FOUNDATION$^+$ (V_2)

Class: IX, MATHEMATICS

3. LIMITS OF TRIGONOMETRIC FUNCTIONS

TEACHING TASK (JEE MAINS)

01. $\lim_{x\to 0} \frac{1-\cos 2x}{x^2} = \lim_{x\to 0} \frac{2\sin^2 x}{x^2} = 2$ Ans: C

02. $\lim_{x\to \frac{\pi}{6}} \frac{2-\sqrt{3}\cos x-\sin x}{(6x-\pi)^2}$ $\left(\frac{0}{0}\right)$

By L-Hospital Rule

$= \lim_{x\to \frac{\pi}{6}} \frac{\sqrt{3}\sin x-\cos x}{2(6x-\pi)\cdot 6}$ $\left(\frac{0}{0}\right)$

$= \lim_{x\to \frac{\pi}{6}} \frac{\sqrt{3}\cos x+\sin x}{2\cdot 6\cdot 6} = \frac{\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)+\frac{1}{2}}{72} = \frac{1}{36}$

Ans: D

03. $\lim_{x\to 0} \frac{\tan 2x - x}{3x-\sin x}$

$= \lim_{x\to 0} \frac{\frac{\tan 2x}{x}-1}{3-\frac{\sin x}{x}}$

$= \frac{2-1}{3-1} = \frac{1}{2}$

Ans: A

04. $\lim_{x\to 2} \frac{\tan(x-2)\left[x^2+(K-2)x-2K\right]}{(x^2-4x+4)} = 5$

$= \lim_{x\to 2} \frac{\tan(x-2)\left[x^2+(K-2)x-2K\right]}{(x-2)^2} = 5$

$= \lim_{x\to 2} \frac{x^2+(K-2)x-2K}{x-2} = 5$ since $\lim_{x\to 0}\frac{\tan x}{x}=1$

$\lim_{x\to 2} \frac{2x+(K-2)}{1} = 5$

$\Rightarrow 4+K-2=5$

$\Rightarrow K=3$

Ans: B

05 We know,

$$-1 \le \sin\left(\frac{2}{x}\right) \le 1$$

$$-x^2 \le x^2 . \sin\left(\frac{2}{x}\right) \le x^2$$

$$\therefore \lim_{x\to 0} (-x^2) = \lim_{x\to 0} (x^2) = 0$$

$$\therefore \lim_{x\to 0} x^2 . \sin\left(\frac{2}{x}\right) = 0$$

Ans: B

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05 We know,

$$-1 \le \sin\left(\frac{1}{x^2}\right) \le 1$$

$$\Rightarrow -x^2 \le x^2 \sin\left(\frac{1}{x^2}\right) \le x^2$$

$$\therefore \lim_{x\to 0} (-x^2) = \lim_{x\to 0} x^2 = 0$$

$$\therefore \lim_{x\to 0} x^2 \sin\left(\frac{1}{x^2}\right) = 0$$

Ans: B

07.

$$\lim_{x\to\infty} \frac{2x^2 - \sin 3x}{x^2+10}$$

$$= \lim_{x\to\infty} \frac{2 - \frac{\sin 3x}{x^2}}{1 + \frac{10}{x^2}}$$

As $x \to \infty$, $\frac{1}{x} \to 0$

$$= \lim_{\frac{1}{x}\to 0} \frac{2 - \frac{1}{x} \cdot \left(\frac{\sin x}{x}\right)}{1 + \frac{10}{x^2}}$$

$$= \frac{2 - 0 \times \text{finite value}}{1+0}$$

$$= 2$$

Ans: A

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08 $\lim_{x\to 0} \dfrac{a^{\tan x} - a^{\sin x}}{\tan x - \sin x}$

$= \lim_{x\to 0} \dfrac{a^{\sin x}\left(a^{\tan x - \sin x} - 1\right)}{(\tan x - \sin x)}$

$= \lim_{x\to 0} a^{\sin x} \cdot \lim_{x\to 0} \dfrac{a^{(\tan x - \sin x)} - 1}{\tan x - \sin x}$

