

VARIATIONS OF GRAPHS

Class: IX, Mathematics

①

IIT ADVANCED.

SOLUTIONS

TEACHING TASK

01. $\sin x \cdot \cos\left(\frac{\pi}{4} - x\right)$
 $= \sin x \left(\frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x \right)$
 $= \frac{1}{\sqrt{2}} (\sin x \cos x + \sin^2 x)$
 $= \frac{1}{\sqrt{2}} \left(\frac{\sin 2x}{2} + \frac{1 - \cos 2x}{2} \right)$
 $= \frac{1}{2\sqrt{2}} (\sin 2x - \cos 2x + 1)$

period of $\sin 2x = \frac{2\pi}{2} = \pi$

period of $\cos 2x = \frac{2\pi}{2} = \pi$

hence period of given function = π

Ans. A

02. $y = \sin^2 \theta + \cos^4 \theta$
 $y = \sin^2 \theta + (1 - \sin^2 \theta)^2$
 $y = \sin^4 \theta - \sin^2 \theta + 1$
 $y = \sin^2 \theta (\sin^2 \theta - 1) + 1$
 $y = -\sin^2 \theta \cos^2 \theta + 1$
 $y = -\frac{1}{4} (2 \sin \theta \cos \theta)^2 + 1$

$$y = -\frac{1}{4} \sin^2 2\theta + 1$$

$$0 \leq \sin^2 2\theta \leq 1$$

$$0 \geq -\frac{1}{4} \sin^2 2\theta \geq -\frac{1}{4}$$

$$1 \geq y \geq -\frac{1}{4} + 1$$

$$\frac{3}{4} \leq y \leq 1$$

Ans. D.

03 $f(\theta) = \frac{\tan \theta - \frac{1}{3} \tan^3 \theta}{\frac{1}{3} - \tan^2 \theta}$

$$= \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$$

$$= \tan 3\theta$$

$\therefore \text{period} = \frac{\pi}{3}$

Ans: A

04 period of $\cos nx \Rightarrow \frac{2\pi}{n}$

period of $\sin\left(\frac{x}{n}\right) \Rightarrow \frac{2\pi}{\left(\frac{1}{n}\right)} = 2\pi n$

L.C.M $\left\{ \frac{2\pi}{n}, 2\pi n \right\} = \frac{2n\pi}{1} = 2n\pi$

Given $2n\pi = 4\pi$

$\Rightarrow n = 2$

Ans:

Ans: C

05 The least value of $\frac{1}{5 \cos x + 12 \sin x + 15}$

max. value of $5 \cos x + 12 \sin x + 15$

$= 15 + \sqrt{25 + 144}$

$= 15 + 13$

$= 28$

Ans: B

$\therefore \text{Ans: } \frac{1}{28}$

06

$$\cos^2 \theta - 6 \sin \theta \cos \theta + 3 \sin^2 \theta + 2$$

$$= (\cos^2 \theta + \sin^2 \theta) - 3 \cdot (2 \sin \theta \cos \theta) + 2 \sin^2 \theta + 2$$

$$= 3 - 3 \sin 2\theta + 2 \sin^2 \theta$$

$$= 4 - 3 \sin 2\theta + 2 \sin^2 \theta - 1$$

$$= 4 - 3 \sin 2\theta + \cos 2\theta$$

$$\text{minimum value} = c - \sqrt{a^2 + b^2}$$

$$= 4 - \sqrt{9 + 1}$$

$$= 4 - \sqrt{10}$$

Ans: B

67.

$$f(x) = \cos^2 x + \sec^2 x$$

$$= (\cos x - \sec x)^2 + 2 \cdot \cos x \cdot \sec x$$

$$= (\cos x - \sec x)^2 + 2$$

$$\text{We know } (\cos x - \sec x)^2 \geq 0$$

$$\therefore f(x) = 0 + 2 = 2 \rightarrow \text{minimum value}$$

$$\text{hence Range} = [2, \infty)$$

Ans: C

08

$$\text{opt: A } f(x) = \sin 3\pi x$$

$$\text{period} = \frac{2\pi}{3\pi} = \frac{2}{3}$$

Ans: A

09.

$$\text{Given } \sin^2 x + \cos^2 x = 1 \quad \forall x \in \mathbb{R}$$

$$\sin^2 \frac{3\pi}{2} + \cos^2 \frac{3\pi}{2} = 1 \quad \forall x \in \mathbb{R}$$

$$\text{hence } \frac{\text{minimum value is } 1}{\text{maximum value is } 1} = 1$$

