COMPOUND ANGLES

SYNOPSIS

Definitions and Formulae:

The algebraic sum of two or more angles is called a compound angle. i.e., $A+B, A-B, A+B+C, A+B-C$, ...etc., are called compound angles.

If A and B are any two angles then

i) sin (A+B) = sin A cos B + cos A sin B

ii) sin (A-B) = sin A cos B - cos A sin B

iii) cos (A+B) = cos A cos B - sin A sin B

iv) cos (A-B) = cos A cos B + sin A sin B

If $A, B, A+B, A-B$ are not odd multiples of $\pi/2$, then

i) $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\ \tan B}$

ii) $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$

If $A, B, A+B$ and $A-B$ are not integral multiples of π, then

i) $\cot(A+B)=\dfrac{\cot A\cot B-1}{\cot B+\cot A}$

ii) $\cot(A-B)=\dfrac{\cot A\cot B+1}{\cot B-\cot A}$

i) $\sin(A+B)+\sin(A-B)=2\sin A\cos B$

ii) $\sin(A+B)-\sin(A-B)=2\cos A\sin B$

iii) $\cos(A+B)+\cos(A-B)=2\cos A\cos B$

iv) $\cos(A+B)-\cos(A-B)=-2\sin A\sin B$

v) $\cos(A-B)-\cos(A+B)=2\sin A\sin B$

i) sin (A+B) sin (A-B) = $\sin^2A - \sin^2B$
= $\cos^2B - \cos^2A$

ii) cos (A+B) cos (A-B) = $\cos^2A - \sin^2B$
= $\cos^2B - \sin^2A$

iii) $\tan(A+B)\tan(A-B)=\dfrac{\tan^2 A-\tan^2 B}{1-\tan^2 A\tan^2 B}$

i) $\tan(45^\circ+\theta)=\dfrac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$

$=\cot(45^\circ-\theta)=\dfrac{1+\tan\theta}{1-\tan\theta}$

ii) $\tan(45^\circ-\theta)=\dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$

$=\cot(45^\circ+\theta)=\dfrac{1-\tan\theta}{1+\tan\theta}$ and

iii) $\tan(45^\circ+\theta).\tan(45^\circ-\theta)=1$

W.E-1: $\sin^2 52\frac{1}{2}^\circ-\sin^2 22\frac{1}{2}^\circ$ **=**

Sol: ***Given*** $=\sin 75^\circ\sin 30^\circ$

= $\left(\dfrac{\sqrt{3}+1}{2\sqrt{2}}\right)\dfrac{1}{2}$ **=** $\dfrac{\sqrt{3}+1}{4\sqrt{2}}$

(i) If $\tan A=\dfrac{m}{n},\tan B=\dfrac{n-m}{m+n}$ then

$(A+B)=\pi/4$

(ii) If $\tan A=\dfrac{m}{n},\tan B=\dfrac{m-n}{m+n}$ then

$(A-B)=\pi/4$

Note :

i) $\sin\theta+\sin(120^\circ+\theta)-\sin(120^\circ-\theta)=0$

ii) $\sin\theta+\sin(240^\circ+\theta)-\sin(240^\circ-\theta)=0$

iii) $\cos\theta+\cos(120^\circ+\theta)+\cos(120^\circ-\theta)=0$

iv) $\cos\theta+\cos(240^\circ+\theta)+\cos(240^\circ-\theta)=0$

If $A+C=B$ then

$\tan B-\tan A-\tan C=\tan A\tan B\tan C$

W.E-3: $\tan 70^\circ - \tan 20^\circ = \_\_$

Sol: since $20^\circ + 50^\circ = 70^\circ$

$\Rightarrow \tan 70^\circ - \tan 20^\circ - \tan 50^\circ = \tan 20^\circ \tan 50^\circ \tan 70^\circ$

$\Rightarrow \tan 70^\circ - \tan 20^\circ = \tan 50^\circ + \tan 50^\circ$

$\Rightarrow \tan 70^\circ - \tan 20^\circ = 2\tan 50^\circ$

If $A + B = 45^\circ$ or 225° then

i) $(1+\tan A)(1+\tan B) = 2$

ii) $(1-\cot A)(1-\cot B) = 2$

iii) $(1+\cot A)(1+\cot B) = 2\cot A \cot B$

If $A + B = 135^\circ$ or 315° then

i) $(1-\tan A)(1-\tan B) = 2$

ii) $(1+\cot A)(1+\cot B) = 2$

iii) $(1+\tan A)(1+\tan B) = 2\tan A \tan B$

i) sin (A+B+C) = sin A cos B cosC + cos A sinBcosC +cosAcosBsinC - sinA sinB sinC

$= \sum \sin A \cos B \cos C - \Pi \sin A$

ii) cos(A+B+C) = cosAcosBcosC-cosA sin B sin C - sin A cosBsinC - sinA sinB cosC

$= \Pi \cos A - \sum \cos A \sin B \sin C$

iii) $\tan(A+B+C) = \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$

$= \frac{\sum \tan A - \Pi \tan A}{1 - \sum \tan A \tan B}$

iv) $\cot(A+B+C) = \frac{\cot A + \cot B + \cot C - \cot A \cot B \cot C}{1 - \cot A \cot B - \cot B \cot C - \cot C \cot A}$

$= \frac{\sum \cot A - \Pi \cot A}{1 - \sum \cot A \cot B}$

i) $\sin(A_1 + A_2 + ... + A_n)$

$= \cos A_1 \cos A_2 ..... \cos A_n (S_1 - S_3 + S_5 - S_7 + ...)$

ii) $\cos(A_1 + A_2 + ... + A_n)$

$= \cos A_1 \cos A_2 .... \cos A_n \left(1 - S_2 + S_4 - S_6 + ...\right)$

iii) $\tan\left(A_1 + A_2 + ... A_n\right) = \dfrac{S_1 - S_3 + S_5 - S_7 + ...}{1 - S_2 + S_4 - S_6 + ...}$

where

$S_1 = \tan A_1 + \tan A_2 + \tan A_3 + ... + \tan A_n$

$S_2 = \tan A_1 \tan A_2 + \tan A_1 \tan A_3 + ...$

$S_3 = \tan A_1 \tan A_2 \tan A_3 + \tan A_2 \tan A_3 \tan A_4 + ...$

Some useful results

If A + B + C = $n\pi, n \in Z$

i) $\sum \tan A = \Pi \tan A$ ii) $\sum \cot A \cot B = 1$

If A + B + C = $(2n+1)\dfrac{\pi}{2}, n \in Z$

i) tan A tan B + tan B tan C + tan C tan A = 1

ii) cot A + cot B + Cot C = cot A cot B cot C

W.E-4: $\tan 25^\circ \tan 31^\circ + \tan 31^\circ \tan 34^\circ + \tan 34^\circ \tan 25^\circ =$

Sol: since $25^\circ + 31^\circ + 34^\circ = 90^\circ$

$\Rightarrow \tan 25^\circ \tan 31^\circ + \tan 31^\circ \tan 34^\circ + \tan 34^\circ \tan 25^\circ = 1$