$= a^{\sin 0} \cdot \log a$

$= \log a$

Ans: A

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09 $\lim_{x\to\infty} 2^{x-1} \cdot \tan\left(\dfrac{a}{2^x}\right)$

$= \lim_{x\to\infty} \dfrac{2^x}{2} \cdot \tan\left(\dfrac{a}{2^x}\right)$

$= \dfrac{1}{2} \cdot \lim_{\frac{1}{2^x}\to 0} \dfrac{\tan\left(\frac{a}{2^x}\right)}{\left(\frac{1}{2^x}\right)}$

$= \dfrac{1}{2} \times a$

$= \dfrac{a}{2}$

Ans: A

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10. $\lim_{x\to 0} \dfrac{(1-\cos 2x)(3+\cos x)}{x \tan 4x}$

$= \lim_{x\to 0} \dfrac{2\sin^2 x \cdot (3+1)}{x^2 \cdot \left(\dfrac{\tan 4x}{x^2}\right)} = \dfrac{2\times 4}{4} = 2$

Ans: C

11.

11. $\lim_{x\to 0} x^n \cdot \sin\left(\frac{1}{x^2}\right)$

Let $n = 2$

$\lim_{x\to 0} x^2 \cdot \sin\left(\frac{1}{x^2}\right) = 0.$

Let $n = -2$

$\lim_{x\to 0} x^{-2} \cdot \sin\left(\frac{1}{x^2}\right)$

$= \lim_{x\to 0} \dfrac{\sin\left(\frac{1}{x^2}\right)}{\left(\frac{1}{x^2}\right)}$

$-1 \leq \sin\left(\frac{1}{x^2}\right) \leq 1$

$-\frac{1}{x^2} \leq \dfrac{\sin\left(\frac{1}{x^2}\right)}{x^2} \leq \frac{1}{x^2}$

$\lim_{x\to 0}\left(\frac{-1}{x^2}\right) = -\infty$

$\lim_{x\to 0}\left(\frac{1}{x^2}\right) = \infty$

$\therefore n<0$, $\lim_{x\to 0} f(x)$

Does not exists

Ans: A, B

12. $\lim_{x\to 0} \frac{\sin x}{x} = 1$, Also $\lim_{x\to 0} \frac{\tan x}{x} = 1$

$\lim_{x\to 0} \cos x = \cos 0 = 1$

Ans: A, B, C

13 Statement I: $\lim_{x\to 0} \frac{\cos x}{x} = 1$ (False)

Statement II: $\lim_{x\to 0} \frac{\sin ax}{a} = a$ (True)

Ans: D

14 Statement I: $\lim_{x\to 0} \frac{\sin x - x}{x^2}$ $\left(\frac{0}{0}\right)$

$= \lim_{x\to 0} \frac{\cos x - 1}{2x}$ $\left(\frac{0}{0}\right)$

$= \lim_{x\to 0} \frac{-\sin x}{2}$

$= \frac{-\sin 0}{2} = 0$ (False)

Statement II:

$$\lim_{x\to 0} \frac{e^{x^2}-\cos x}{x^2} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to 0} \frac{e^{x^2}\cdot 2x + \sin x}{2x}$$

$$= \lim_{x\to 0} \left(e^{x^2} + \frac{\sin x}{2x}\right)$$

$$= e^0 + \frac{1}{2}$$

$$= 1+\frac{1}{2}$$

$$= \frac{3}{2} \text{ (True)}$$

Ans: D

15. Assertion: $\lim_{x\to 0} \frac{\sin^2 3x}{x^2} = (3)^2 = 9$ (True)

Reason: $\lim_{x\to 0} \frac{\sin^2 x}{x^2} = 1$ (True)

Ans: A

16. Assertion: $\lim_{x\to 0} \frac{\sin mx}{\sin nx}$

$$= \frac{\left(\lim_{x\to 0} \frac{\sin mx}{x}\right)}{\left(\lim_{x\to 0} \frac{\sin nx}{x}\right)} = \frac{m}{n} \text{ (True)}$$