Ans: A

10. A) $f(x) = \sin x + |\sin x|$

$$f(x) = \sin(2\pi + x) + |\sin(2\pi + x)|$$

$$f(x) = \sin x + |\sin x|$$

hence $f(x)$ is periodic period with period 2π

b) $g(x) = \sin x + \cos x$

$$g(x) = \sin(2\pi + x) + \cos(2\pi + x) \\ = \sin x + \cos x$$

hence $g(x)$ is periodic with period 2π

c) $h(x) = \tan x$ is periodic

d) $p(x) = \sin^4 x + \cos^4 x$

$$p(\pi + x) = \sin^4(\pi + x) + \cos^4(\pi + x) \\ = \sin^4 x + \cos^4 x$$

hence, it is periodic with period π

11

$$\begin{aligned} u^2 &= a^2 + b^2 + 2 \sqrt{(a^2 \cos^2 \theta + b^2 \sin^2 \theta)(a^2 \sin^2 \theta + b^2 \cos^2 \theta)} \\ &= a^2 + b^2 + 2 \sqrt{a^4 \cos^2 \theta \sin^2 \theta + b^4 \sin^2 \theta \cos^2 \theta + a^2 b^2 (\sin^4 \theta + \cos^4 \theta)} \\ &= a^2 + b^2 + 2 \sqrt{(a^4 + b^4) \sin^2 \theta \cos^2 \theta + a^2 b^2 (1 - 2 \sin^2 \theta \cos^2 \theta)} \\ &= a^2 + b^2 + 2 \sqrt{\sin^2 \theta \cos^2 \theta (a^2 - b^2) + a^2 b^2} \\ &= a^2 + b^2 + 2 \sqrt{a^2 b^2 + (a^2 - b^2) \cdot \sin^2 2\theta / 4} \end{aligned}$$

$$u_{\max}^2 = a^2 + b^2 + 2 \sqrt{a^2 b^2 (a^2 - b^2) / 4} = 2(a^2 + b^2)$$

$$u_{\min}^2 = a^2 + b^2 + 2 \sqrt{a^2 b^2} = (a+b)^2$$

$$\text{Difference} = 2(a^2 + b^2) - (a+b)^2 = (a-b)^2 \quad \text{Ans. B}$$

13. Statement I:

$$\text{period of } \sin 8x = \frac{2\pi}{8} = \frac{\pi}{4}$$

$$\text{period of } \cos 2x = \frac{2\pi}{2} = \pi$$

$$\text{period} \equiv \text{L.C.M.} \left\{ \frac{\pi}{4}, \pi \right\} = \pi. \quad (\text{True})$$

Ans: A

Statement II: Conceptual (True)

14 $3 \cos x + 4 \sin x + 5$

$$\text{Max. value} = 5 + \sqrt{3^2 + 4^2} = 5 + 5 = 10 \quad \text{Ans: C}$$

15 $5 \sin^2 \frac{\theta}{2} + 4 \cos^2 \frac{\theta}{2}$

$$= 5 \sin^2 \frac{\theta}{2} + 4 (1 - \sin^2 \frac{\theta}{2})$$

$$= 5 \sin^2 \frac{\theta}{2} + 4 - 4 \sin^2 \frac{\theta}{2}$$

$$= \sin^2 \frac{\theta}{2} + 4.$$

$$0 \leq \sin^2 \frac{\theta}{2} \leq 1$$

$$4 \leq 4 + \sin^2 \frac{\theta}{2} \leq 5$$

$$\text{minimum value} = 4.$$

Ans: D

16 $5|\sec x| - 7|\csc x|$

$$a \neq b \Rightarrow \text{hence period} = \pi$$

Ans: A

17 $29|\tan x| + \sqrt{841}|\cot x|$

$$29 \neq \sqrt{841}$$

$$\text{hence period} \rightarrow \pi$$

Ans: A

18 $\sqrt{13} (|\sin x| + |\cos x|)$

here $\sqrt{13} = \sqrt{13}$ i.e. $a=b$

period = $\frac{\pi}{2}$

Ans: D

19 $\sin^2 \theta + 5 \sin \theta \cos \theta + 5 \cos^2 \theta$

$= (\sin^2 \theta + \cos^2 \theta) + \frac{5}{2} (2 \sin \theta \cos \theta) + 4 \cos^2 \theta$

