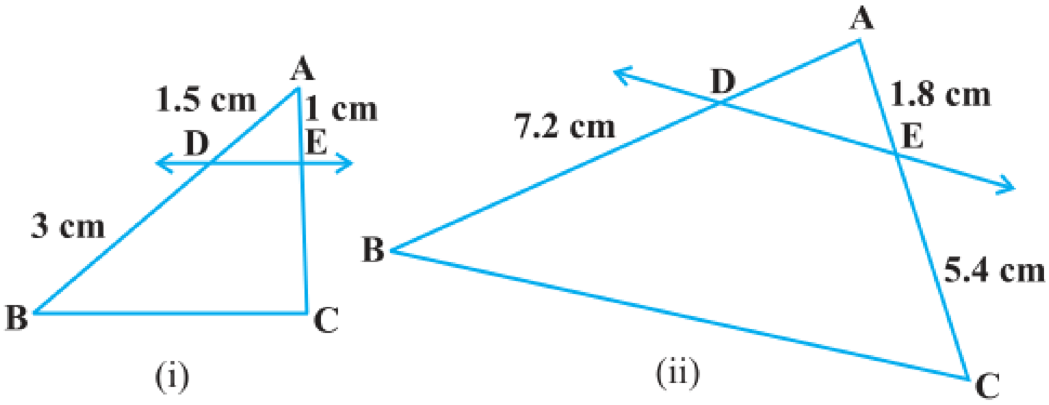
CHAPTER – 6: Triangles

Exercise 6.2

1. In figure. (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).



Solution:

(i) Given, in △ABC, DE∥BC

∴ AD/DB = AE/EC [From Theorem 6.1, Basic proportionality theorem or Thales Theorem]

⇒1.5/3 = 1/EC

⇒EC = 3/1.5

EC = 3×10/15 = 2 cm

Therefore, EC = 2 cm.

(ii) Given, in △ ABC, DE∥BC

∴ AD/DB = AE/EC [From Basic proportionality theorem]

⇒ AD/7.2 = 1.8 / 5.4

⇒ AD = 1.8 ×7.2/5.4 = (18/10) × (72/10) × (10/54) = 24/10

⇒ AD = 2.4

Therefore, AD = 2.4 cm.

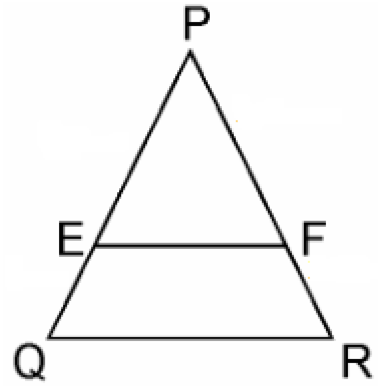
2. E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether EF || QR.
(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm
(iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.63 cm

Solution:

Given, in ΔPQR, E and F are two points on side PQ and PR respectively.

Look at the figure below:



(i) Given, PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2,4 cm

Therefore, by using Basic proportionality theorem, we get,

PE/EQ = 3.9/3 = 1.3

And PF/FR = 3.6/2.4 = 3/2 = 1.5

So, we get, PE/EQ ≠ PF/FR

Therefore, EF is not parallel to QR.

(ii) Given, PE = 4 cm, QE = 4.5 cm, PF = 8cm and RF = 9cm

Therefore, by using Basic proportionality theorem, we get,

PE/QE = 4/4.5 = 8/9

And, PF/RF = 8/9

So, we get here,

PE/QE = PF/RF

Therefore, EF is parallel to QR.

(iii) Given, PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm

From the figure,

EQ = PQ – PE = 1.28 – 0.18 = 1.10 cm

And, FR = PR – PF = 2.56 – 0.36 = 2.20 cm

So, PE/EQ = 0.18/1.10 = 18/110 = 9/55………… **(i)**

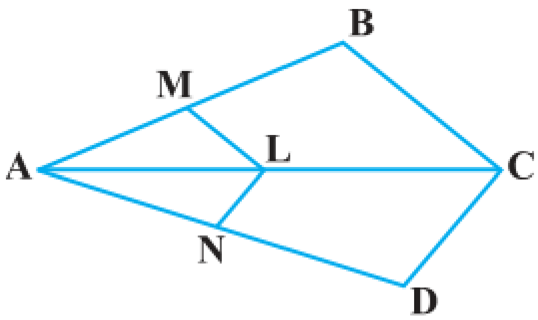
And, PE/FR = 0.36/2.20 = 36/220 = 9/55………… **(ii)**

So, we get here,

PE/EQ = PF/FR

Therefore, EF is parallel to QR.

