 = (x+2)^2+(0-9)^2$

$\Rightarrow x = -7$ (q)

d) $OA^2 = 3^2+4^2 = 25$ (r)

Ans: —, p, q, r

24 A) A(1,2), B(3,6), C(5,10)

slope of $AB = \frac{6-2}{3-1} = \frac{4}{2} = 2$

slope of $BC = \frac{10-6}{5-3} = \frac{4}{2} = 2$

∴ A, B, C are Collinear (1)

B) O(0,0), A(2,3), B(4,5)

$OA = \sqrt{4+9} = \sqrt{13}$, $OB = \sqrt{4^2+5^2} = \sqrt{16+25} = \sqrt{41}$

$AB = \sqrt{(4-2)^2+(5-3)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

A, B, C are not collinear (2)

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C. A(2,−1), B(4,3), C(6,7)

$AB = \sqrt{(4-2)^2 + (3+1)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$BC = \sqrt{(6-4)^2 + (7-3)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$CA = \sqrt{(6-2)^2 + (7+1)^2} = \sqrt{16+64} = \sqrt{80} = 4\sqrt{5}$

clearly AB + BC = CA

also, AB = BC (3)

D) A(−1,−2), B(1,2), C(3,6)

$AB = \sqrt{(1+1)^2 + (2+2)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$BC = \sqrt{(3-1)^2 + (6-2)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$CA = \sqrt{(3+1)^2 + (6+2)^2} = \sqrt{16+64} = \sqrt{80} = 4\sqrt{5}$

clearly, $AB + BC = 2\sqrt{5} + 2\sqrt{5} = 4\sqrt{5} = CA$

(2) A, B, C are collinear

AB = BC (3)

Ans: 1, 2, 3, 3

LEARNER'S TASK (CUQ'S)

01. A(a,0), B(b,0)

$AB = \sqrt{(b-a)^2} = \sqrt{(a-b)^2} = |a-b|$ Ans. D

02. A(0,a), B(0,b)

$AB = \sqrt{(0-0)^2 + (b-a)^2} = |a-b|$ Ans: D

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03. $A(-3,4)$, $B(0,0)$

$AO = \sqrt{(-3)^2+4^2} = \sqrt{16+9} = \sqrt{25} = 5$ Ans B

04. Conceptual Ans: B

05 $AO = \sqrt{a^2+b^2}$ Ans: B

06. $A = (a,0)$

Distance from Y-axis $= |x| = |a|$ Ans: C

07 $A = (0,b)$

Distance from Y-axis $= |0| = 0$ Ans: A

08 $A(a,0)$, $B(0,b)$
x_1, y_1 x_2, y_2

$AB = \sqrt{(0-a)^2+(b-0)^2} = \sqrt{a^2+b^2}$ Ans: A

09 $PQ = |x_2 - x_1|$ Ans: D

10 $A = (-3,4)$

Distance from Y-axis $= |x| = |-3| = 3$ Ans. A

JEE MAINS LEVEL

01. $P(x,y)$ $A(a+b, a-b)$, $B(a-b, a+b)$

Given $PA = PB$

$\Rightarrow PA^2 = PB^2$

$\Rightarrow [x-(a+b)]^2 + [y-(a-b)]^2 = [(x-(a-b))^2 + (y-(a+b))^2]$

$\Rightarrow ax+by = 0$ Ans. D

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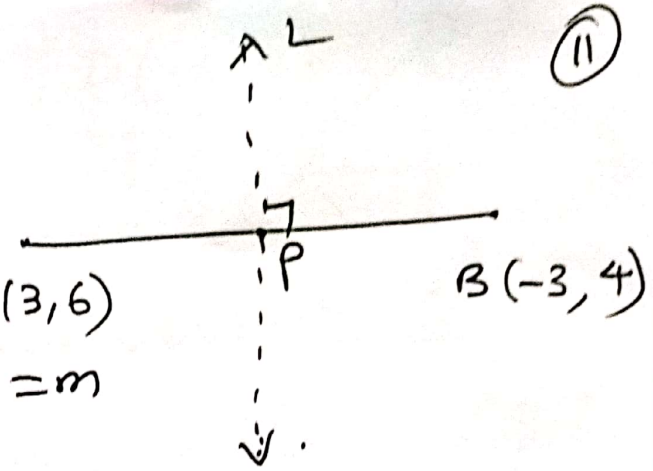
02. ~~P(x,y), A(~~