$= 1 + \frac{5}{2} (\sin 2\theta) + 2 (2 \cos^2 \theta - 1) + 2$

$= 3 + \frac{5}{2} (\sin 2\theta) + 2 \cos 2\theta$

max. value = $3 + \sqrt{\left(\frac{5}{2}\right)^2 + 2^2}$

$= 3 - \sqrt{\frac{41}{4}} = 3 - \frac{\sqrt{41}}{2} = \frac{6 - \sqrt{41}}{2}$

$\therefore \text{max value of } f(x) = \frac{1}{\left(\frac{6 - \sqrt{41}}{2}\right)} = \frac{2}{6 - \sqrt{41}}$

20

20

(07)

$$a) \frac{7 + 6 \tan \theta - \tan^2 \theta}{1 + \tan^2 \theta}$$

$$= \frac{7 + 6 \tan \theta - \tan^2 \theta}{\sec^2 \theta}$$

$$= 7 \cos^2 \theta + 6 \sin \theta \cos \theta - \sin^2 \theta$$

$$= 6 \cos^2 \theta + \cos 2\theta + 3 \sin 2\theta$$

$$= 3(2\cos^2 \theta - 1) + \cos 2\theta + 3\sin 2\theta + 3$$

$$= 3 \cos 2\theta + \cos 2\theta + 3\sin 2\theta + 3$$

$$= 3 + 3\sin 2\theta + 4\cos 2\theta$$

$$\max = 3 + \sqrt{3^2 + 4^2} = 3 + 5 = 8 \rightarrow \lambda$$

$$\min = 3 - \sqrt{3^2 + 4^2} = 3 - 5 = -2 \rightarrow \mu$$

$$\therefore \lambda + \mu = 8 - 2 = 6$$

$$b) 5 \cos \theta + 3 \cos \left(\theta + \frac{\pi}{3} \right) + 3$$

$$= 5 \cos \theta + 3 \left(\cos \theta \cdot \frac{1}{2} - \sin \theta \cdot \frac{\sqrt{3}}{2} \right) + 3$$

$$= 5 \cos \theta + \frac{3}{2} \cos \theta - \frac{3\sqrt{3}}{2} \sin \theta + 3$$

$$= 3 - \frac{3\sqrt{3}}{2} \sin \theta + \frac{13}{2} \cos \theta$$

$$\max = 3 + \sqrt{\left(\frac{3\sqrt{3}}{2} \right)^2 + \left(\frac{13}{2} \right)^2}$$

$$= 3 + 7 = 10 \rightarrow \lambda$$

$$\min = 3 - 7 = -4 \rightarrow \mu$$

$$\lambda + \mu = 10 - 4 = 6$$

⑥

$$\begin{aligned} c) & 1 + \sin\left(\frac{\pi}{4} + \theta\right) + 2 \cos\left(\frac{\pi}{4} - \theta\right) \\ &= 1 + \left(\frac{1}{\sqrt{2}} \cos \theta + \frac{1}{\sqrt{2}} \sin \theta\right) + 2 \left(\frac{1}{\sqrt{2}} \cos \theta - \frac{1}{\sqrt{2}} \sin \theta\right) \\ &= 1 + \left(\frac{1}{\sqrt{2}} + \sqrt{2}\right) \cos \theta + \left(\frac{1}{\sqrt{2}} - \sqrt{2}\right) \sin \theta \end{aligned}$$

$$\text{Max. value} = 1 + \sqrt{\left(\frac{1}{\sqrt{2}} + \sqrt{2}\right)^2 + \left(\frac{1}{\sqrt{2}} - \sqrt{2}\right)^2}$$

$$= 1 + \sqrt{2 \cdot \left[\left(\frac{1}{\sqrt{2}}\right)^2 + (\sqrt{2})^2\right]}$$

$$= 1 + \sqrt{5} \rightarrow N$$

$\therefore N =$

$$\text{min v. alue} = 1 - \sqrt{5} \rightarrow M$$

$$N + M = 2$$

$$\begin{aligned} d) & \sin^2(120^\circ + \theta) + \sin^2(120^\circ - \theta) \\ &= \left[\sin(120^\circ + \theta) + \sin(120^\circ - \theta)\right]^2 - 2 \sin(120^\circ + \theta) \cdot \sin(120^\circ - \theta) \\ &= \left[2 \sin 120^\circ \cdot \cos \theta\right]^2 - 2 \left[\sin^2 120^\circ - \sin^2 \theta\right] \\ &= \left[2 \cdot \frac{\sqrt{3}}{2} \cdot \cos \theta\right]^2 - 2 \left[\frac{3}{4} - \sin^2 \theta\right] \\ &= 3 \cos^2 \theta - \frac{3}{2} + 2 \sin^2 \theta \\ &= 2 + \cos^2 \theta - \frac{3}{2} \\ &= \frac{1}{2} + \cos^2 \theta \end{aligned}$$