Some standard trigonometric values

i) $\sin 15^\circ = \dfrac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75^\circ$

ii) $\cos\ 15^\circ = \dfrac{\sqrt{3}+1}{2\sqrt{2}} = \sin\ 75^\circ$

iii) $\tan\ 15^\circ = 2 - \sqrt{3} = \cot\ 75^\circ$

iv) $\cot\ 15^\circ = 2 + \sqrt{3} = \tan\ 75^\circ$

v) $\sec 15^\circ = \sqrt{6} - \sqrt{2} = \operatorname{cosec} 75^\circ$

vi) $\operatorname{cosec} 15^\circ = \sqrt{6} + \sqrt{2} = \sec 75^\circ$

TEACHING TASK

LEVEL - I

1. $\sin 40^\circ 35' \cos 19^\circ 25' + \cos 40^\circ 35' \sin 19^\circ 25' =$

A) 1 B) $\frac{\sqrt{3}}{2}$ C)0 D) -1

2. $\cos(n+1)\alpha\cos(n-1)\alpha+\sin(n+1)\alpha\sin(n-1)\alpha=$

A) $\cos 2n\alpha$ B) $\sin 2n\alpha$ C) $\cos 2\alpha$ D) $\sin 2\alpha$

3. If $0 < \theta < \frac{\pi}{2}$ and $2\sin\theta=\sqrt{3}\cos 10^\circ+\sin 10^\circ$ then $\theta =$

A) 70° B) 50° C) 60° D) 40°

4. $\tan 40^\circ + \tan 80^\circ - \sqrt{3}\tan 40^\circ \tan 80^\circ =$

A) $\sqrt{3}$ B) $-\sqrt{3}$ C) $\frac{1}{\sqrt{3}}$ D) $\frac{-1}{\sqrt{3}}$

5. $\tan 100^\circ + \tan 125^\circ + \tan 100^\circ \tan 125^\circ =$

A) -1 B) 1 C) $\frac{1}{\sqrt{3}}$ D) $\frac{1}{2}$

6. $\frac{\tan 225^\circ - \cot 81^\circ \cot 69^\circ}{\cot 261^\circ + \tan 21^\circ} =$

A) 1 B) $\frac{1}{\sqrt{2}}$ C) $\sqrt{3}$ D) $\frac{1}{\sqrt{3}}$

7. If $\tan(A+B) = p, \tan(A-B) = q$ then $\cot 2B =$

A) $\frac{1+pq}{p-q}$ B) $\frac{1+pq}{p+q}$ C) $\frac{1-pq}{p-q}$ D) $\frac{1-pq}{p+q}$

8. If $0 < \theta < \frac{\pi}{2}, \tan\theta = \frac{\cos 29^\circ + \sin 29^\circ}{\cos 29^\circ - \sin 29^\circ}$, then $\theta =$

A) 16° B) 74° C) 37° D) 8°

9. $\operatorname{cosec} 15^\circ + \sec 15^\circ =$

A) $2\sqrt{2}$ B) $\sqrt{6}$ C) $2\sqrt{6}$ D) $\sqrt{6}+\sqrt{2}$

10. If $\cot 20^\circ = p$ then $\frac{\tan 160^\circ - \tan 110^\circ}{1+\tan 160^\circ \tan 110^\circ} =$

A) $\frac{p^2-1}{2p}$ B) $\frac{p^2+1}{2p}$ C) $\frac{1-p^2}{2p}$ D) $\frac{2p}{1+p^2}$

11. $\cos^2 52\frac{1}{2}^\circ - \sin^2 22\frac{1}{2}^\circ =$

A) $\frac{\sqrt{3}+1}{4\sqrt{2}}$ B) $\frac{\sqrt{3}-1}{4\sqrt{2}}$ C) $\frac{3+\sqrt{3}}{4\sqrt{2}}$ D) $\frac{3-\sqrt{3}}{4\sqrt{2}}$

12. In ΔABC, $\sum\left[\frac{\sin(A-B)}{\cos A\cos B}\right] =$

A) 1 B) 2 C) 0 D) $\frac{1}{2}$

13. $\frac{\tan 80^\circ - \tan 10^\circ}{\tan 70^\circ} =$

A) 0 B) 1 C) 2 D) 3

14. $\cot\left(\frac{\pi}{4}+\theta\right)\cot\left(\frac{\pi}{4}-\theta\right) =$

A) 0 B) -1 C) 1 D) 1/2

15. $\frac{(1+\tan 13^\circ)(1+\tan 32^\circ)}{(1+\tan 12^\circ)(1+\tan 33^\circ)} =$

A) 1 B) 2 C) 3 D) 4

16. $A+B=135^\circ \Rightarrow (1+\cot A)(1+\cot B) =$

A) 1 B) 2 C) 3 D) 4

17. $(1-\cot 200^\circ)(1-\cot 25^\circ) =$

A) 0 B) 1 C) 2 D) $\frac{1}{2}$

18. $\sin 20^\circ + \sin 40^\circ - \sin 80^\circ =$

A) -1 B) 1 C) 2 D) 0

19. $\cos A + \cos(120^\circ + A) + \cos(120^\circ - A) =$

A) 1 B) -1 C) 0 D) 2

20. $\cos 20^\circ + \cos 100^\circ + \cos 140^\circ =$

A) 0 B) -1 C) 1 D) 2

21. $\sin 10^\circ + \sin 130^\circ - \sin 110^\circ =$

A) 0 B) -1 C) 1 D) $\frac{1}{2}$

22. $\tan 27^\circ \tan 32^\circ + \tan 32^\circ \tan 31^\circ + \tan 31^\circ \tan 27^\circ =$

A) 1 B) 2 C) 3 D) 4

23. $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{a+b}{a-b} \Rightarrow \frac{\tan\alpha}{\tan\beta} =$

A) a/b B) b/a C) $a/2b$ D) $-a/b$

24. If $\sin A = \frac{12}{13}, \cos B = \frac{-3}{5}; 0 < A < \frac{\pi}{2}, \pi < B < \frac{3\pi}{2}$ then $\sin(A+B) =$

A) $\frac{33}{65}$ B) $-\frac{1}{63}$ C) $-\frac{56}{65}$ D) $-\frac{63}{65}$

25. If $B, A+B$ are acute angles , $\sin(A+B) = \frac{12}{13}, \sin B = \frac{5}{13}$ then $\sin A =$

A) $-\frac{119}{169}$ B) $\frac{119}{169}$ C) $\frac{169}{119}$ D) $-\frac{169}{119}$

26. If $270^\circ < A < 360^\circ$, $90^\circ < B < 180^\circ$, $\cos A = \frac{5}{13}, \tan B = -\frac{15}{8}$ then $\sin(A+B) =$

A) $\frac{140}{221}$ B) $\frac{171}{221}$ C) $\frac{140}{171}$ D) $\frac{221}{171}$

27. If A, B are acute angles. $\tan A = \frac{5}{12}, \cos B = \frac{3}{5}$ then $\cos(A+B) =$

A) $\frac{16}{65}$ B) $\frac{65}{16}$ C) $\frac{65}{63}$ D) $\frac{56}{65}$

28. If $\tan(A-B) = \frac{7}{24}, \tan A = \frac{4}{3}$ where A, B acute then A+B=

A) $\frac{\pi}{5}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{3}$ D) $\frac{\pi}{2}$

29. If $0 < \alpha, \beta < \frac{\pi}{4}$, $\cos(\alpha+\beta) = \frac{4}{5}$, $\sin(\alpha-\beta) = \frac{5}{13}$ then $\tan 2\alpha =$