$P = \left(\frac{3-3}{2}, \frac{6+4}{2}\right)$

$P = (0, 5)$



Slope of $AB = \frac{4-6}{-3-3} = \frac{1}{3} = m$

Slope of $L = \frac{-1}{m} = -3$

Eqy. $y - y_1 = m(x - x_1)$

$\Rightarrow y - 5 = -3(x - 0)$

$\Rightarrow 3x + y - 5 = 0$

Ans:- B

03 Points on Y-axis $P(0, y)$, $A(6, 5)$, $B(-4, 3)$

$PA = PB \Rightarrow PA^2 = PB^2$

$\Rightarrow (6-0)^2 + (5-y)^2 = (-4-0)^2 + (3-y)^2$

$\Rightarrow y = 9 \quad \therefore (0, 9)$

Ans:- D

04 Let $P(2x, x)$, $Q(2, -5)$, $R(-3, 6)$

$\therefore PQ = PR$

$\Rightarrow PQ^2 = PR^2$

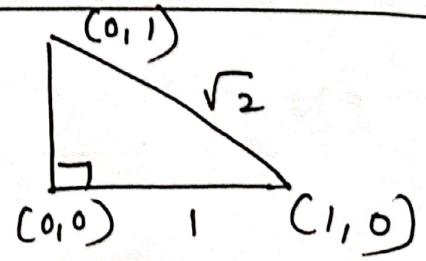
$\Rightarrow (2x-2)^2 + (x+5)^2 = (2x+3)^2 + (x-6)^2$

$\Rightarrow x = 8 \quad \therefore P(16, 8)$

Ans:- B

05 Perimeter $= 1 + 1 + \sqrt{2}$

$= 2 + \sqrt{2}$



Ans:- D

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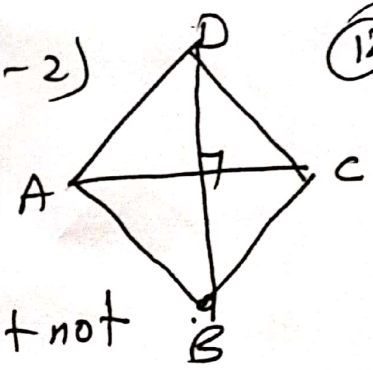
06 $A(2,-1)$, $B(3,4)$, $C(-2,3)$, $D(-3,-2)$