$$\text{max} \rightarrow \frac{3}{2} \rightarrow N$$

$$\text{min} \rightarrow \frac{1}{2} \rightarrow M$$

$$N + M = \frac{3}{2} + \frac{1}{2} = 2$$

$$\therefore 0 \leq \cos^2 \theta \leq 1$$

$$\frac{1}{2} \leq \frac{1}{2} + \cos^2 \theta \leq 1 + \frac{1}{2}$$

$$\frac{1}{2} \leq \frac{1}{2} + \cos^2 \theta \leq \frac{3}{2}$$

Ans: $\gamma, \gamma, \rho, \rho$

21 a) $2\sin^2 x + \cos x$
 $= \sin^2 x + 1$
 $0 \leq \sin^2 x \leq 1$
 $1 \leq 1 + \sin^2 x \leq 2$

21 a) $2\sin^2 x + \cos x$
 $f(\pi+x) = 2\sin^2(\pi+x) + \cos(\pi+x)$
 $= 2\sin^2 x + \cos x \rightarrow \text{period } \pi$

b) $f(0) = \sin(\pi \sin 0)$
 $f(2\pi+0) = \sin(\pi \sin(2\pi+0))$
 $= \sin(\pi \sin 0) \rightarrow \text{period } 2\pi$

c) $\sin x \cdot \sin(120^\circ+x) \cdot \sin(120^\circ-x)$
 $= \sin x \cdot (\sin^2 120^\circ - \sin^2 x)$
 $= \sin x \left(\frac{3}{4} - \sin^2 x \right)$
 $= \frac{3\sin x - 4\sin^3 x}{4} = \frac{1}{4} \sin 3x$
 $\text{period} = \frac{2\pi}{3}$

d) $f(x) = |\sin x| \cdot |\cos x|$
 $f\left(\frac{\pi}{2}+x\right) = \left|\sin\left(\frac{\pi}{2}+x\right)\right| \cdot \left|\cos\left(\frac{\pi}{2}+x\right)\right|$
 $= |\cos x| |\sin x| = f(x)$
 $\text{period} = \frac{\pi}{2}$

Ans: SP, 1, 2

LEARNERS TASK

(10)

CUQ'S

01. period of $\sin(ax+b) = \frac{2\pi}{|a|}$

Ans: C

02. $\sin 2x \rightarrow \text{period} = \frac{2\pi}{2} = \pi$

Ans: C

03. $\tan kx, \cot kx \rightarrow \text{period} = \frac{\pi}{|k|}$

Ans: C

04. $-\cot x \rightarrow \text{period} = \pi$

Ans: D

05. Amplitude $\rightarrow \frac{M-m}{2}$

Ans: B

07. minimum value $= n - \sqrt{l^2 + m^2}$

Ans: B

08. $(a \sin x)^2 + (b \cos x)^2$

$$= (a \sin x - b \cos x)^2 + 2(a \sin x)(b \cos x)$$

$$= (a \sin x - b \cos x)^2 + 2ab \geq 2ab$$

minimum value $= 2ab$

Ans: A

09. period of $k \cdot f(x) \rightarrow \text{period of } f(x)$

Ans: A

10. $\tan 3x + \tan x \rightarrow \text{period } \tan 3x \rightarrow \frac{\pi}{3}$

~~$= 3 \tan x + 4 \tan^3 x$~~

period $\tan x \rightarrow \pi$

period L.C.M $\left\{ \frac{\pi}{3}, \pi \right\} = \frac{\pi}{1} = \pi$

Ans: D.