A) $\frac{33}{56}$ B) $\frac{56}{33}$ C) $\frac{16}{63}$ D) $\frac{14}{63}$

30. If $x\cos\theta = y\cos\left(\theta+\frac{2\pi}{3}\right) = z\cos\left(\theta+\frac{4\pi}{3}\right)$ then the value of $\frac{1}{x}+\frac{1}{y}+\frac{1}{z} =$

A) 1 B) 2 C) 0 D) $3\cos\theta$

31. $\cot A \cot B = 2, \cos(A+B) = 3/5 \Rightarrow \sin A . \sin B =$

A) $2/5$ B) $1/5$ C) $4/5$ D) $3/5$

32. If $\sin x\cos y=\frac{1}{4}$ and $3\tan x=4\tan y$ then $\sin(x-y)=$

A) $\frac{1}{16}$ B) $\frac{7}{16}$ C) $\frac{3}{4}$ D) $\frac{3}{16}$

33. $\sin^2\alpha+\cos^2(\alpha+\beta)+2\sin\alpha\sin\beta\cos(\alpha+\beta)=$

A) $sin^2\alpha$ B) $sin^2\beta$ C) $\cos^2\alpha$ D) $\cos^2\beta$

34. If $\sin A+\sin B=\sqrt{3}(\cos B-\cos A)$ then $\sin 3A+\sin 3B=$

A) 0 B) 2 C) 1 D) -1

35. If $\frac{\cos A}{\cos B}=n$ and $\frac{\sin A}{\sin B}=m$ then $(m^2-n^2)\sin^2 B=$

A) $1-n^2$ B) $1+n^2$ C) $1-n$ D) $1+n$

36. If $2\tan A+\cot A=\tan B$ then $\cot A+2\tan(A-B)=$

A) -1 B) 0 C) 1 D) $1/2$

37. In a ΔABC if $\tan B=\frac{2\sin A\sin C}{\sin(A+C)}$ then $\tan A,\tan B,\tan C$ are in

A) A.P B) G.P C) H.P D) A.G.P.

38. If $A+B+C=\frac{\pi}{2}$ then $\sum\frac{\cos(B+C)}{\cos B\cos C}=$

A) 1 B) 2 C) 3 D) 4

39. If $x^2+y^2+z^2=r^2$ and $\tan\alpha=\frac{xy}{zr}$, $\tan\beta=\frac{yz}{xr},\tan\gamma=\frac{zx}{yr}$ then $\alpha+\beta+\gamma=$

A) $\frac{\pi}{4}$ B) π C) $\pi/2$ D) $\pi/3$

40. In ΔABC if $\cot A+\cot B+\cot C=\sqrt{3}$ then ΔABC is

A) equilateral triangle B) right angled triangle
C) isosceles D) scalene triangle.

41. $\tan A+\tan B+\tan C-\dfrac{\sin(A+B+C)}{\cos A\cos B\cos C}=$

A) $\tan A\tan B\tan C$ B) $\sin A\sin B\sin C$

C) $\cos A\cos B\cos C$ D) $\sum\tan A\tan B$

42. $\dfrac{\cos x}{\cos(x-2y)}=\lambda\Rightarrow\tan(x-y)\tan y=$

A) $\dfrac{1+\lambda}{1-\lambda}$ B) $\dfrac{1-\lambda}{1+\lambda}$ C) $\dfrac{\lambda}{1+\lambda}$ D) $\dfrac{\lambda}{1-\lambda}$

43 If $\tan A=\dfrac{x\sin B}{1-x\cos B}$ and $\tan B=\dfrac{y\sin A}{1-y\cos A}$ then $\dfrac{\sin A}{\sin B}=$

A) x / y B) y / x C) $x+y$ D) $x-y$

KEY

01) B 02) C 03) A 04) B 05) B 06) C 07)A 8) B 9) C 10) A
11) D 12) C 13) C 14) C 15) A 16) B 17) C 18) D 19) C 20)A
21) A 22)A 23)A 24) C 25) B 26) B 27) A 28) D 29) B 30) C
31) D 32) A 33) D 34) A 35)C 36) B 37)C 38) B 39) C 40) A
41) A 42) B 43) A

HINTS

1. $\sin 40^\circ 35' \cos 19^\circ 25' + \cos 40^\circ 35' \sin 19^\circ 25'$

$=\sin(40^\circ 35' + 19^\circ 25')$ $=\sin 60^\circ=\dfrac{\sqrt{3}}{2}$

2. $\cos(n\alpha+\alpha-n\alpha+\alpha)$ $=\cos 2\alpha$

3. $\sin\theta=\dfrac{\sqrt{3}}{2}\cos 10^\circ+\dfrac{1}{2}\sin 10^\circ$

$=\sin(60^\circ+10^\circ)$ $=\sin 70^\circ$

4. $40^\circ+80^\circ=120^\circ$

$\Rightarrow\tan 120^\circ-\tan 80^\circ-\tan 40^\circ=\tan 120^\circ\tan 40^\circ\tan 80^\circ$

$\Rightarrow-\sqrt{3}-\tan 80^\circ-\tan 40^\circ=-\sqrt{3}\tan 40^\circ\tan 80^\circ$

5. $100^\circ+125^\circ=225^\circ$

$\tan 225^\circ - \tan 100^\circ - \tan 125^\circ = \tan 225^\circ \tan 100^\circ \tan 125^\circ$

$\Rightarrow \tan 100^\circ + \tan 125^\circ + \tan 100^\circ \tan 125^\circ = 1$

6. $\dfrac{1-\cot 81^\circ \cot 69^\circ}{\cot 81^\circ + \cot 69^\circ} = -\cot\left(81^\circ + 69^\circ\right)$

$= -\cot 150^\circ = \sqrt{3}$

7. $\cot 2B = \cot\left[\left(A+B\right)-\left(A-B\right)\right]$

$= \dfrac{\frac{1}{p}\frac{1}{q}+1}{\frac{1}{q}-\frac{1}{p}} = \dfrac{1+pq}{p-q}$

8. $\tan\theta = \tan\left(\dfrac{\pi}{4}+29^\circ\right) = \tan 74^\circ$

9. $\sqrt{6}+\sqrt{2}+\sqrt{6}-\sqrt{2} = 2\sqrt{6}$

10. $\dfrac{-\tan 20^\circ + \cot 20^\circ}{1+\tan 20^\circ \cot 20^\circ} = \dfrac{\frac{-1}{p}+p}{2} = \dfrac{p^2-1}{2p}$

11. $\cos\left(52\frac{1}{2}^\circ + 22\frac{1}{2}^\circ\right)\cos\left(52\frac{1}{2}^\circ - 22\frac{1}{2}^\circ\right)$

$= \cos 75^\circ \cos 30^\circ = \left(\dfrac{\sqrt{3}-1}{2\sqrt{2}}\right)\dfrac{\sqrt{3}}{2}$

12. $\sum \dfrac{\sin A\cos B - \cos A \sin B}{\cos A \cos B}$

$= \sum\left(\tan A - \tan B\right) = 0$

13. $70^\circ + 10^\circ = 80^\circ$

$\Rightarrow \tan 80^\circ - \tan 70^\circ - \tan 10^\circ = \tan 10^\circ \tan 70^\circ \tan 80^\circ$

$\Rightarrow \tan 80^\circ - \tan 10^\circ = 2\tan 70^\circ$

14. $\dfrac{\cot\theta - 1}{\cot\theta + 1}, \dfrac{\cot\theta + 1}{\cot\theta - 1} = 1$