3. In the figure, if LM || CB and LN || CD, prove that AM/MB = AN/AD



Solution:

In the given figure, we can see, LM || CB,

By using basic proportionality theorem, we get,

AM/MB = AL/LC…………………….**(i)**

Similarly, given, LN || CD and using basic proportionality theorem,

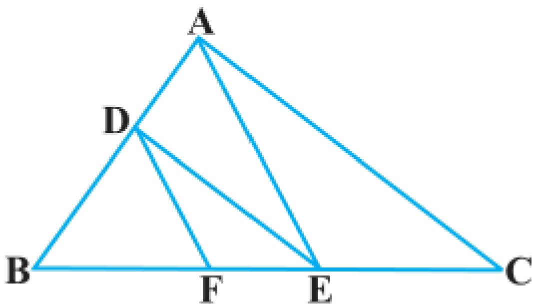
∴AN/AD = AL/LC…………………………**(ii)**

From equation **(i)** and **(ii)**, we get,

AM/MB = AN/AD

Therefore, proved.

4. In the figure, DE||AC and DF||AE. Prove that BF/FE = BE/EC



Solution:

In ΔABC, given as, DE || AC

Thus, by using Basic Proportionality Theorem, we get,

∴BD/DA = BE/EC ………………………………………………**(i)**

In ΔABC, given as, DF || AE

Thus, by using Basic Proportionality Theorem, we get,

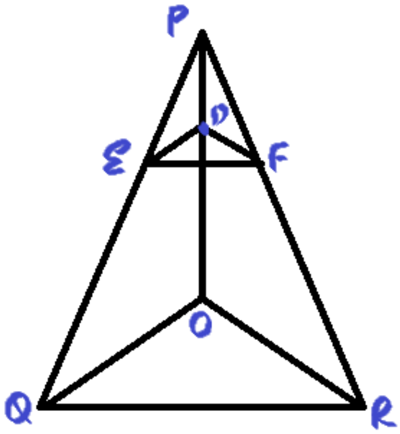
∴BD/DA = BF/FE ………………………………………………**(ii)**

From equation **(i)** and **(ii)**, we get

BE/EC = BF/FE

Therefore, proved.

5. In the figure, DE||OQ and DF||OR, show that EF||QR.



Solution:

Given,

In ΔPQO, DE || OQ

By using Basic Proportionality Theorem,

PD/DO = PE/EQ……………… **..(i)**

Again given, in ΔPQO, DE || OQ ,

So by using Basic Proportionality Theorem,

PD/DO = PF/FR………………… **(ii)**

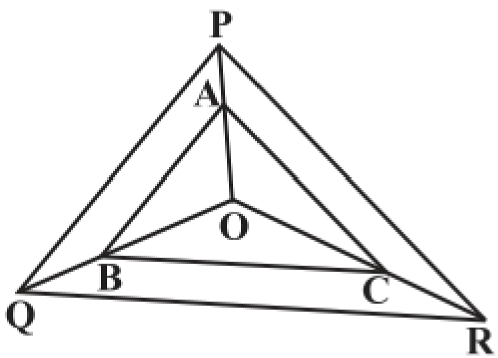
From equation **(i)** and **(ii)**, we get,

PE/EQ = PF/FR

Therefore, by converse of Basic Proportionality Theorem,

EF || QR, in ΔPQR.

6. In the figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.



Solution:

Given here,

In ΔOPQ, AB || PQ

By using Basic Proportionality Theorem…..Theorem 6.1,

Since AB parallel to PQ, It divides the other two sides in the same ratio.

OA/AP = OB/BQ……………**(i)**

Also given,

In ΔOPR, AC || PR

By using Basic Proportionality Theorem

∴ OA/AP = OC/CR……………(ii)

From equation **(i)** and **(ii)**, we get,

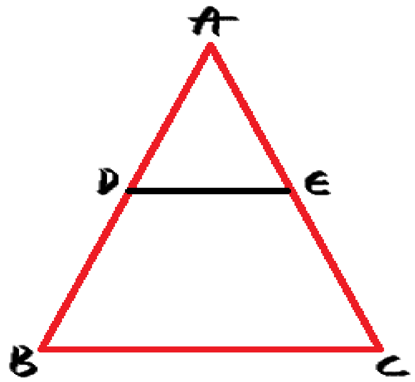
OB/BQ = OC/CR

Therefore, by converse of Basic Proportionality Theorem or Theorem 6.2,

Because…BC divided OQ and OR in the same ratio, we can say that…

In ΔOQR, BC || QR.

7. Using Basic proportionality theorem, prove that a line drawn through the mid-points of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).



Solution:

Given, in ΔABC, D is the midpoint of AB such that AD=DB.