JEE MAINS LEVEL

(11)

01. $\sin \frac{\pi}{2} x + \cos \frac{\pi}{3} x$

period $\sin \frac{\pi}{2} x \rightarrow \frac{2\pi}{(\frac{\pi}{2})} = 4$

period $\cos \frac{\pi}{3} x \rightarrow \frac{2\pi}{(\frac{\pi}{3})} = 6$

period L.C.M $\{4, 6\} = 12$

Ans: C

02. $\frac{1}{2}(2 \sin x \cos x) = \frac{1}{2} \sin 2x$

max. of $\sin 2x \rightarrow 1$

max. of $\frac{1}{2} \sin 2x \rightarrow \frac{1}{2}$

Ans: C

03. $\cos \theta + \cos 2\theta$

$= \cos \theta + 2\cos^2 \theta - 1$

$= 2\cos^2 \theta + \cos \theta - 1$

$= 2 \left[\cos^2 \theta + \frac{1}{2} \cos \theta - \frac{1}{2} \right] - 1$

$= 2 \left(\cos^2 \theta + 2 \cdot \cos \theta \cdot \frac{1}{4} + \frac{1}{16} - \frac{1}{16} - \frac{1}{2} \right) - 1$

$= 2 \left(\left(\cos \theta + \frac{1}{4} \right)^2 - \frac{9}{8} \right)$

minimum value $= -\frac{9}{8}$

Ans: A

04. period of $\sin 2x = \frac{2\pi}{2} \rightarrow \pi$

period of $\cos 4x = \frac{\pi}{4}$

period L.C.M. of $(\pi, \frac{\pi}{4}) = \frac{\pi}{1} = \pi$

Ans: B

5. $\tan(x+2x+3x+\dots+nx)$

(12)

$$= \tan(1+2+3+\dots+n)x$$

$$= \tan\left[\frac{n(n+1)}{2}\right]x$$

$$\text{period} \rightarrow \frac{\pi}{\left(\frac{n(n+1)}{2}\right)} = \frac{2\pi}{n(n+1)}$$

Ans: B

06

$$f(x) = \frac{1}{5\sin x - 6}$$

$$-1 \leq \sin x \leq 1$$

$$-5 \leq 5\sin x \leq 5$$

$$-5-6 \leq 5\sin x - 6 \leq 5-6$$

$$-11 \leq 5\sin x - 6 \leq -1$$

$$-\frac{1}{11} \geq \frac{1}{5\sin x - 6} \geq -1$$

$$\therefore -1 \leq \frac{1}{5\sin x - 6} \leq -\frac{1}{11}$$

$$\text{Hence Range} = \left[-1, -\frac{1}{11}\right]$$

Ans: B

07

$$\tan(x+2x+3x+\dots+nx)$$

$$= \tan(1+2+3+\dots+n)x$$

$$= \tan\left[\frac{n(n+1)}{2}\right]x$$

$$\text{period} = \frac{\pi}{\left(\frac{n(n+1)}{2}\right)} = \frac{2\pi}{n(n+1)}$$

$$= \frac{\pi}{\sum n}$$

Ans: B, C

08 Statement I:

(13)

$$2 \cos^2 \theta + \sqrt{5} \sin \theta \cos \theta + 4 \sin^2 \theta$$

$$a \cos^2 \theta + b \sin \theta \cos \theta + c \sin^2 \theta$$

$$\therefore \text{maximum value} = \frac{1}{2}(a+c) + \frac{1}{2}\sqrt{(a-c)^2 + b^2}$$

$$= \frac{1}{2}(2+4) + \frac{1}{2}\sqrt{(2-4)^2 + 5}$$

$$= 3 + \frac{3}{2} = \frac{9}{2}$$

$$= \frac{9}{2} \text{ (True)}$$

Statement II: The maximum value of

$$a \cos^2 \theta + b \sin \theta \cos \theta + c \sin^2 \theta$$

$$\frac{1}{2}(a+c) + \frac{1}{2}\sqrt{(a-c)^2 + b^2} \text{ (False)}$$

09.