15. $13^\circ + 32^\circ = 45^\circ \Rightarrow \left(1+\tan 13^\circ\right)\left(1+\tan 32^\circ\right) = 2$

16. If $A + B = 135^\circ \Rightarrow \left(1+\cot A\right)\left(1+\cot B\right) = 2$

17. $200^\circ + 25^\circ = 225^\circ$

$\Rightarrow (1-\cot 200^\circ)(1-\cot 25^\circ)=2$

18. $\sin 20^\circ+\sin(60^\circ-20^\circ)-\sin(60^\circ+20^\circ)=0$

19. $\cos A+2\cos 120^\circ\cos A=0$

20. $\cos 20^\circ+\cos(120^\circ-20^\circ)+\cos(120^\circ+20^\circ)=0$

21. $\sin 10^\circ+\sin(120^\circ+10^\circ)-\sin(120^\circ-10^\circ)=0$

22. $27^\circ+32^\circ+31^\circ=90^\circ$

$\Rightarrow \sum \tan 27^\circ \tan 32^\circ=1$

23. $\dfrac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\sin(\alpha+\beta)-\sin(\alpha-\beta)}=\dfrac{a+b+a-b}{a+b-a+b}$

$\Rightarrow \dfrac{2\sin\alpha\cos\beta}{2\cos\alpha\sin\beta}=\dfrac{a}{b}\Rightarrow \dfrac{\tan\alpha}{\tan\beta}=\dfrac{a}{b}$

24. $\sin(A+B)=\dfrac{12}{13}\left(\dfrac{-3}{5}\right)+\dfrac{5}{13}\left(\dfrac{-4}{5}\right)=\dfrac{-56}{65}$

25. $\sin A=\sin(A+B-B)$

$=\sin(A+B)\cos B-\cos(A+B)\sin B$

$=\dfrac{12}{13}\dfrac{12}{13}-\dfrac{5}{13}\dfrac{5}{13}=\dfrac{119}{169}$

26. $\cos(A+B)=\dfrac{12}{13}\dfrac{3}{5}-\dfrac{5}{13}\dfrac{4}{5}=\dfrac{16}{65}$

27. $\tan(A-B)=\dfrac{\frac{4}{3}-\tan B}{1+\frac{4}{3}\tan B}$

$\dfrac{7}{24}=\dfrac{4-3\tan B}{3+4\tan B}\Rightarrow \tan B=3/4$

$\therefore A+B=\dfrac{\pi}{2}$

28 $\tan 2\alpha=\tan\big((\alpha+\beta)+(\alpha-\beta)\big)$

29. $x\cos\theta=y\cos\left(\theta+\dfrac{2\pi}{3}\right)=z\cos\left(\theta+\dfrac{4\pi}{3}\right)=k$

$$\frac{k}{x}=\cos\theta.\frac{k}{y}=\cos\left(\theta+\frac{2\pi}{3}\right).\frac{k}{z}=\cos\left(\theta+\frac{4\pi}{3}\right)$$

$$\therefore k\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=\cos\theta+\cos\left(\frac{2\pi}{3}+\theta\right)+\cos\left(\frac{4\pi}{3}+\theta\right)$$

$$=\cos\theta+\cos\left(120^{\circ}+\theta\right)+\cos\left(120^{\circ}-\theta\right)=0$$

30. $\cos A\cos B=2\sin A\sin B$ and $\cos\left(A+B\right)=\frac{3}{5}$

$$\Rightarrow \cos A\cos B-\sin A\sin B=\frac{3}{5}$$

$$\Rightarrow 2\sin A\sin B-\sin A\sin B=3/5$$

31. $\sin x\cos y=1/4$, $\frac{3\sin x}{\cos x}=\frac{4\sin y}{\cos y}\Rightarrow 3\sin x\cos y=4\cos x\sin y$

$$\Rightarrow \frac{3}{16}=\cos x\sin y$$

$$\therefore \sin\left(x-y\right)=\frac{1}{4}-\frac{3}{16}=\frac{1}{16}$$

32. $\sin\alpha+\cos^2\alpha\cos^2\beta+\sin^2\alpha\sin^2\beta$

$-2\sin\alpha\sin\beta\cos\alpha\cos\beta+$
$2\cos\alpha\cos\beta\sin\alpha\sin\beta-2\sin^2\alpha\sin^2\beta$

$$=\sin^2\alpha+\cos^2\alpha\cos^2\beta-\sin^2\alpha\sin^2\beta$$

$$=\sin^2\alpha\cos^2\beta+\cos^2\alpha\cos^2\beta=\cos^2\beta$$

33. $\sin A+\sqrt{3}\cos A=\sqrt{3}\cos B-\sin B$

$$\Rightarrow \frac{1}{2}\sin A+\frac{\sqrt{3}}{2}\cos A=\frac{\sqrt{3}}{2}\cos B-\frac{1}{2}\sin B$$

$$\Rightarrow \sin\left(A+60^{\circ}\right)=\sin\left(60^{\circ}-B\right)$$

$A+60^{\circ}=60^{\circ}-B$, $\therefore \sin 3\left(-B\right)+\sin 3B=0$

34. $\left(m^2-n^2\right)\sin^2 B=\left(\frac{\sin^2 A}{\sin^2 B}-\frac{\cos^2 A}{\cos^2 B}\right)\sin^2 B$

$$= \frac{\sin(A+B)\sin(A-B)}{\cos^2 B}$$

$$= \frac{\cos^2 B - \cos^2 A}{\cos^2 B} = 1 - \frac{\cos^2 A}{\cos^2 B} = 1 - n^2$$

35. $2\tan A + \cot A = \tan B$

$$\therefore \cot A + 2\left(\frac{\tan A - \tan B}{1 + \tan A \tan B}\right)$$

$$= \frac{\cot A + \tan B + 2\tan A - 2\tan B}{1 + \tan A \tan B}$$

$$= \frac{\cot A - \cot A}{1 + \tan A \tan B} = 0$$

36. $\dfrac{\sin A \cos C + \cos A \sin C}{\sin A \sin C} = \dfrac{2}{\tan B}$

$\Rightarrow \cot C + \cot A = 2\cot B$

$\therefore \cot A, \cot B, \cot C$ are in A.P

$\Rightarrow \tan A, \tan B, \tan C$ are in H.P.