Reason: $\lim_{x\to 0} \frac{\sin ax}{x} = a$ (True)

Ans. A

17. We know, $-1 \le \cos\left(\frac{2}{x}\right) \le 1$

$$\Rightarrow -x^3 \le x^3 \cos\left(\frac{2}{x}\right) \le x^3$$

$$\therefore \lim_{x\to 0} (-x^3) = \lim_{x\to 0} x^3 = 0$$

$$\therefore \lim_{x\to 0} x^3 \cos\left(\frac{2}{x}\right) = 0$$

Ans: A

18 We know $-1 \le \sin\left(\frac{1}{x}\right) \le 1$

$$\Rightarrow -x^2 \le x^2 \sin\left(\frac{1}{x}\right) \le x^2$$

$$\because \underset{x \to 0}{Lt} (-x^2) = \underset{x \to 0}{Lt}\ x^2 = 0$$

$$\therefore \underset{x \to 0}{Lt}\ x^2 \sin\left(\frac{1}{x}\right) = 0$$

Ans. A

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19. We know, $-1 \le \sin\left(\frac{4}{x}\right) \le 1$

$$\Rightarrow -x^2 \le x^2 \sin\left(\frac{4}{x}\right) \le x^2$$

$$\therefore \underset{x \to 0}{Lt} (-x^2) = \underset{x \to 0}{Lt}\ x^2 = 0$$

$$\therefore \underset{x \to 0}{Lt}\ x^2 \sin\left(\frac{4}{x}\right) = 0$$

Ans. A

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20. $\underset{x \to 0}{Lt} \frac{\sin x}{x}$

Differentiate w.r.t x both Numerator & Denominator

Ans. A

$$\therefore \underset{x \to 0}{Lt} \frac{\cos x}{1}$$

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21) $\underset{x \to 0}{Lt} \frac{\sin x}{x} = 1$

Ans. B

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22 $\underset{x \to 0}{Lt} \frac{1 - \cos 4x}{x^2}$ $\left(\frac{0}{0}\right)$

$$= \underset{x \to 0}{Lt} \frac{2\sin^2 2x}{x^2} = 2(2)^2 = 8$$

Ans. 8

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23 $\underset{x \to 0}{Lt}\left(\frac{\sin 3x}{x} - \frac{\sin 2x}{x}\right) = 3 - 2 = 1$

Ans. 1

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24 a) $\underset{x \to 0}{Lt} \frac{\sin x^\circ}{x}$

$$= \underset{x \to 0}{Lt} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$= \frac{\pi}{180} \quad (S)$$

b) $\underset{x \to 0}{Lt} \frac{\tan 2x - x}{3x - \sin x}$ $\left(\frac{0}{0}\right)$

$$= \underset{x \to 0}{Lt} \frac{\sec^2 2x \cdot 2 - 1}{3 - \cos x}$$

$$= \frac{2 - 1}{3 - 1} = \frac{1}{2} \quad (P)$$

c) $\lim_{x\to 0} \frac{\sin(\pi\cos^2 x)}{x^2}$

$= \lim_{x\to 0} \frac{\sin(\pi(1-\sin^2 x))}{x^2}$

$= \lim_{x\to 0} \frac{\sin(\pi - \pi\sin^2 x)}{x^2}$

$= \lim_{x\to 0} \frac{\sin(\pi\sin^2 x)}{x^2}$

$= \lim_{x\to 0} \underbrace{\frac{\sin(\pi\sin^2 x)}{\pi\sin^2 x}}_{1} \times \frac{\pi\sin^2 x}{x^2}$

$= 1 \times \pi$

$= \pi$ (q)

d) $\lim_{x\to\infty} 2^{x-1} \cdot \tan\left(\frac{a}{2^x}\right)$

$= \lim_{x\to\infty} \frac{2^x}{2} \cdot \tan\left(\frac{a}{2^x}\right)$

$= \frac{1}{2} \cdot \lim_{x\to\infty} \frac{\tan\left(\frac{a}{2^x}\right)}{\left(\frac{1}{2^x}\right)}$