$$p \cos \frac{\pi}{4} + q \sin \frac{\pi}{4}$$

$$\text{maximum value} = c + \sqrt{a^2 + b^2}$$

$$= 0 + \sqrt{p^2 + q^2}$$

$$= (p^2 + q^2)^{1/2}$$

Ans: D

10

$$f(x) = 3 \sin^2 x + 5 \sin x \cos x + 4 \cos^2 x$$

$$f(x) = a \sin^2 x + b \sin x \cos x + c \cos^2 x$$

$$\text{max. value} = \frac{1}{2}(a+c) + \frac{1}{2}\sqrt{(a-c)^2 + b^2}$$

$$= \frac{1}{2}(3+4) + \frac{1}{2}\sqrt{(3-4)^2 + 5^2}$$

$$= \frac{7}{2} + \frac{\sqrt{26}}{2}$$

$$= \frac{7 + \sqrt{26}}{2}$$

Ans: D

$$11. f = 7 + 10 \sin 2x + 8 \sin^2 x$$

$$f = 7 + 20 \sin x \cos x + 8 \sin^2 x$$

$$11. f = 7 + 10 \sin 2x + 8 \sin^2 x \quad (14)$$

$$f = 7 + 10 \sin 2x - 4(1 - 2 \sin^2 x) + 4$$

$$f = 11 + 10 \sin 2x - 4 \cos 2x$$

$$\text{minimum value} = 11 - \sqrt{100 + 16}$$

$$= 11 - \sqrt{116}$$

$$= 11 - 2\sqrt{29}$$

Ans: B

$$12. f(x) = 11 + 5\sqrt{1 - \cos 2x} + 10 \cos x$$

$$f(x) = 11 + 5\sqrt{\sin^2 x} + 10 \cos x$$

$$f(x) = 11 + 5|\sin x| + 10 \cos x$$

$$= 11 + 5 + |\sin 2x| + 5(2 \cos x - 1) + 5$$

$$= 16 + 5|\sin 2x| + 5 \cos 2x$$

$$\text{minimum value} = 16 - \sqrt{5^2 + 5^2}$$

$$= 16 - \sqrt{50}$$

$$= 16 - 5\sqrt{2}$$

Ans: A

$$13. f(x) = 1 - 8 \sin^2 x \cos^2 x$$

$$= 1 - 2(2 \sin x \cos x)^2$$

$$= 1 - 2$$

$$f(x) = 1 - 8 \sin^2 x \cdot \cos^2 x \quad (15)$$

$$= 1 - 2(2 \sin^2 x \cos^2 x)^2$$

$$= 1 - 2 \sin^4 x$$

$$0 \leq \sin^2 x \leq 1$$

$$\geq -2 \sin^2 x \geq -2$$

$$1 \geq 1 - 2 \sin^2 x \geq 1 - 2$$

$$-1 \leq 1 - 2 \sin^2 x \leq 0$$

$$\text{maxi value} = 0 \quad \text{Ans: } 0$$

$$f(x) = \sin^2 x$$

$$\text{We know } 0 \leq \sin^2 x \leq 1$$

$$\text{minimum value} = 0$$

$$\text{Ans: } 0$$

$$a) 2 \sin^2 x + \cos^2 x$$

$$\Rightarrow 2 \sin^2 x + 1 - \sin^2 x$$



15 a) $2\sin^2 x + \cos^2 x$ (16)

$$= 2\sin^2 x + 1 - \sin^2 x$$

$$= \sin^2 x + 1$$

$$0 \leq \sin^2 x \leq 1$$

$$1 \leq 1 + \sin^2 x \leq 2 \rightarrow \text{Range } [1, 2]$$

b) $1 + 8\sin^2 x \cdot \cos^2 x$

$$= 1 + 2(2\sin x \cos x)^2$$

$$= 1 + 2\sin^2 2x$$

$$0 \leq \sin^2 2x \leq 1$$

$$0 \leq 2\sin^2 2x \leq 2$$

$$1 \leq 1 + 2\sin^2 2x \leq 3 \rightarrow \text{Range } [1, 3]$$

c) $\cos^2 x + \sin^4 x$

$$= 1 - \sin^2 x + \sin^4 x$$

$$= 1 - \sin^2 x (1 - \sin^2 x)$$

$$= 1 - \sin^2 x \cos^2 x$$

$$= 1 - \frac{1}{4}(2\sin x \cos x)^2$$

$$= 1 - \frac{1}{4}\sin^2 2x$$

$$0 \leq \sin^2 2x \leq 1$$

$$0 \geq -\frac{1}{4}\sin^2 2x \geq -\frac{1}{4}$$

$$1 \geq 1 - \frac{1}{4}\sin^2 2x \geq 1 - \frac{1}{4}$$

$$\text{Range} = \left[\frac{3}{4}, 1\right]$$

d) $3\sin^2 x + 4\cos^2 x$

$$= 3\sin^2 x + 4 - 4\sin^2 x$$

$$= -\sin^2 x + 4$$

$$0 \leq \sin^2 x \leq 1$$

$$0 \geq -\sin^2 x \geq -1$$

$$4 \geq 4 - \sin^2 x \geq 3$$

$$\text{Range} = [3, 4]$$

THE END