37. $\sum(1 - \tan B \tan C)$

$= \sum 1 - \sum \tan B \tan C = 3 - 1 = 2$

39. put x = y = z

40. Take $A = B = C = \pi/3$

41. Take $A = B = C = \pi/3$

42 Use Com. & Div. rule.

43. $\dfrac{\sin A}{\cos A} = \dfrac{x \sin B}{1 - x\cos B}$

$\sin A - x \sin A \cos B = x \cos A \sin B$

$\sin A = x \sin(A + B)$

similary $\sin B = y \sin(A + B)$

LEARNER'S TASK

BEGINNERS (Level - I)

1. $\sin 11^{o}19^{|} \cos 18^{o}41^{|} + \cos 11^{o}19^{|} \sin 18^{o}41^{|} =$

A)1 B)$\frac{\sqrt{3}}{2}$ C)$\frac{1}{2}$ D)0

2. $\cos\left(\frac{\pi}{4}+A\right)\cos\left(\frac{\pi}{4}-B\right)-\sin\left(\frac{\pi}{4}+A\right)\sin\left(\frac{\pi}{4}-B\right)=$

A)cos(A+B) B) cos(A-B) C)sin(A+B) D)sin(B-A)

3. If $0<\theta<\frac{\pi}{2}$ and $2\cos\theta=\sqrt{3}\ \cos 10^{0}-\sin 10^{0}$ $then\ \theta =$

A)20^{0} B) $70^{0|}$ C)40^{0} D)10^{0}

4. If $\tan 22^{0} + \tan 38^{0} - \sqrt{3} = k \tan 22^{0}\ \tan 38^{0}$ then k =

A)-1 B)$-\sqrt{3}$ C)0 D)1

5. $\tan 20^{0}+\tan 25^{0} + \tan 20^{0} \tan 25^{0} =$

A)-1 B)0 C)1 D)2

6. If $\frac{1-\tan 2^{0}\cot 62^{0}}{\tan 152^{0}-\cot 88^{0}} = k\sqrt{3}$ then k =

A) -1 B) $\frac{1}{2}$ C) 1 D) $\frac{-1}{2}$

7. $\tan 75^{0} - \cot 75^{0} =$

A)$3\sqrt{2}$ B)$2\sqrt{3}$ C)1 D)4

8. If $\tan 20^{0} = p$ then $\frac{\tan 160^{0}-\tan 110^{0}}{1+\tan 160^{0}\tan 110^{0}}$

A)$\frac{2p}{1+p^{2}}$ B)$\frac{1-p^{2}}{2p}$ C)$\frac{1+p^{2}}{2p}$ D)$\frac{2p}{1-p^{2}}$

9. $\cos^{2} 48^{o} - \sin^{2} 12^{o} =$

A)$\frac{\sqrt{5}-1}{4}$ B)$\frac{\sqrt{5}+1}{8}$ C)$\frac{\sqrt{3}-1}{4}$ D)$\frac{\sqrt{3}+1}{2\sqrt{2}}$

10. $\cos^2\left(\frac{\pi}{4}+x\right)-\sin^2\left(\frac{\pi}{4}-x\right)=$

A)1 B)0 C)1 D)2

11. $\tan 40^\circ+2\tan 10^\circ=\cot x$ then x =

A) 75° B) 85° C) 30° D) 40°

12. $\tan 35^\circ+2\tan 20^\circ=\tan x$ then x =

A) 15° B) 5° C) 55° D) 50°

13. **17.** $\tan\left(\frac{\pi}{4}+\theta\right)\tan\left(\frac{3\pi}{4}+\theta\right)=$

A) 0 B) -1 C) 3 D) 1

14. $\frac{(1+\tan 33^\circ)(1+\tan 12^\circ)}{(1+\tan 18^\circ)(1+\tan 27^\circ)}=$

A) 1 B) 2 C) 4 D) $\sqrt{3}$

15. If $A+B=315^\circ$ then $(1-\tan A)(1-\tan B)=$

A) 0 B) 1 C) 2 D) $\frac{1}{2}$

16. $(1+\cot 78^\circ)(1+\cot 57^\circ)=$

A) 0 B) 1 C) 2 D) $\frac{1}{2}$

17. $\sin 40^\circ-\sin 80^\circ+\sin 160^\circ=$

A) 0 B) -1 C) 1 D) 2

18. $\cos 40^\circ+\cos 80^\circ+\cos 160^\circ+\cos 240^\circ=$

A) 0 B) 1 C) $\frac{1}{2}$ D) $-\frac{1}{2}$

19. $\cos 35^\circ+\cos 85^\circ+\cos 155^\circ=$

A) -1 B) 0 C) 1 D) 2

20. $\tan 29^\circ\tan 30^\circ+\tan 30^\circ\tan 31^\circ+\tan 31^\circ\tan 29^\circ=$

A) 1 B) 0 C) 3 D) 2

21. In a ΔABC, A is obtuse, $\sin A=\frac{3}{5}$, $\sin B=\frac{5}{13}$ then $\sin C=$

A) $\frac{33}{65}$ B) $\frac{16}{65}$ C) $\frac{4}{5}$ D) $\frac{12}{13}$

22. If A, B are acute angles, $\sin A=\frac{4}{5}$, $\tan B=\frac{5}{12}$ then $\sin(A+B)=$

A) $\frac{36}{65}$ B) $\frac{65}{56}$ C) $\frac{65}{63}$ D) $\frac{63}{65}$

23. If $\tan A=\frac{17}{18}$, $\tan B=\frac{1}{35}$ then $\cos(A+B)=$

A)1 B) $\sqrt{2}$ C) -1 D) $\frac{1}{\sqrt{2}}$

24 If $\cos\alpha=-\frac{12}{13},\cot\beta=\frac{24}{7},90^o<\alpha<180^o$ and $180^o<\beta<270^o$ then the quadrant in which $\alpha+\beta$ lies

A) I B) II C) III D) IV

25. If $\tan\alpha=\frac{1}{7}$ and $\sin\beta=\frac{1}{\sqrt{10}}$ where $0<\alpha,\beta<\frac{\pi}{2}$ then 2β is equal to

A) $\frac{\pi}{4}-\alpha$ B) $\frac{3\pi}{4}-\alpha$ C) $\frac{\pi}{8}-\alpha$ D) $\frac{3\pi}{8}-\frac{\alpha}{2}$

26. $\cos\alpha\sin(\beta-\gamma)+\cos\beta\sin(\gamma-\alpha)+\cos\gamma\sin(\alpha-\beta)=$

A) 0 B) 1 C) -1 D) $4\cos\alpha\cos\beta\cos\gamma$

27. If $\cos(A-B)=3/5$ and $\tan A\tan B=2$ then which one of the following is true

A) $\sin(A+B)=1/5$ B) $\sin(A+B)=-1/5$

C) $\cos(A-B)=1/5$ D) $\cos(A+B)=-1/5$

28. If $\cos\alpha+\cos\beta=0=\sin\alpha+\sin\beta$ then $\cos(\alpha-\beta)=$

A) 0 B) 1 C) -1 D) 2

29. $\cos^2(A-B)+\cos^2 B-2\cos(A-B)\cos A\cos B=$

A) $\sin^2 A$ B) $\sin^2 B$ C) $\cos^2 A$ D) $\cos^2 B$

30. If $\sin x+\sin y=3(\cos y-\cos x)$ then $\frac{\sin 3x}{\sin 3y}=$

A) 0 B) 1 C) -1 D) 2

31. If $\sin B=\frac{1}{5}.\sin(2A+B)$ then $\frac{\tan(A+B)}{\tan A}=$

A) $5/3$ B) $2/3$ C) $3/2$ D) $3/5$

32. $\cot B = 2\tan(A-B) \Rightarrow 2\tan B + \cot B =$

A) $\tan A$ B) $\cot A$ C) $2\tan A$ D) $2\cot A$

33. In a ΔABC, $\sum \frac{\cos(B-C)}{\sin B \sin C} =$

A) 0 B) 2 C) 3 D) 4

34. If $\tan A = 1, \tan B = 2, \tan C = 3$ then A+B+C =

A) $\frac{n\pi}{2}, n \in z$ B) $n\pi, n \in z$ C) $\frac{n\pi}{4}, n \in z$ D) $\frac{2n\pi}{3}, n \in z$