$= \frac{1}{2} \cdot a$

$= \frac{a}{2}$ (r)

Ans: s, p, q, r

25. a) $\lim_{x\to 0} \frac{\sqrt{1+\tan x} - \sqrt{1-\tan x}}{\sin x}$ $\left(\frac{0}{0}\right)$

L-Hospital Rule

$= \lim_{x\to 0} \frac{\frac{1}{2\sqrt{1+\tan x}} \cdot \sec^2 x - \frac{1}{2\sqrt{1-\tan x}} \cdot (-\sec^2 x)}{\cos x}$

$= \frac{1}{2} + \frac{1}{2}$

$= 1$ (e)

b) $\underset{x\to 0}{Lt} \dfrac{1-\cos 2x^\circ}{x^2}$

$= \underset{x\to 0}{Lt} \dfrac{2\sin^2 x^\circ}{x^2}$

$= 2 . \underset{x\to 0}{Lt} \dfrac{\sin^2\left(\frac{\pi x}{180^\circ}\right)}{x^2}$

$= 2\left(\frac{\pi}{180}\right)^2$ (p)

c) $\underset{x\to 0}{Lt} \dfrac{1-\cos x}{x . \log(1+x)}$

$= \underset{x\to 0}{Lt} \dfrac{2\sin^2\left(\frac{x}{2}\right)}{x^2 . \frac{\log(1+x)}{x}}$

$= 2\left(\frac{1}{2}\right)^2$

$= \frac{1}{2}$ (r)

d) $\underset{x\to 0}{Lt} \dfrac{e^x - e^{\sin x}}{2(x-\sin x)}$

$= \frac{1}{2} \underset{x\to 0}{Lt} \dfrac{e^{\sin x}\left(e^{x-\sin x}-1\right)}{x-\sin x}$

$= \frac{1}{2} . e^{\sin 0} \times 1$

$= \frac{1}{2}$ (r)

Ans: e, p, r, r

LEARNERS TASK (CUQS)

01	$\underset{(x-a)\to 0}{Lt} \dfrac{\tan(x-a)}{x-a} = 1$	Ans: A
02	$\underset{x\to 0}{Lt} \dfrac{\sin 3x}{x} = 3$	Ans: C
03	$\underset{x\to 0}{Lt} \dfrac{\sin ax}{\sin bx} = \dfrac{a}{b}$	Ans. D
04	$\underset{x\to 0}{Lt} \dfrac{1-\cos 4x}{1-\cos 6x}$ $\left(\frac{0}{0}\right)$ $= \underset{x\to 0}{Lt} \dfrac{\sin 4x}{\sin 6x} = \dfrac{4}{6} = \dfrac{2}{3}$	Ans.. A

05 | 1 — Ans: B

06 | Conceptual — Ans: C

07. | Conceptual — Ans: C

08 | $\lim_{x \to 0} \cos x = \lim_{x \to 0} \cos 0 = 1$ — Ans. D

09. | 1 — Ans: C

10. | $\lim_{x \to 0} \frac{\sin x}{x} = 1$ — Ans.. A

JEEMAINS LEVEL

01. $\lim_{x \to 0} \frac{\sin 7x + \sin 5x}{\tan 5x - \tan 2x} = \frac{7+5}{5-2} = \frac{12}{3} = 4$

Ans. B

02. $\lim_{x \to 2} \frac{x^2 - x - 2}{2x - \sin(x-2)} \quad \left(\frac{0}{0}\right)$

$= \lim_{x \to 2} \frac{2x - 1}{2 - \cos(x-2)} = \frac{2.2 - 1}{2 - \cos(2-2)} = \frac{3}{1} = 3$

Ans D

03 $\lim_{x \to 0} \frac{\sec ax - \sec bx}{x^2} \quad \left(\frac{0}{0}\right)$

$= \lim_{x \to 0} \frac{\sec ax \cdot \tan ax \cdot a - \sec bx \cdot \tan bx \cdot b}{2x}$