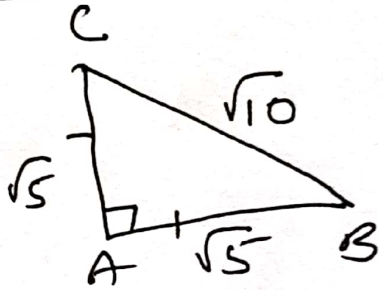
We observe $AB = BC = CD = DA$

Also, $AC \neq BD$

Hence, ABCD is a Rhombus, but not square

Ans. B

07 $A(0,-1)$, $B(-2,0)$, $C(-1,-3)$



We observe

$AB = \sqrt{4+1} = \sqrt{5}$

$BC = \sqrt{1+9} = \sqrt{10}$

$CA = \sqrt{1+4} = \sqrt{5}$

$\therefore BC^2 = AB^2 + BC^2$

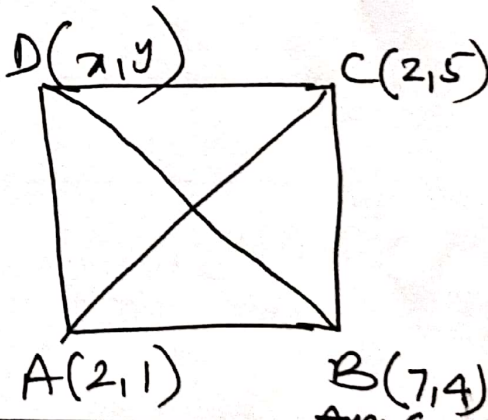
It is a right angled isosceles triangle

Ans. B

08 $x = 2+2-7 = -3$

$y = 1+5-4 = 2$

$\therefore D(x,y) = (-3,2)$



Ans: C

09 $A(\underset{x_1}{-4}, \underset{y_1}{0})$, $B(\underset{x_2}{4}, \underset{y_2}{0})$ → Equilateral triangle

$$\text{Third Vertex} = \left(\frac{x_1 + x_2 \pm \sqrt{3}(y_2 - y_1)}{2}, \frac{y_1 + y_2 \mp \sqrt{3}(x_2 - x_1)}{2} \right)$$

$$= (0, \pm\sqrt{2})$$

Ans:

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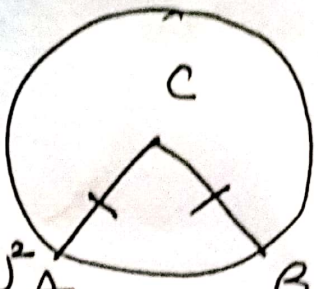
10 $C(2,-3y)$, $A(-1,y)$, $B(5,7)$



$CA = CB$

$\Rightarrow CA^2 = CB^2$

$\Rightarrow (2+1)^2 + (-3y-y)^2 = (2-5)^2 + (-3y-7)^2$

$\Rightarrow y^2 - 6y - 7 = 0$

$\Rightarrow (y-7)(y+1) = 0$

$\Rightarrow y = -1, 7$

let $y = -1$, $C = (2,3)$, $A(-1,-1)$

$r = CA = \sqrt{(2+1)^2 + (3+1)^2} = \sqrt{9+16} = 5$

let $y = 7$ $C = (2,-21)$, $A(-1,7)$

$r = CA = \sqrt{(2+1)^2 + (-21-7)^2}$

$= \sqrt{9+784} = \sqrt{793}$ Ans. D

JEE ADVANCED

11. Conceptual

Ans A, B, C, D

12. $A(0,0)$, $B(8,0)$, $C(4,4)$

A) $AB = \sqrt{8^2+0^2} = \sqrt{64} = 8$ ✓

B) $BC = \sqrt{(8-4)^2 + (4-0)^2} = \sqrt{16+16} = \sqrt{32}$

$AC = \sqrt{4^2+4^2} = \sqrt{16+16} = \sqrt{32}$ ✓ $\therefore BC = AC$

C) clearly $BC^2 + AC^2 = AB^2$ ✓

Ans. A, B, C

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13. Statement I: $P(2,2)$, $A(-2,K)$, $B(-2K,-3)$

$PA = PB$

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (-2-2)^2 + (K-2)^2 = (2+2K)^2 + (2+3)^2$

$\Rightarrow K = -1$ or -3 (True)

Statement II: Not always true (False)

Ans: C

14. Statement I: $M(2,3)$, $N(5,7)$

$MN = \sqrt{(5-2)^2 + (7-3)^2} = \sqrt{9+16} = 5$ (True)

Statement II: True. Let $A(x_1,y_1)$, $B(x_2,y_2)$

Since, let $|x_1 - x_2| = |y_1 - y_2| = K$.

Now $AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{K^2+K^2} = \sqrt{2K^2} = \sqrt{2}K$.