35. In a ΔABC, $\tan A + \tan B + \tan C = 6$ and $\tan A \tan B = 2$ then the triangle ABC is

A) right angled isosceles B) acute angled isosceles

C) acute angled D) equilateral

36. $\cot A + \cot B + \cot C + \frac{\cos(A+B+C)}{\sin A \sin B \sin C} =$

A) $\cot A \cot B \cot C$ B) $\tan A \tan B \tan C$

C) $\cos A \cos B \cos C$ D) $\sin A \sin B \sin C$

37. In a ΔABC, if $\sin A \sin B \sin C = \frac{1}{3}$ then the valueof $\cot A\cot B + \cot B\cot C + \cot C\cot A$ is

A) 1 B) 4/3 C) 4 D) 3

38. $\cos(x-y) + \cos(y-z) + \cos(z-x) = -\frac{3}{2} \Rightarrow \sum(\cos x) =$

A) 0 B) 1 C) 2 D) 3

KEY

1) C	02)D	3)C	4) B	5)C	6) A	7) B	8) B	9) B	10) B
11) D	12) C	13) B	14) A	15) C	16) C	17) A	18) D	19) B	20) A
21) B	22) D	23) D	24) D	25) A	26) A	27) D	28) C	29) A	30) C
31)C	32) A	33) D	34) B	35) C	36) A	37) A	38) A		

HINTS & SOLUTIONS

1. $\sin 11^\circ 19' \cos 18^\circ 41' + \cos 11^\circ 19' \sin 18^\circ 41'$

$= \sin\left(11^\circ 19' + 18^\circ 41'\right)$ $= \sin 30^\circ = \frac{1}{2}$

2. $\cos\left(\frac{\pi}{4} + A + \frac{\pi}{4} - B\right) = \cos\left(\frac{\pi}{2} + A - B\right)$

$= -\sin(A - B) = \sin(B - A)$

3. $\cos\theta = \frac{\sqrt{3}}{2}\cos 10^\circ - \frac{1}{2}\sin 10^\circ$

$= \cos\left(30^\circ + 10^\circ\right) = \cos 40^\circ$

4. $22^\circ + 38^\circ = 60^\circ$

$\Rightarrow \tan 60^\circ - \tan 22^\circ - \tan 38^\circ = \tan 60^\circ \tan 22^\circ \tan 38^\circ$

$\Rightarrow -\sqrt{3}\tan 22^\circ \tan 38^\circ = \tan 22^\circ + \tan 38^\circ - \sqrt{3}$

5. $20^\circ + 25^\circ = 45^\circ$; $\frac{\tan 20^\circ + \tan 25^\circ}{1 - \tan 20^\circ \tan 25^\circ} = 1$

6. $\frac{1 - \cot 88^\circ \cot 62^\circ}{-\cot 62^\circ - \cot 88^\circ} = \frac{\cot 88^\circ \cot 62^\circ - 1}{\cot 88^\circ + \cot 62^\circ}$

$= \cot 150^\circ = -\sqrt{3}$

7. $\left(2 + \sqrt{3}\right) - \left(2 - \sqrt{3}\right) = 2\sqrt{3}$

8. $\frac{-\tan 20^\circ + \cot 20^\circ}{1 + \tan 20^\circ \cot 20^\circ} = \frac{-p + \frac{1}{p}}{2} = \frac{1 - p^2}{2p}$

9. $\cos\left(48^\circ + 12^\circ\right)\cos\left(48^\circ - 12^\circ\right) = \frac{1}{2}\left(\frac{\sqrt{5} + 1}{4}\right)$

10. $\cos\left(\frac{\pi}{4} + x + \frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} + x - \frac{\pi}{4} + x\right) = 0$

11. $40^\circ + 10^\circ = 50^\circ$

$\Rightarrow \tan 50^\circ - \tan 40^\circ - \tan 10^\circ = \tan 10^\circ \tan 40^\circ \tan 50^\circ$

$\Rightarrow \tan 50^\circ - \tan 40^\circ = 2\tan 10^\circ$

$\Rightarrow \cot 40^\circ = 2\tan 10^\circ + \tan 40^\circ$

12. $20^\circ+35^\circ=55^\circ$

$\Rightarrow \tan 55^\circ-\tan 20^\circ-\tan 35^\circ=\tan 20^\circ\tan 35^\circ\tan 55^\circ$

$\Rightarrow \tan 55^\circ=\tan 20^\circ+2\tan 35^\circ$

13. $\tan\left(\frac{\pi}{4}+\theta\right)\tan\left(\pi-\left(\frac{\pi}{4}-\theta\right)\right)$

$=\tan\left(\frac{\pi}{4}+\theta\right)\left(-\tan\left(\frac{\pi}{4}-\theta\right)\right)=-1$

14. $12^\circ+33^\circ=45^\circ\Rightarrow(1+\tan 12^\circ)(1+\tan 33^\circ)=2$

15. $A+B=315^\circ\Rightarrow(1-\tan A)(1-\tan B)=2$

16. $78^\circ+57^\circ=135^\circ$

$(1+\cot 78^\circ)(1+\cot 57^\circ)=2$

17. $\sin 40^\circ-\sin(120^\circ-40^\circ)+\sin(120^\circ+40^\circ)=0$

18. $\cos 40^\circ+\cos(120^\circ-40^\circ)+\cos(120^\circ+40^\circ)+\cos(180^\circ+60^\circ)$

$=0-\cos 60^\circ=-1/2$

19. $\cos 35^\circ+\cos(120^\circ-35^\circ)+\cos(120^\circ+35^\circ)=0$

20. $29^\circ+30^\circ+31^\circ=90^\circ\Rightarrow\sum\tan 29^\circ\tan 30^\circ=1$

21. $A+B+C=180^\circ\Rightarrow A+B=180^\circ-C$

$\Rightarrow\sin C=\left(\frac{3}{5}\right)\left(\frac{12}{13}\right)+\left(\frac{-4}{5}\right)\left(\frac{5}{13}\right)=\frac{16}{65}$

22. $\sin(A+B)=\left(\frac{4}{5}\right)\left(\frac{12}{13}\right)+\left(\frac{3}{5}\right)\left(\frac{5}{13}\right)=\frac{63}{65}$

23. $\cos(A+B)=\frac{18}{\sqrt{613}}\frac{35}{\sqrt{2}\sqrt{613}}-\frac{17}{\sqrt{613}}\frac{1}{\sqrt{2}\sqrt{613}}=\frac{613}{(613)\sqrt{2}}=1/\sqrt{2}$

24. $270^\circ<\alpha+\beta<450^\circ$

$\tan(\alpha+\beta)=\frac{\frac{-5}{12}+\frac{7}{24}}{1+\frac{5}{12}\frac{7}{24}}=\frac{-36}{323}$

25. $\tan 2\beta = \dfrac{2\left(\frac{1}{3}\right)}{1-\frac{1}{9}} = \dfrac{3}{4}, \quad \therefore \tan(2\beta+\alpha) = \dfrac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\frac{1}{7}} = 1$

$2\beta + \alpha = \dfrac{\pi}{4} \Rightarrow 2\beta = \dfrac{\pi}{4} - \alpha$

26. Take $\alpha = \beta = \gamma$

27. $\cos A\cos B + \sin A\sin B = 3/5$ and

$\sin A\sin B = 2\cos A\cos B$

$\Rightarrow \cos A\cos B = 1/5, \sin A\sin B = 2/5$

$\therefore \cos(A+B) = \dfrac{1}{5} - \dfrac{2}{5} = -1/5$

28. $\cos\alpha + \cos\beta = 0$ ..... (1)