$= \frac{1}{2} \cdot \lim_{x \to 0} \left(\sec ax \cdot \frac{\tan ax}{x} \cdot a - \sec bx \cdot \frac{\tan bx}{x} \cdot b \right)$

$= \frac{a^2 - b^2}{2}$

Ans A

04. $$\lim_{x\to 5} \frac{\sin^2(x-5)\cdot\tan(x-5)}{(x^2-25)(x-5)}$$

$$= \lim_{x\to 5} \frac{\sin^2(x-5)\cdot\tan(x-5)}{(x+5)(x-5)(x-5)}$$

$$= \lim_{x\to 5} \frac{\sin^2(x-5)}{(x-5)^2} \cdot \lim_{x\to 5} \frac{\tan(x-5)}{(x+5)}$$

$$= 1 \times \frac{\tan(5-5)}{5+5} = 0$$

Ans: C

05 $$\lim_{x\to \frac{\pi}{4}} \frac{1-\cot^3 x}{2-\cot x-\cot^3 x} \quad \left(\frac{0}{0}\right)$$

$$= \lim_{x\to \frac{\pi}{4}} \frac{-3\cot^2 x\cdot(-\operatorname{cosec}^2 x)}{-(-\operatorname{cosec}^2 x) - 3\cdot\cot^2 x\,(-\operatorname{cosec}^2 x)}$$

$$= \lim_{x\to \frac{\pi}{4}} \frac{3\cot^2 x\cdot\operatorname{cosec}^2 x}{\operatorname{cosec}^2 x + 3\cot^2 x\cdot\operatorname{cosec}^2 x}$$

$$= \frac{3(1)^2\cdot(\sqrt{2})^2}{(\sqrt{2})^2+3(1)^2\cdot(\sqrt{2})^2} = \frac{6}{8} = \frac{3}{4}$$

Ans B

6 (Repeated) $$\lim_{x\to\infty} 2^x - 1\cdot \tan\left(\frac{a}{2^x}\right) = \frac{a}{2}$$

Ans. A

7.

07. $$\lim_{x\to\infty} x\left(\tan^{-1}\left(\frac{x+1}{x+4}\right)-\frac{\pi}{4}\right)$$

$$=\lim_{x\to\infty} x\left(\tan^{-1}\left(\frac{x+1}{x+4}\right)-\tan^{-1}(1)\right)$$

$$=\lim_{x\to\infty} x\left[\tan^{-1}\left(\frac{\frac{x+1}{x+4}-1}{1+\frac{x-1}{x+4}}\right)\right]$$

$$=\lim_{x\to\infty} x\left(\tan^{-1}\left(\frac{-3}{2x+5}\right)\right]$$

$$=\lim_{x\to\infty}\frac{\tan^{-1}\left(\frac{-3}{2x+5}\right)}{\left(\frac{-3}{2x+5}\right)}\times\lim_{x\to\infty}\frac{-3x}{2x+5}$$

$$=1\times\lim_{\frac{1}{x}\to 0}\frac{-3}{2+\left(\frac{5}{x}\right)}=\frac{-3}{2+0}=\frac{-3}{2}$$

Ans. C

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08 $$\lim_{x\to 1}\frac{a^{x-1}-1}{\sin\pi x}\quad\left(\frac{0}{0}\right)$$

$$=\lim_{x\to 1}\frac{a^{x-1}\cdot\log a}{\cos\pi x\cdot\pi}$$

$$=-\frac{\log a}{\pi}$$

Ans. C

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09. $$\lim_{x\to 2}\frac{\sin\left(e^{x-2}-1\right)}{\log(x-1)}\quad\left(\frac{0}{0}\right)$$

$$=\lim_{x\to 2}\frac{\cos\left(e^{x-2}-1\right)\cdot e^{x-2}}{\frac{1}{x-1}}=1$$

Ans. A

10 $\lim_{x\to 0} \dfrac{a^{\tan x} - a^{\sin x}}{\tan x - \sin x} = \ln a$ Ans. A