Ans.. A

15. Assertion: $A(1,2)$, $B(3,4)$, $C(5,6)$

$AB = \sqrt{(3-1)^2 + (4-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$BC = \sqrt{(5-3)^2 + (6-4)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$CA = \sqrt{(5-1)^2 + (6-2)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

Clearly $AB + BC = AC \rightarrow A, B, C$ Collinear

Reason: Conceptual (True)

Ans.. A

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16. Assertion: Conceptual (True)
Reason: Conceptual (True)
Ans. A

17. $P(x,y)$, $A(3,6)$, $B(-3,4)$

$PA = PB$

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$

$\Rightarrow 3x + y - 5 = 0$

Ans. B

18. Conceptual $|x| = |y|$

Ans. D

19. Conceptual

Ans. A, B, C

20. $A(0,0)$, $C(4,4)$

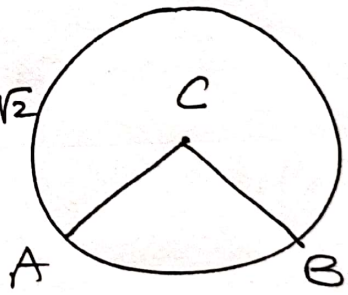
$AC = \sqrt{4^2+4^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

Ans. B

21. $C(2a, a-7)$, $A(11,-9)$

$r = CA = \sqrt{(2a-11)^2 + (a-7+9)^2} = 5\sqrt{2}$

$\Rightarrow a = 3, 5$



Ans: 5

22. $A = (2,1)$, $C = (8,9)$

$AC = \sqrt{(8-2)^2 + (9-1)^2} = \sqrt{36+64} = \sqrt{100} = 10$

Ans. 10

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23 a) $X(x,0)$, $A(5,9)$, $B(-4,6)$

$XA = XB \Rightarrow XA^2 = XB^2$

$\Rightarrow (x-5)^2 + (0-9)^2 = (x+4)^2 + (0-6)^2$

$\Rightarrow x = 3$ $\therefore (3,0)$ (S)

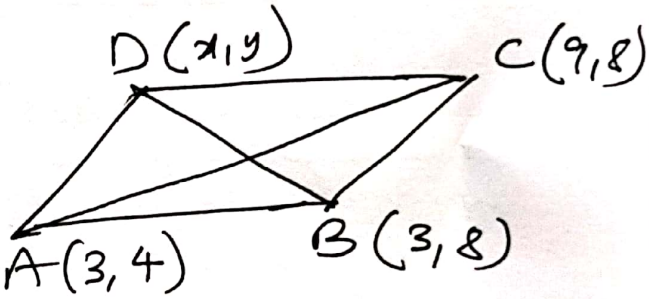
b) $Y(0,y)$, $A(2,3)$, $B(-4,1)$

$YA = YB \Rightarrow YA^2 = YB^2$

$\Rightarrow (0-2)^2 + (y-3)^2 = (0+4)^2 + (y-1)^2$

$\Rightarrow y = -1$ $\therefore (0,-1)$ (P)

c)



$x = 3+9-3 = 9$

$y = 4+8-8 = 4$

$\therefore D(x,y) = (9,4)$ (Q)

d) $PA = PB \Rightarrow PA^2 = PB^2$

$\Rightarrow (3-0)^2 + (k-2)^2 = (k-0)^2 + (5-2)^2$

$\Rightarrow k = 1$ (T)

Ans S, P, Q, T

24 a) $O(0,0)$, $A(1,1)$, $B(2,2)$

slope of AO = slope of AB = 1 (1)

b) Not collinear (2)

c) $A(-1,-3)$, $B(0,-1)$, $C(1,1)$

slope of $AB = \frac{-1+3}{0+1} = \frac{2}{1} = 2$

slope of $BC = \frac{1+1}{1-0} = 2$ (3)

D) slope of AB ≠ slope of BC ⇒ Not collinear (2)

THE END

Ans: 1, 2, 3, 2

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