$\sin\alpha + \sin\beta = 0$ ..... (2)

(1)² + (2)² $\Rightarrow 2 + 2\cos(\alpha-\beta) = 0$

$\therefore \cos(\alpha-\beta) = -1$

29. Take $A = 60^\circ \; B = 45^\circ$ (or)

$\cos^2 A\cos^2 B + \sin^2 A\sin^2 B + 2\cos A\cos B\sin A\sin B$

$+\cos^2 B - 2\cos^2 A\cos^2 B - 2\cos A\cos B\sin A\sin B$

$= \sin^2 A\sin^2 B + \cos^2 B - \cos^2 A\cos^2 B$

$= \sin^2 A\sin^2 B + \sin^2 A\cos^2 B \;\; = \sin^2 A$

30. $2\sin\dfrac{x+y}{2}\cos\dfrac{x-y}{2} = 3\left(2\sin\left(\dfrac{x+y}{2}\right)\sin\left(\dfrac{x-y}{2}\right)\right)$

$\Rightarrow \sin\left(\dfrac{x+y}{2}\right)\left[\cos\left(\dfrac{x-y}{2}\right) - 3\sin\left(\dfrac{x-y}{2}\right)\right] = 0$

$\Rightarrow \sin\dfrac{x+y}{2} = 0 \Rightarrow x+y = 0 \Rightarrow x = -y$

$\therefore \dfrac{\sin 3x}{\sin 3y} = \dfrac{\sin(-3y)}{\sin 3y} = -1$

31. $\frac{\sin(2A+B)}{\sin B}=\frac{5}{1}$ using C & D

$\Rightarrow \frac{\sin(2A+B)+\sin B}{\sin(2A+B)-\sin B}=\frac{5+1}{5-1}$

$\Rightarrow \frac{\tan(A+B)}{\tan A}=\frac{3}{2}$

32. $\frac{\cot B}{2}=\frac{\tan A-\tan B}{1+\tan A\tan B}$

$\Rightarrow \cot B+\tan A=2\tan A-2\tan B$

$\Rightarrow 2\tan B+\cot B=\tan A$

33. $\sum\frac{\cos B\cos C+\sin B\sin C}{\sin B\sin C}$

$\sum\cot B\cot C+\sum 1 \quad =1+3=4$

34. $\tan(A+B+C)=\frac{1+2+3-1(2)(3)}{1-1.2-2.3-3.1}=0$

$\therefore A+B+C=n\pi / n\in z$

35. Take $\tan A=1,\ \tan B=2,\ \tan C=3$

$A = \pi/4,\ B < \pi/2,\quad C < \pi/2$

$\therefore \Delta ABC$ is acute angled triangle.

36. Take A = B = C = 60º

$\Rightarrow \sqrt{3}+\frac{-8}{3\sqrt{3}}=\frac{1}{3\sqrt{3}}$

verify option (1) $\cot A\cot B\cot C=\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}=\frac{1}{3\sqrt{3}}$

37. $\frac{\sum\cos A\cos B\sin C}{\sin A\sin B\sin C}=\frac{\sin(A+B+C)+\sin A\sin B\sin C}{\sin A\sin B\sin C} \quad =\frac{0+\frac{1}{3}}{\frac{1}{3}}=1$

38. $x-y=y-z=z-x=120^\circ$

$y=x-120^\circ,\ z=x+120^\circ$

$\therefore\sum\cos x=\cos x+\cos(x-120^\circ)+\cos(x+120^\circ)=0$

ACHIEVERS (Level - II)

I. MCQ's with one or more than one correct answer

1. If $\tan\left(\frac{\pi}{4}+\theta\right)+\tan\left(\frac{\pi}{4}-\theta\right)=a;\ then\ \tan^3\left(\frac{\pi}{4}+\theta\right)+\tan^3\left(\frac{\pi}{4}-\theta\right)=....?$

A) $a(-3+a^2)$ B) a C)3a D) a^3-3a

2. If $\cos\alpha+\cos\beta=0=\sin\alpha+\sin\beta\ then\ \cos(\alpha-\beta)=.....?$

A)-1 B) $\log_2^{\frac{1}{2}}$ C) -2 D) $\frac{1}{2}$

3. If cotA . cotB =2; cos (A+B) = 3/5 then sinA . sinB =...?

A) $\frac{9}{15}$ B) $\frac{12}{20}$ C) $\frac{4}{5}$ D) $\frac{3}{5}$

4. If $\frac{COSA}{COSB}=n$ and $\frac{\sin A}{\sin B}=m$ then $(m^2-n^2)\sin^2 B=-----$

A) $1-n^2$ B) $\frac{n^2-n^4}{n^2}$ C) $n(\frac{1}{n}-n)$ D) n^2-1

5. $(1-\cot 200^0)(1-\cot 25^0)$ = ... ?

A)0 B)1 C)2 D) $\log_2^4$

6. $\sin^2\alpha+\cos^2(\alpha+\beta)+2\sin\alpha\sin\beta\cos(\alpha+\beta)=...$

A) $1-\sin^2\beta$ B) $\frac{1}{\sec^2\beta}$ C) $\cos^2\alpha$ D) $\cos^2\beta$

7. If tan (A- B) = $\frac{7}{24}$; $\tan A=\frac{4}{3}$ where A, B acute then A + B=

A) $\frac{\pi}{2}$ B) $\frac{\pi}{5}$ C) $\frac{\pi}{4}$ D) $\frac{2\pi}{4}$

II. Assertion and Reasoning

A) both A and R are true R is correct explantion of A

B) both A & R are true R is not correct explantion of A

C) A is true R is false **D) A is false R is true**

1. A : $\tan 40^0+\tan 80^0-\sqrt{3}$ $\tan 40^0\tan 80^0-\sqrt{3}$

R : $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$

2. Assertion A : If $\tan A = \frac{2015}{2016}; \tan B = \frac{1}{4031}$ tan (A+B) =1

R : If $\tan A = \frac{m}{n}; \tan B = \frac{n-m}{m+n}$ *then* $A + B = \frac{\pi}{4}$

3. A : $\sin^2 52\frac{1}{2}^0 - \sin^2 22\frac{1}{2}^0 = \frac{\sqrt{3}-1}{4\sqrt{2}}$

R : $\sin^2 - \sin^2 B = \sin(A+B) . \sin(A-B)$

4. A : If $0 < \theta < \frac{\pi}{2}, \tan\theta = \frac{\cos 29^0 + \sin 29^0}{\cos 29^0 - \sin 29^0}$, *then* $\theta = 74^0$

R : If $0 < \theta < \frac{\pi}{2}$ *then* $\tan(45 - \theta) = \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$