JEE ADVANCED

11 $\lim_{x\to 0} \dfrac{\sin x - 2\sin 3x + \sin 5x}{x} = 1 - 2.3 + 5$

$= 0$ Ans:

Also, $\lim_{x\to \frac{\pi}{2}} \cos x = 0$

Also, $\lim_{x\to 0} \dfrac{x^2 - 2x}{\cos x} = 0$ Ans. A, B, C

12 $\lim_{x\to \frac{\pi}{2}} \dfrac{\tan 2x}{x - \frac{\pi}{2}}$ $\left(\frac{0}{0}\right)$

$= \lim_{x\to \frac{\pi}{2}} \dfrac{\sec^2 2x . 2}{1} = 2$

Also, $\lim_{x\to 0} \dfrac{\sin 2x}{x} = 2$ Ans: A D

13. Statement I: $\lim_{x\to 0} \dfrac{\sin x}{x} = 1$ (True)

Statement II: Conceptual (True) Ans. A

14. Statement I: $\lim_{x\to 0} \dfrac{1-\cos x}{x^2}$ $\left(\frac{0}{0}\right)$

$\lim_{x\to 0} \dfrac{2\sin^2\frac{x}{2}}{x^2} = 2\left(\frac{1}{2}\right)^2 = \frac{1}{2}$ (True)

Statement II: Conceptual (True) Ans: A

15. Assertion: $\lim_{x \to 0} \frac{\sin^{-1} 3x}{\tan^{-1} 6x} = \frac{3}{6} = \frac{1}{2}$ (True)

Reason: Conceptual (True)

Ans.. A

16. Assertion: $\lim_{x \to 0} \frac{\sin^{-1} x}{x} = 1$ (True)

Reason: $\lim_{x \to 0} \frac{\sin (ax)}{x} = a$ (True)

Ans. A

17. $\frac{5+1}{1+3} = \frac{6}{4} = \frac{3}{2}$ Ans. D

18. $\frac{3+4}{3+1} = \frac{7}{4}$ Ans. A

19. $\frac{3-4}{3+1} = \frac{-1}{4}$ Ans: D

20. $\frac{3+7}{3+7} = 1$ Ans: C

21. $\frac{3}{4}$ Ans.. A

22. $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$ Ans: B

23. 2 Ans: 2

24. $(3)^2 = 9$ Ans ~~B~~ 9

25 a) $\lim_{x\to 0} \sin x = \sin 0 = 0$ (s)

b) $\lim_{x\to 0} \frac{\sin x^\circ}{x} = \frac{\pi}{180}$ (p)

c) $\lim_{x\to 0} \frac{x^2}{x} = \lim_{x\to 0} x = 0$ (s)

d) $\lim_{x\to a} \sin^{-1} x = \sin^{-1} a = \cancel{0} = \sin^{-1}(a) \cancel{\Rightarrow a = 0}$ ($\cancel{s}$)

$\Rightarrow -1 \leq a \leq 1$

$\Rightarrow |a| < 1$ (q)

Ans: s, p, s, q

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26 a) $\left(\frac{2}{4}\right)^2 = \frac{1}{4}$ (s)

b) $\lim_{x\to\pi} \frac{\sin x}{x-\pi}$ $\left(\frac{0}{0}\right)$

$= \lim_{x\to\pi} \frac{\cos x}{1} = \cos\pi = -1$ (p)

c) $\lim_{x\to 0} \frac{\sin x}{\sqrt{x+1} - \sqrt{1-x}} \times \frac{\sqrt{x+1} + \sqrt{1-x}}{\sqrt{x+1} + \sqrt{1-x}}$

$= \lim_{x\to 0} \frac{\sin x\left(\sqrt{x+1} + \sqrt{1-x}\right)}{2x}$

$= 1$ (r)

d) $\lim_{x\to 0} |\cos x| = |\cos 0| = |1| = 1$ (r)

Ans: s, p, r, r

$\Rightarrow$ THE END $\Leftarrow$