A line parallel to BC intersects AC at E as shown in above figure such that DE || BC.

We have to prove that E is the mid-point of AC.

Since, D is the mid-point of AB.

∴ AD=DB

⇒AD/DB = 1 ………………………… **(i)**

In ΔABC, DE || BC,

By using Basic Proportionality Theorem,

Therefore, AD/DB = AE/EC

From equation (i), we can write,

⇒ 1 = AE/EC

∴ AE = EC

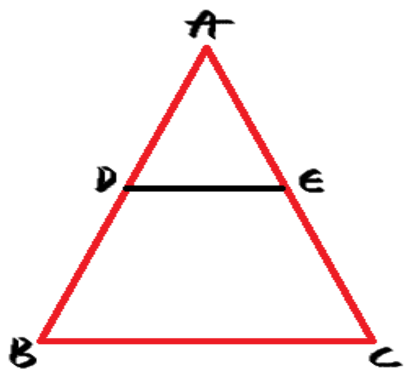
Hence, proved, E is the midpoint of AC.

8. Using Converse of basic proportionality theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Solution:

Given, in ΔABC, D and E are the mid points of AB and AC respectively, such that,

AD=BD and AE=EC.



We have to prove that: DE || BC.

Since, D is the midpoint of AB

∴ AD=DB

⇒AD/BD = 1…………………………….. **(i)**

Also given, E is the mid-point of AC.

∴ AE=EC

⇒ AE/EC = 1

From equation **(i)** and **(ii)**, we get,

AD/BD = AE/EC

By converse of Basic Proportionality Theorem,

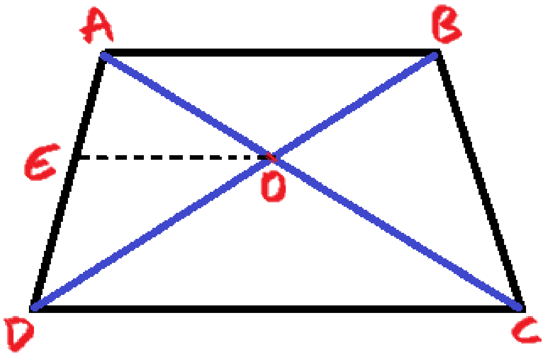
DE || BC

Hence, proved.

9. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that AO/BO = CO/DO.

Solution:

Given, ABCD is a trapezium where AB || DC and diagonals AC and BD intersect each other at O.



We have to prove, AO/BO = CO/DO

From the point O, draw a line EO touching AD at E, in such a way that,

EO || DC || AB

In ΔADC, we have OE || DC

Therefore, By using Basic Proportionality Theorem

AE/ED = AO/CO ……………..**(i)**

Now, In ΔABD, OE || AB

Therefore, By using Basic Proportionality Theorem

DE/EA = DO/BO…………….**(ii)**

From equation **(i)** and **(ii)**, we get,

AO/CO = BO/DO

⇒AO/BO = CO/DO

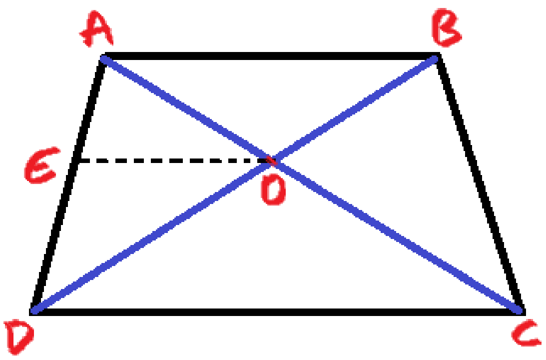
Hence, proved.

10. The diagonals of a quadrilateral ABCD intersect each other at the point O such that AO/BO = CO/DO. Show that ABCD is a trapezium.

Solution:

Given, Quadrilateral ABCD where AC and BD intersects each other at O such that,

AO/BO = CO/DO.



We have to prove here, ABCD is a trapezium

From the point O, draw a line EO touching AD at E, in such a way that,

EO || DC || AB

In ΔDAB, EO || AB

Therefore, By using Basic Proportionality Theorem

DE/EA = DO/OB ……………………(i)

Also, given,

AO/BO = CO/DO

⇒ AO/CO = BO/DO

⇒ CO/AO = DO/BO

⇒DO/OB = CO/AO **…………………………..(ii)**

From equation **(i)** and **(ii)**, we get

DE/EA = CO/AO

Therefore, By using converse of Basic Proportionality Theorem,

EO || DC also EO || AB

⇒ AB || DC.

Hence, quadrilateral ABCD is a trapezium with AB || CD.