5. A : $\frac{(1+\tan 13^0)(1+\tan 32^0)}{(1+\tan 12^0)(1+\tan 33^0)} = 1$

R : If A + B = 45^0 (or) 225^0 then (1+tanA) (1+tanB) =2

6. A : $\sin 1^0 + \sin 121^0 - \sin 119^0 = 0$

R : $\sin\theta + \sin(120^0 + \theta) - \sin(120^0 - \theta) = 0$

III. Match the following

	List - I	List - II
1.	1) $\cot\left(\frac{\pi}{4}+\theta\right).\cot\left(\frac{\pi}{4}-\theta\right)$	a) 0
	2) $\sin(45^0+\theta) - \cos(45^0-\theta)$	b) $\tan 56^0$
	3) $\frac{\cos 11^0 + \sin 11^0}{\cos 11^0 - \sin 11^0}$	c) $\frac{\sqrt{3}}{2}$
	4) $\sin^2 75^0 - \sin^2 15^0$	d) 1
	A) 1-d, 2-a, 3-b, 4-c	B) 1-a, 2-b, 3-c, 4-d
	C) 1-c, 2-b, 3-d, 4-c	D) 1-b, 2-c, 3-a, 4-d
2.	1) $\sin 15^0$	a) $\sin 75^0$
	2) $\cos 15^0$	b) $\cot 75^0$
	3) $\tan 15^0$	c) $\tan 75^0$
	4) $\cot 15^0$	d) $\cos 75^0$
	A) 1-d, 2-a, 3-c, 4-b	B) 1-a, 2-d, 3-b, 4-c
	C) 1-d, 2-a, 3-b, 4-c	D) 1-c, 2-a, 3-b, 4-d
3.	1) $\frac{\tan 80^0 - \tan 10^0}{\tan 70^0}$	a) 0
	2) cosec 15^0 + sec15^0	b) 2

3) $\sin 20^0 + \sin 40^0 - \sin 80^0$ c) $2\sqrt{6}$

4) $\sin 10^0 + \sin 130^0 - \sin 110^0$ d) 1

A) 1-b, 2-c, 3-a, 4-a B) 1-c, 2-b, 3-a, 4-d

C) 1-b, 2-c, 3-d, 4-a D) 1-d, 2-b, 3-a, 4-c

IV. comprehension type

1. If A, B, A+B, A-B are not odd multiples of $\frac{\pi}{2}$ then

i) $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A.\tan B}$ ii) $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A.\tan B}$

using the above information find the following

i) $\tan 40^0 + \tan 80^0 - \sqrt{3}\ \tan 40^0 \tan 80^0 =$

A) $\sqrt{3}$ B) $-\sqrt{3}$ C) $\frac{1}{\sqrt{3}}$ D) $-\frac{1}{\sqrt{3}}$

ii) $\frac{\tan 225^0 - \cot 81^0.\cot 69^0}{\cot 261^0 + \tan 21^0} =$

A) 1 B) $\frac{1}{\sqrt{2}}$ C) $\sqrt{3}$ D) $\sqrt{3}$

2. If $\tan A = \frac{m}{n}, \tan B = \frac{n-m}{m+n}$ *then* $A + B = \frac{\pi}{4}$

If $\tan A = \frac{m}{n}, \tan B = \frac{m-n}{m+n}$ *then* $A + B = \frac{\pi}{4}$

using the above information find the following

i) $\tan A = \frac{2015}{2016}, \tan B = \frac{1}{4031}$ then tan(A+B) =

A) 1 B) 0 C) -1 D) 1/2

ii) $\tan A = \frac{112}{111}, \tan B = \frac{1}{223}$ then tan (A- B) =

A)1/2 B)-1 C)1 D)1/3

V. Integer type

1. If $\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$ *then* $\cos(\alpha - \beta)$

A)0 B)1 C)-1 D)2

2. If sinx +siny =3 (cosy- cosx) then $\frac{\sin 3x}{\sin 3y}$

A)1 B)0 C)-1 D)2

3. If $\alpha+\beta+\delta=\frac{\pi}{2}$ and $\cot\alpha,\cot\beta,\cot\delta$ are in A.P then $\cot\alpha.\cot\delta$

A) 1 B) 2 C) 3 D) 4

4. In a ΔABC, $\sum\frac{\cos(B-C)}{\sin B.\sin C}=$

A) 0 B)2 C) 3 D) 4

5. $\cot\left(\frac{\pi}{4}+\theta\right).\cot\left(\frac{\pi}{4}-\theta\right)=$

A)0 B) -1 C) 1 D) 1/2

EXPLORERS (Level - III)

1. If $0<\alpha,\beta<\frac{\pi}{4}$, $\cos(\alpha+\beta)=\frac{4}{5}$, $\sin(\alpha-\beta)=\frac{5}{13}$ then $\tan 2\alpha=$ [AIEEE-2010]

A) $\frac{33}{56}$ B) $\frac{56}{33}$ C) $\frac{16}{63}$ D) $\frac{14}{63}$

2. In a ΔPQR, $\angle R=\frac{\pi}{2}$ if $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of the equation $ax^2+bx+c=0\ (a\neq 0)$ then [AIEEE-2012]

A) $a+b=c$ B) $b+c=a$ C) $a+c=b$ D) $b=c$

3. $\frac{\cos x}{\cos(x-2y)}=\lambda\Rightarrow\tan(x-y)\tan y=$ [EAM-2009]

A) $\frac{1+\lambda}{1-\lambda}$ B) $\frac{1-\lambda}{1+\lambda}$ C) $\frac{\lambda}{1+\lambda}$ D) $\frac{\lambda}{1-\lambda}$

4. If A, B are acute angles. $\tan A=\frac{5}{12},\cos B=\frac{3}{5}$ then $\cos(A+B)=$

A) $\frac{16}{65}$ B) $\frac{65}{16}$ C) $\frac{65}{63}$ D) $\frac{56}{65}$

5. $\cot B=2\tan(A-B)\Rightarrow 2\tan B+\cot B=$

A) $\tan A$ B) $\cot A$ C) $2\tan A$ D) $2\cot A$

6. $\tan 100^\circ+\tan 125^\circ+\tan 100^\circ\tan 125^\circ=$

A) -1 B) 1 C) $\frac{1}{\sqrt{3}}$ D) $\frac{1}{2}$

7. $\operatorname{cosec}15^\circ + \sec 15^\circ =$

A) $2\sqrt{2}$ B) $\sqrt{6}$ C) $2\sqrt{6}$ D) $\sqrt{6}+\sqrt{2}$

8. $\cos^2 52\frac{1}{2}^\circ - \sin^2 22\frac{1}{2}^\circ =$

A) $\frac{\sqrt{3}+1}{4\sqrt{2}}$ B) $\frac{\sqrt{3}-1}{4\sqrt{2}}$ C) $\frac{3+\sqrt{3}}{4\sqrt{2}}$ D) $\frac{3-\sqrt{3}}{4\sqrt{2}}$

9. $\frac{\tan 80^\circ - \tan 10^\circ}{\tan 70^\circ} =$

A) 0 B) 1 C) 2 D) 3

10. **In a** ΔABC, $\sum \frac{\cos(B-C)}{\sin B \sin C} =$

A) 0 B) 2 C) 3 D) 4

KEY

ΦΦ LEARNER'S TASK :

☐ **ACHIEVERS :**

I) 1-AD, 2-AB, 3- ABD, 4- ABC, 5- CD, 6- ABD, 7- AD

II) 1- A, 2- A, 3- D, 4- C, 5-A, 6- A

III) 1-A, 2-C,3-A

IV) 1) i -B ii) C 2) i-A, ii -C

☐ **EXPLORERS :**

1-C, 2- C, 3-C, 4- D, 5-C, 6-B, 7-C, 8-D, 9-C, 10-D